CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – M W 10:30am
Lab – F 3:10pm

Lecture Meeting Location: SP 105
Lab Meeting Location: SP 309

Business

• HW5 out, due April 19 / April 20 (see HW sheet)
• HW6 out April 20, due May 1 / May 2

• (Please bring exams to class until we can finish going over them!)

• Reading: Makinson, Ch. 5
• Document on structural induction available soon from course website
  (follow the Additional Notes link)
Business, pt. 2

• Lab5, lab6, and course scheduling:
  – Lab5 is posted already; can be worked on anytime
  – Lab6 will be held as usual this Friday, April 22 (Coaches will run lab in my absence)
  – Both labs due by the end of the day Thursday, April 28

• Class session on Monday, April 25 will be held in the Asprey Lab, SP307 (where Coaching Hours are held)
  – (Coaches will run this class session)
  – That class session will be exclusively for work on labs (completing them, getting them checked off, etc.)
  – Attendance is not mandatory on April 25 (though it is at all other lab sessions)

Counting

• Counting things is an important part of Computer Science problem solving
  – The number of elements in a set, or subsets of a set
  – The number of routes through a map that visit every city
  – The number of words of length $L$ formed from alphabet $A$

  – Can be important for: brute-force solutions (i.e., exhaustively enumerating every possibility under some circumstances), probability, etc.
Count On It!

- More principles for counting elements of sets
- Subtraction principle for (finite set) difference
  - $|A - B| = \ldots$ ?
- Addition principle for finite sets
  - $|A \cup B| = \ldots$ ?

• Example application: A logic class has 20 students who also take calculus, 9 who also take philosophy, 11 who take neither, and 2 who take both calculus and philosophy. How many students are in the class?
Count On It!

- Subtraction principle for (finite set) difference
  - $|A - B| = |A| - |A \cap B|
- Addition principle for finite sets
  - $|A \cup B| = |A| + |B| - |A \cap B|
  - If $A, B$ are disjoint, then $|A \cup B| = |A| + |B|$ (do you see why?)

- Multiplication principle for finite sets: $|A \times B| = |A| \cdot |B|
  - This generalizes to a set product of any finite number of sets

- Example application:
  - How many distinct license plates are there consisting of two letters followed by four digits?
  - (This is easier than counting the number of plates consisting of two letters and four digits, without the restriction that letters come first)

Selection: Order and Repetition

- Sometimes it’s useful to consider the number of ways to select (or choose) $k$ items out of $n$ items

- Two factors to consider:
  - Order: does the order matter when considering the elements? (E.g., is selecting $a, b$ in that order different from selecting $b, a$ in that order?)
  - Is repetition permitted? (Can the same element be selected multiple times)

- How many ways can we choose 2 letters from $\{a, b, c, d, e\}$?
  - If order matters and repetition is permitted?
  - If order matters and repetition is not permitted?
  - If order doesn’t matter and repetition is permitted?
  - If order doesn’t matter and repetition is not permitted?
Selection: Permutations

- The operations for selecting \( k \) out of \( n \) items (for given \( k \leq n \)) are particularly important when repetition is not permitted
  - Corresponds to working with sets, where elements are not repeated
- Those operations are often called *combination* and *permutation*
  - Difference: the order in which they’re selected matters in permutations, not in combinations
- Permutations:
  - Number of ways to select \( k \) out of \( n \) items, where order matters = \( \ldots \)?

\[ P(n, k) = \frac{n!}{(n-k)!} \]

It may not be necessary to memorize this formula; it could be derived when needed.
Selection: Combinations

- The operations for selecting \( k \) out of \( n \) items (for given \( k \leq n \)) are particularly important when repetition is not permitted
  - Corresponds to working with sets, where elements are not repeated
- Those operations are often called combination and permutation
  - Difference: the order in which they’re selected matters in permutations, not in combinations
- Combinations:
  - Number of ways to select \( k \) out of \( n \) items, where order doesn’t matter = …?

It may not be necessary to memorize this formula; it could be derived when needed.
Counting 1

• Using the counting ideas from combinations and permutations…

• Exercises
  – Your investment advisor gives you a list of 8 stocks that seem like good investments. You decide to invest in 3 of them. How many different selections are possible?
  – Same scenario, except you decide to invest $1,000 in one stock, $2,000 in another, and $4,000 in a third. How many different selections are possible?

Counting 2—Repetition

• It can be useful to think about situations in terms of possibly repeated elements being counted, as well as possible orderings of elements

• Using the counting ideas from combinations and permutations…

• Exercises
  – A restaurant has five flavors of ice cream, and you can order one, two, or three scoops. How many different ice cream orders could you place?