CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – M W 1:30pm
Lab – F 1:30pm

Lecture Meeting Location: SP 105
Lab Meeting Location: SP 309

Business

• HW1 out, due Wednesday February 10
• Reading: Please finish Ch. 1 from your textbook
  – Your book talks about sets and diagrams in Ch.1.2.3; it’s interesting, but I won’t cover it in class
• Lab0 due by the end of the day, Thursday, February 11
  – Must be submitted online (using the submit145 command)
  – Must be checked off by me or a Coach
• Coaching Hours
  – 2 Coach Staffing: Sun. 4-6pm and 7-9pm; Mon. 8-9:30pm; Tu. 4:30-6pm and 7-10pm; Wed. 9-10pm; Thu. 5-6pm and 8-9pm
  – 1 or 2 Coach Staffing: Mon. 4:30-6pm, 7-8pm, and 9:30-10pm; Wed. 8-9pm; Thu. 7-8pm
  – Might change slightly; if so, will be announced in lecture and revised hours will be posted on course website
Before We Hit Empty:
A Logical Progression

• Basic ideas from propositional logic are showing up in “logic boxes” in your reading

• Before we go further into sets, an overview of these basics of boolean expressions and propositional reasoning

Propositions

• Defn: proposition – a statement that has the property of truth or falsity

• Propositions are the key elements to represent, analyze, or explain declarative knowledge

<table>
<thead>
<tr>
<th>Propositions:</th>
<th>Non-Propositions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Washington, D.C. is the capital of the USA.</td>
<td>• What time is it?</td>
</tr>
<tr>
<td>• Poughkeepsie is the capital of New York.</td>
<td>• Pass the salt.</td>
</tr>
<tr>
<td>• $1 + 1 = 2$</td>
<td>• $x + 1 = 2$</td>
</tr>
<tr>
<td>• $2 + 2 = 3$</td>
<td>presuming values for $x$, $y$, $z$ are not given / known</td>
</tr>
</tbody>
</table>

The first and third of these are true; the second and fourth are false.
Propositional operators

- Recall: proposition – a statement that has the property of truth or falsity
  - Often, we use propositional letters (or variables) to represent propositions: e.g., \( p \) stands for “Poughkeepsie is the capital of NY”
- There are several operators (sometimes called boolean operators) that can construct new propositions from old ones
  - Negation (“not”): if \( P \) is a proposition, \( \neg P \) is a proposition
  - Conjunction (“and”): \( P \land Q \)
  - Disjunction (“or”): \( P \lor Q \)
  - Implication (“if – then”): \( P \rightarrow Q \)
  - Equivalence (“is equal / equivalent to”): \( P \iff Q \)
    * Equivalence can also be written as “if and only if”

Propositional operator: Negation

- Whatever the value of \( p \), True or False, the value of proposition not \( p \) (written \( \neg p \)) is the opposite
  - If \( p \) is “Today is Monday,” \( \neg p \) is “It is not the case that today is Monday,” or more simply “Today is not Monday.”
- Negation can be expressed with a truth table

<table>
<thead>
<tr>
<th>( p )</th>
<th>( \neg p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
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Propositional operator: Conjunction

- Conjunction—the “and” operator
  - Whatever the values of propositions \( p, q \), conjunction \( p \text{ and } q \) (written \( p \wedge q \) or \( p \& \& q \)) is also a proposition
  - If \( p \) is “Today is Monday” and \( q \) is “It is snowing today,” then \( p \wedge q \) is “Today is Monday and it is snowing today.”
    - \( p \wedge q \) is true on snowy Mondays and false on any day that is not Monday, and on any day that is Monday but not snowing
  - Conjunction values as a truth table

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</table>

Propositional operator: Disjunction

- Disjunction—the “or” operator
  - Whatever the values of propositions \( p, q \), disjunction \( p \text{ or } q \) (written \( p \vee q \) or \( p \| q \)) is also a proposition
  - If \( p \) is “Today is Monday” and \( q \) is “It is snowing today,” then \( p \vee q \) is “Today is Monday or it is snowing today.”
    - \( p \vee q \) is true on any day that is a Monday or on which it is snowing — including snowy Mondays (it is not exclusive) — and false only on days that are not Mondays on which it is not snowing
  - Disjunction values as a truth table

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<th>( p \vee q )</th>
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The non-exclusive sense of “or” can be a bit subtle

Exercise: What would the exclusive-or operator’s truth table look like? It turns out there is such an operator, and it’s commonly used in logic! The English word “or” is a complicated thing to understand!
Propositional operator: Implication

- Implication—the “if...then” operator (also called conditional)
  - Whatever the values of propositions $p$, $q$, implication $p \rightarrow q$ (written $p \rightarrow q$) is also a proposition
  - If $p$ is “Today is Monday” and $q$ is “It is snowing today,” then $p \rightarrow q$ is “If today is Monday then it is snowing today.”
  - Vocabulary: in $p \rightarrow q$, $p$ is called the hypothesis (or antecedent) and $q$ is called the conclusion (or consequent)
- Implication values as a truth table

<table>
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Really? These are the truth values for implication?
They look like the values for $(\neg p \lor q)$! (Exercise: Check for yourselves!!)

Sounds if-y: Material Implication

- Meaning for implication symbol $\rightarrow$ in propositional logic is referred to as material implication
  - It says that $p \rightarrow q$ is False exactly when $p$ is True and $q$ is False
  - Not the same as every meaning of “if...then” in English, but it’s what’s used in logic
- Examples of material implication and natural language usage:
  - Politician says: “If I am elected, then I will fix the environment”
    - False if the speaker is elected and doesn’t fix the environment
    - True if, e.g., the speaker doesn’t get elected
  - "If today is Friday, then $2 + 2 = 4$"
    - True no matter what day it is
    - True except on Fridays, even though $2 + 2 = 5$ is false!
Properties of operators

• Logical operators have an *order of operations* just like mathematical operators
  – From high to low: negation; conjunction; disjunction; implication
    • Conjunction is kinda like multiplication; disjunction is kinda like addition
    • Math: \(-k \cdot (x + y)\)
    • Logic: \(\neg p \land (q \lor r)\)
  • Also similarly, disjunction and conjunction are *commutative* and *associative*
    – Associative: e.g., \(p \land q \land r\) is \((p \land q) \land r\)
    – Commutative: e.g., \(p \land q\) is \(q \land p\)
      • similar with disjunction
  • Implication is *right-associative*
    – \(p \implies q \implies r\) is \(p \implies (q \implies r)\)

The *biconditional (or equivalence)* operator

• The *biconditional (or equivalence)* operator:
  – If \(p\) and \(q\) are propositions, then \(p \iff q\) is a proposition, read as “\(p\) if and only if \(q\)”
  – \(p \iff q\) is true exactly when \(p\) and \(q\) have the same truth values
  – What does the truth table for \(\iff\) look like?
  – How could we define the biconditional in terms of operators we already know (not, and, or, if... then)?

The equivalence operator can also be written as \(\equiv\) or \(==\) in other contexts.
Exercise: Evaluating boolean expressions

• Defn: Propositions are boolean-valued expressions—i.e., their values are either True or False
• Boolean expressions are evaluated like any other mathematical expressions

Examples: Let $p = \text{True}$, $q = \text{False}$, $r = \text{True}$. What do the following expressions evaluate to?

1. $p \land \neg r$
2. $q \lor \text{False}$
3. $p \to q$
4. $q \leftrightarrow p$
5. $q \leftrightarrow \neg \text{True}$
6. $r \lor (p \land q)$
7. $(p \lor r) \to ((p \lor q) \land r)$
8. $\text{True} \to r$