CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – M W 1:30pm
Lab – F 1:30pm

Lecture Meeting Location: SP 105
Lab Meeting Location: SP 309

Business

• HW4 out, extended due dates April 10 & 11
  – Programming due April 10, non-programming and printouts the 11th

• HW5-Lookahead out today
  – HW5 due April 19 & 20 (see HW sheet)

• Reading: Makinson, Ch.4.1-4.6
  – Our coverage of the material will be different from that in the textbook, but it’s good to see the textbook’s presentation, as well

• Reading: Prof. Hunsberger’s document “The Natural Numbers, Induction, and Numeric Recursion”
  – Posted on the Additional Notes / Readings page of the CS145 website
Multiplication

• Multiplication on the naturals can be defined recursively, similarly to addition on the naturals
• Definition of addition:
  1. Case $z=0$ -- For all $n$ in $\mathbb{N}$, $n + 0 = n$
  2. Case $z=S(m)$ -- For all $n$ in $\mathbb{N}$, $z = S(m)$ for some $m$ in $\mathbb{N}$: $n + S(m) = S(n + m)$
• Definition of multiplication:
  3. Base case: $x * 0 = 0$
  4. Inductive case: $x * S(y) = (x*y) + x$
• Some notes about the phrasing of that definition
  – Abbreviations and condensed forms are used, but it really means what we would expect, based on the definition of addition
  – $Sy$ is a shorthand for $S(y)$
  – Variables are implicitly universally quantified over the naturals. E.g., the base case is “For all $x$ in $\mathbb{N}$, $x * 0 = 0$”. (cf. definition of addition)

Theorem: $0 * x = 0$. Prove (for all $x$ in $\mathbb{N}$) by induction
– Let $P(n)$ be the proposition $0 * n = 0$
– Base case: $P(0)$. To prove…
– Inductive case: Assume $P(k)$, prove $P(Sk)$. To prove…
Multiplication

- Multiplication on the naturals can be defined recursively, similarly to addition on the naturals
- Definition of addition:
  1. Case $z=0$ -- For all $n$ in $N$, $n + 0 = n$
  2. Case $z=S(m)$ -- For all $n$ in $N$, $z = S(m)$ for some $m$ in $N$: $n + S(m) = S(n + m)$
- Definition of multiplication:
  3. Base case: $x * 0 = 0$
  4. Inductive case: $x * S(y) = (x * y) + x$
- Theorem: Multiplication is commutative: $x * y = y * x$
  - What proposition should we use for $P(n)$?
  - Base case: $P(0)$. To prove…
  - Inductive case: Assume $P(k)$, prove $P(Sk)$. To prove…

This proof might be subtle in places… if there are some small details that need proof, be sure to prove them!

Multiplication (Theorem: Jove)

- Multiplication on the naturals can be defined recursively, similarly to addition on the naturals
- Definition of addition:
  1. Case $z=0$ -- For all $n$ in $N$, $n + 0 = n$
  2. Case $z=S(m)$ -- For all $n$ in $N$, $z = S(m)$ for some $m$ in $N$: $n + S(m) = S(n + m)$
- Definition of multiplication:
  3. Base case: $x * 0 = 0$
  4. Inductive case: $x * S(y) = (x * y) + x$
- Theorem: Jove—$x = 1 * x$ (We use 1 as a common shorthand for $S0$)
  - What proposition should we use for $P(n)$?
  - Base case: $P(0)$. To prove…
  - Inductive case: Assume $P(k)$, prove $P(Sk)$. To prove…
Exponentiation

• Exponentiation on the naturals can be defined recursively, similarly to multiplication on the naturals
• Definition of multiplication:
  3. Base case: \( x \times 0 = 0 \)
  4. Inductive case: \( x \times Sy = (x\times y) + x \)
• Definition of exponentiation:
  5. \( x^0 = 1 \)
  6. \( x^{Sk} = (x^k) \times x \)
• Theorem: \( 0^n = 0 \)
  – What proposition should we use for \( P(n) \)?
  – Base case: \( P(0) \). To prove…
  – Inductive case: Assume \( P(k) \), prove \( P(Sk) \). To prove…

Exponentiation

• Exponentiation on the naturals can be defined recursively, similarly to multiplication on the naturals
• Definition of multiplication:
  3. Base case: \( x \times 0 = 0 \)
  4. Inductive case: \( x \times Sy = (x\times y) + x \)
• Definition of exponentiation:
  5. \( x^0 = 1 \)
  6. \( x^{Sk} = (x^k) \times x \)
• Theorem: \( 0^n = 0 \)
  – Be careful! This “Theorem” is actually false!