CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – T Th 9:00am
Lab – F 10:30am

Lecture Meeting Location: SP 309
Lab Meeting Location: SP 309

Business

• HW5 out—extended deadline: Nov. 16 / Nov. 17 (extended from Nov. 11 / Nov. 12, on the HW sheet)
  – HW5 Programming exercises build upon lab4
  – I will post lab4 solutions tonight or tomorrow morning
  – Until then, feel free to use your own lab4 solutions (and asmt-helper file) for HW5…
  – … but before submitting your HW5, please make sure your code works with the posted solution files
Business, pt. 2

- Exam back today

- Reading: Makinson, Ch.4.1-4.6
  - Our coverage of the material will be different from that in the textbook, but it’s good to see the textbook’s presentation, as well
- Also Reading: Makinson, Ch. 5
  - Chapter 5.3 is especially useful for lab tomorrow!

- Lab tomorrow: Please bring your textbook to lab tomorrow!

Odd Numbers

- It is important to be able to prove low-level properties (such as \( x = 1 \times x \)) from low-level principles (such as the Peano axioms and the recursive definition of multiplication)
- … but eventually, we treat those low-level properties as already proved, and we use them to prove higher-level properties

- One way of defining odd numbers:
  - The first (i.e., 0’th) odd number is 1
  - If \( q \) is an odd number, the next odd number after \( q \) is \( S(S(q)) = q + 2 \)
- Theorem: The \( n \)’th odd number is \( 2n + 1 \)
- Proof: By induction!
  - What proposition should we use for \( P(n) \)?
  - Base case: \( P(0) \). To prove…
  - Inductive case: Assume \( P(k) \), prove \( P(Sk) \). To prove…

For this proof, assume we know common commutativity, associativity, and distributivity properties
Sometimes One Just Isn’t Enough…

• In standard induction:
  – Inductive hypothesis is of the form $P(k)$
  – Then, assuming the inductive hypothesis, show $P(k+1)$

• Sometimes the inductive hypothesis isn’t strong enough to prove what we need

• Example:
  – Claim: Every natural number $n \geq 2$ is the product of one or more prime numbers
  – Proof: By induction!
    – What’s the base case?
    – What’s the inductive hypothesis? The inductive case?

Sometimes One Just Isn’t Enough…

• Sometimes the standard inductive hypothesis ($P(k)$) isn’t strong enough to prove what we need

• Example:
  – Claim: Every natural number $n \geq 2$ is either prime or the product of prime numbers.
  – Proof: By induction! Show
    – What’s the base case? $n = 2$. Proof: 2 is prime.
    – We could try to continue inductively by assuming arbitrarily chosen $k$ is either prime or the product of primes, then proving $(k+1)$ is…
    – … but what’s the difficulty with that?
Sometimes One Just Isn’t Enough…

- Sometimes the standard inductive hypothesis ($P(k)$) isn’t strong enough to prove what we need
- Example:
  - Claim: Every natural number $n \geq 2$ is either prime or the product of prime numbers.
  - Proof: By induction! Show
  - What’s the base case? $n = 2$. Proof: 2 is prime.
  - We could try to continue inductively by assuming arbitrarily chosen $k$ is either prime or the product of primes, then proving $(k+1)$ is…
  - … but what’s the difficulty with that?

Sometimes One Just Isn’t Enough: The Strong Induction Story

- *Strong induction* is an alternative to standard induction
  - If the base case is some number $b$ (i.e., Base case: show $P(b)$ is true)
  - Then in strong induction, the I.H. is Assume $P(n)$ for all $b \leq n < k$
  - And the inductive case is Show $P(k)$
  - Do you see how this follows from the same principles as standard induction? Think of dominoes falling….
- Example:
  - Claim: Every natural number $n \geq 2$ is either prime or the product of prime numbers.
  - Proof: By strong induction! Let $P(n)$ be “$n$ is either prime or the product of primes”.
  - What’s the base case? $n = 2$; prove $P(2)$. Proof: 2 is prime.
  - Inductive hypothesis: Assume $P(n)$ for all $2 \leq n < k$
  - Inductive case: Show $P(k)$. What’s the proof?