CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – M W 1:30pm
Lab – F 1:30pm

Lecture Meeting Location: SP 105
Lab Meeting Location: SP 309

Business

• HW5 out, due April 19 / April 20 (see HW sheet)
• HW6 out April 20, due May 1 / May 2

• (Please bring exams to class until we can finish going over them!)
• Reading: Makinson, Ch. 5
• Document on structural induction available soon from course website (follow the Additional Notes link)
Business, pt. 2

• Lab5, lab6, and course scheduling:
  – Lab5 is posted already; can be worked on anytime
  – Lab6 will be held as usual this Friday, April 22 (Coaches will run
    lab in my absence)
  – Both labs due by the end of the day Thursday, April 28

• Class session on Monday, April 25 will be held in the Asprey Lab,
  SP307 (where Coaching Hours are held)
  – (Coaches will run this class session)
  – That class session will be exclusively for work on labs (completing
    them, getting them checked off, etc.)
  – Attendance is not mandatory on April 25 (though it is at all other
    lab sessions)

Counting

• Counting things is an important part of Computer Science
  problem solving
  – The number of elements in a set, or subsets of a set
  – The number of routes through a map that visit every city
  – The number of words of length $L$ formed from alphabet $A$

  – Can be important for: brute-force solutions (i.e., exhaustively
    enumerating every possibility under some circumstances),
    probability, etc.
Count On It!

- More principles for counting elements of sets
- Subtraction principle for (finite set) difference
  - $|A - B| = ...$ ?
- Addition principle for finite sets
  - $|A \cup B| = ...$ ?

Count On It!

- More principles for counting elements of sets
- Subtraction principle for (finite set) difference
  - $|A - B| = |A| - |A \cap B|
- Addition principle for finite sets
  - $|A \cup B| = |A| + |B| - |A \cap B|
  - If $A, B$ are disjoint, then $|A \cup B| = |A| + |B|$ (do you see why?)

Example application: (calling it that sounds better than calling it “Practice Counting”)
- A logic class has 20 students who also take calculus, 9 who also take philosophy, 11 who take neither, and 2 who take both calculus and philosophy. How many students are in the class?
**Count On It!**

- Subtraction principle for (finite set) difference
  - $|A - B| = |A| - |A \cap B|

- Addition principle for finite sets
  - $|A \cup B| = |A| + |B| - |A \cap B|
  - If $A$, $B$ are disjoint, then $|A \cup B| = |A| + |B|$ (do you see why?)

- Multiplication principle for finite sets: $|A \times B| = |A| \times |B|
  - This generalizes to a set product of any finite number of sets

- Example application:
  - How many distinct license plates are there consisting of two letters followed by four digits?
  - (This is easier than counting the number of plates consisting of two letters and four digits, without the restriction that letters come first)

**Selection: Order and Repetition**

- Sometimes it’s useful to consider the number of ways to select (or choose) $k$ items out of $n$ items

- Two factors to consider:
  - Order: does the order matter when considering the elements? (E.g., is selecting $a$, $b$ in that order different from selecting $b$, $a$ in that order?)
  - Is repetition permitted? (Can the same element be selected multiple times)

- How many ways can we choose 2 letters from $\{a, b, c, d, e\}$?
  - If order matters and repetition is permitted?
  - If order matters and repetition is not permitted?
  - If order doesn’t matter and repetition is permitted?
  - If order doesn’t matter and repetition is not permitted?
Selection: Permutations

• The operations for selecting \( k \) out of \( n \) items (for given \( k \leq n \)) are particularly important when repetition is not permitted.
  – Corresponds to working with sets, where elements are not repeated.
• Those operations are often called *combination* and *permutation*.
  – Difference: the order in which they’re selected matters in permutations, not in combinations.
• Permutations:
  – Number of ways to select \( k \) out of \( n \) items, where order matters = \( \ldots \)?
  
  It may not be necessary to memorize this formula; it could be derived when needed.
Selection: Combinations

• The operations for selecting k out of n items (for given $k \leq n$) are particularly important when repetition is not permitted
  – Corresponds to working with sets, where elements are not repeated
• Those operations are often called *combination* and *permutation*
  – Difference: the order in which they’re selected matters in permutations, not in combinations
• Combinations:
  – Number of ways to select k out of n items, where order doesn’t matter = ...

How is this formula derived?

It may not be necessary to memorize this formula; it could be derived when needed.

$C(n, k) = \frac{n!}{k! (n-k)!}$
Counting 1

• Using the counting ideas from combinations and permutations…

• Exercises
  – Your investment advisor gives you a list of 8 stocks that seem like good investments. You decide to invest in 3 of them. How many different selections are possible?
  – Same scenario, except you decide to invest $1,000 in one stock, $2,000 in another, and $4,000 in a third. How many different selections are possible?

Counting 2—Repetition

• It can be useful to think about situations in terms of possibly repeated elements being counted, as well as possible orderings of elements

• Using the counting ideas from combinations and permutations…

• Exercises
  – A restaurant has five flavors of ice cream, and you can order one, two, or three scoops. How many different ice cream orders could you place?