CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – M W 10:30am
Lab – F 3:10pm

Lecture Meeting Location: SP 105
Lab Meeting Location: SP 309

Business

• HW1 out, due Wednesday February 10
• Coaching hours TBD very soon
  – Temp. Coaching Hours for this week: Wed. 8-10pm; Thu. 7-9pm

• Reading: Please finish Ch. 1 from your textbook
  – Your book talks about sets and diagrams in Ch.1.2.3; it’s interesting, but I won’t cover it in class

• Probably lab this Friday
Subset (cont.)

• More terminology / notation:
  – Recall, notation $A \subseteq B$ indicates that $A$ is a subset of $B$ (and possibly equal to $B$—the horizontal line looks like part of an $=$ sign)
  – Notation $A \subsetneq B$ explicitly indicates that $A$ is a subset of $B$ and not equal to $B$
  – In the case that $A \subsetneq B$, we say $A$ is a proper subset of $B$

• Exercises:
  – The notation $A \not\subset B$ indicates … ? How about $A \not\subseteq B$?

• In your textbook, notation $A \subset B$ also indicates that $A$ is a proper subset of $B$, but …
• ... in other sources, sometimes $A \subset B$ indicates that $A$ is a subset of $B$, but doesn’t say anything about whether or not they are equal
• For now, our default will be the convention of your textbook (which your Prof. will try not to forget!)

Subset and Membership, cont.

• Membership / inclusion symbols can exist in both directions, and with negated forms, such as:
  – for membership: $\in, \notin, \ni, \notni$
  – for inclusion: $\subset, \supset, \not\subset, \not\supset, \subseteq, \supseteq, \not\subseteq, \not\supseteq$

  Do you see similarities between (1) inclusion symbols and (2) math symbols for less-than and greater-than?

• If $A \subseteq B$ and $B \subseteq A$, what else can we say about a relationship between sets $A$ and $B$?
• Questions / exercises:
  – For a set $A$, is it necessarily true that $A \subseteq A$?
  – How about $A \subset A$? Or $A \supseteq A$? Or $A \in A$?
Exercises

• True or false?
  (a) Whenever \(A \subseteq B\) and \(B \subseteq C\), \(A \subseteq C\)
  (b) Whenever \(A \subseteq B\) and \(C \subseteq B\), \(A \subseteq C\)
  (c) Whenever \(A_1 \subseteq A_2 \subseteq A_3 \ldots \subseteq A_n\), and also \(A_n \subseteq A_1\), then \(A_i = A_j\) for all \(i, j\) in \([1..n]\)
  (d) \(A = B\) iff neither \(A \subset B\) nor \(B \subset A\)
  (e) Whenever \(A \subset B\) and \(B \subseteq C\), \(A \subset C\)

Exercise

• True or false: Whenever \(A \subset B\) and \(B \subseteq C\), \(A \subset C\)
  – Answer: True
• Proof idea:
  – \(A \subset B\) means two things:
    • P1) For all elements \(x\), if \(x \in A\) then \(x \in B\); and
    • P2) there exists element \(y\) in \(B\) that is not in \(A\)—that is, there exists \(y\) s.t. \(y \notin A\) and \(y \in B\)
  – \(B \subseteq C\) means:
    • P3) For all elements \(x\), if \(x \in B\) then \(x \in C\)
  – From these 3 premises, we need to show \(A \subset C\)—that is, show that
    • C1) For all elements \(x\), if \(x \in A\) then \(x \in C\); and
    • C2) There exists element \(y\) s.t. \(y \notin A\) and \(y \in C\)
  – To show C1)....? [Hint: use P1 and P3]
    • For the “for all”, consider arbitrarily chosen element...
    • For the “if—then”, assume the antecedent, show the consequent...
  – To show C2)....? [Hint: use witness \(y\) from P2, and P3]
Defining Sets

• Intuitively, there are multiple ways of defining a set
  – Enumerate all of its individual members
    • E.g., \{2, 3, 5, 7\}, \{1, 2, 3, 5, 8, 13\}
  – Provide a common, defining property
    • E.g., \{n \mid n \text{ is a prime number less than 10}\},
      \{n \mid n \text{ is a Fibonacci number less than 20}\}
  – Either of these is fine, as long as the definition is complete
    and clear in context

(There are other ways, too, but we’ll focus on these for now)

Intersection

• There are some fundamental set operations, i.e., ways of constructing sets
  from other sets
• Example: Intersection
  – Intuitively, an intersection is what two (or more) things have in common
  – For sets A, B, the intersection A \cap B is the set of elements that A and B have in
    common. More formally…
• Definition: The intersection A \cap B of sets A and B is defined by:
  \( x \in A \cap B \iff x \in A \text{ and } x \in B \)
  – Also written as A \cap B = \{x \mid x \in A \text{ and } x \in B\}
• Examples:
  – What’s \{1,2,3,4\} \cap \{2,4,6,8\}?  
  – What’s \{x \mid x \text{ is odd between 0 and 10}\} \cap \{x \mid x \text{ is prime between 0 and 10}\}
Union

- Another set operation: Union
  - Intuitively, a union is the combination of what two (or more) things have when they're put together
  - For sets A, B, the union $A \cup B$ is the set of elements that either A or B contain. (Or both! This is a non-exclusive or.) More formally…
- Definition: The union $A \cup B$ of sets A and B is defined by: $x \in A \cup B$ iff $x \in A$ or $x \in B$
  - Also written as $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Examples:
  - What’s $\{1,2,3,4\} \cup \{2,4,6,8\}$?
  - What’s $\{x \mid x \text{ is odd between 0 and 10}\} \cup \{x \mid x \text{ is prime between 0 and 10}\}$

Definitions like the definition of union or of intersection can be important parts of proofs—they can be reasons for proof steps. In your proofs, think about what definitions can be used to justify your proof steps!