CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – M W 10:30am
Lab – F 3:10pm

Lecture Meeting Location: SP 105
Lab Meeting Location: SP 309

Business

• HW5 out, due April 19 / April 20 (see HW sheet)

• Exam grading update

• Reading: Makinson, Ch.4.1-4.6
  – Our coverage of the material will be different from that in the textbook, but it’s good to see the textbook’s presentation, as well

• Also Reading: Makinson, Ch. 5

• Document on structural induction available soon from course website (follow the Additional Notes link)
Proving Statements about Recursively Defined Sets

- A related kind of induction is called *structural induction*, which can be used to prove claims about all items constructed by a recursive definition.

- To prove property $P$ holds for all elements of a recursively defined set:
  - Base case(s): Show that $P$ holds for every element in the basis for the recursive definition.
  - Inductive case(s): Show that every *constructor* in the definition preserves property $P$.

- Recall the definition of *transitive closure*:
  - $R_0 = R$;
  - $R_{i+1} = R_i \cup \{ (a, c) \mid \exists x \text{ s.t. } (a, x) \in R_i \text{ and } (x, c) \in R \}$;

- Claim: In the above definition, $R$ is a subset of $R_i$ for all $i$. Prove by structural induction.

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Proving Statements about Recursively Defined Sets

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  - Base case(s): Show that $P$ holds for every element in the basis for the recursive definition.
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- Recall our recursive definition of propositional logic expressions
  - Base: Given an initial set $A$ of propositional letters (e.g., $p, q, r, \ldots$), all elements of $A$ are propositional logic expressions
  - Induction: If $P, Q$ are propositional logic expressions, then the following are also propositional logic expressions (note that the parentheses are part of the expressions)
    - $\neg P$; $P \land Q$; $P \lor Q$; $P \implies Q$; $P \iff Q$  
      *(Note: 5 constructors)*

- Claim: All propositional logic expressions contain an even number of parentheses. (We consider 0 to be an even number.) Prove by structural induction.