CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – M W 1:30pm
Lab – F 1:30pm

Lecture Meeting Location: SP 105
Lab Meeting Location: SP 309

Business

• A reminder about Academic Integrity

• HW6, Lab7 due already
• HW7-Lookahead: Out now
  – HW7 will be due May 8 / May 9 (see HW sheet)
  – (We may not have covered the probability material for some exercises by the end of class today, but it’s coming soon!)
• Reading: Makinson, Ch. 6
  – We may not cover all the material, but it’s worth reading anyway
• Grading update
• (Please bring exams to class until we can finish going over them!)

• Office Hours by appt. only today, tomorrow (due to illness) and may be canceled
  – Please email me with any questions or concerns!
Counting Full Houses

• What’s the number of 5-card hands can be dealt from a standard 52-card deck (standard 4 suits, 13 cards each; no jokers)?
  – \((52\text{-choose-5}) = \frac{52!}{5! 47!} = 2598960\)

• Number of 5-card hands with a full house?
  – One way to think of it: choose rank r1 for the triple, then choose 3 cards for it; then choose rank r2 for the pair, then choose 2 cards for it
    • So: \(13\text{-choose-3} \times 12\text{-choose-2} = 13 \times 4 \times 12 \times \frac{(4 \times 3)}{2} = 3744\)

• Probability of a 5-card hand being a full house?
  – \((\text{number of full houses}) / (\text{number of 5-card hands}) = 3744 / 2598960\)

What are the odds?

Some other probability-related exercises:

• Imagine that there are 10 envelopes in a mailbox, exactly 2 of which contain bills. The owner of the mailbox picks 3 envelopes to open. What is the probability that none of them is a bill?
What are the odds?

Some other probability-related exercises:

• Imagine that there are 10 envelopes in a mailbox, exactly 2 of which contain bills. The owner of the mailbox picks 3 envelopes to open. What is the probability that none of them is a bill?

• There are 7 index cards in a bag, each identical to the others except for the number printed on it: 4 cards have even numbers, 3 cards have odd numbers. (No two numbers are the same.) Someone then picks the cards out of the bag in random order, one at a time. What is the probability that all of the odd numbers are picked before any of the even numbers?

Probability

• To talk about probability more rigorously, some terminology / definitions:
  – We will discuss discrete probability, i.e., probability over finite domains (rather than continuous, infinite domains)
  – Probabilities are considered w.r.t. a sample space, i.e., a finite (non-empty) set S
  – Probabilities of elementary events (i.e., single elements of S) are described by functions called distributions

See Makinson, Ch.6
Probability Distributions

• Given a sample space \( S \), a **probability distribution** on \( S \) is a function \( p:S \rightarrow [0,1] \) such that \( \sum \{ p(s) | s \in S \} = 1 \)
  - \([0,1]\) is the **real interval**, the set of all real numbers from 0 to 1, inclusive
  - The summation \( \sum \{ p(s) | s \in S \} = 1 \) constrains function \( p \) so that the sum of the values it assigns add up to exactly 1
    • (Can think of it as the probability of *something* occurring is 1)

• Exercise: Let \( S = \{a,b,c,d,e\} \). In each case below, is function \( p \) a probability distribution?
  1. \( p(a) = 0.1, p(b) = 0.2, p(c) = 0.3, p(d) = 0.4, p(e) = 0.5 \)
  2. \( p(a) = 0.1, p(b) = 0.2, p(c) = 0.3, p(d) = 0.4, p(e) = 0 \)
  3. \( p(a) = 0, p(b) = 0, p(c) = 0, p(d) = 0, p(e) = 1 \)
  4. \( p(a) = -1, p(b) = 0, p(c) = 0, p(d) = 1, p(e) = 1 \)
  5. \( p(a) = 0.2, p(b) = 0.2, p(c) = 0.2, p(d) = 0.2, p(e) = 0.2 \)

Probability Function

• A probability distribution assigns a probability (in \([0, 1]\)) to each single element of \( S \) (i.e., each **elementary event**)

• A **probability function** is an extension of a probability distribution, so it applies to all events—i.e., all subsets of \( S \)—not just all elements of \( S \)
  - Probability function \( p^*:P(S) \rightarrow [0,1] \) assigns values to elements of the power set of \( S \)
  - \( p^*(A) = \sum \{ p(a) | a \in A \} \) for non-empty subsets \( A \) of \( S \)
  - \( p^*(\emptyset) = 0 \)

• Terminology
  - The pair of a sample space \( S \) and a probability function \( p^* \) over \( S \), \( (S, p^*) \), is called a **probability space**
Probability Function

• For simplicity’s sake, we might call a probability function $p$ instead of $p^+$, when it is not harmfully ambiguous to do so.

• Exercises: Let $p: \mathcal{P}(S) \rightarrow [0,1]$ be a probability function over $S$. Then, show that $p$ has the following properties:
  
  - $p(S) = 1$
  - $p(A \cup B) = p(A) + p(B) - p(A \cap B)$ (A, B are arbitrary subsets of S)
  - when $A$, $B$ are disjoint, $p(A \cup B) = p(A) + p(B)$
  - $p(A \cap B) = p(A) + p(B) - p(A \cup B)$
  - $p(S - A) = 1 - p(A)$

Probability Spaces

• Terminology
  
  - The pair of a sample space $S$ and a probability function $p^*$ over $S$, $(S, p^*)$, is called a probability space.

  We abbreviate $p^*$ as $p$, when it’s not harmfully ambiguous to do so.

• For each of the following examples, what is the relevant probability space? And what are the relevant probabilities?
  
  - Throw a fair 6-sided die. What is the probability that the number is divisible by 3?
  - Throw a loaded die, with probability distribution
    
    $p(1) = 0.1, p(2) = 0.2, p(3) = 0.1, p(4) = 0.2, p(5) = 0.1, p(6) = 0.3$

    Which is more probable, that the die lands with an even number, or that it lands with a number greater than 3?