CMPU 241 - Algorithmics

Spring 2020

Assignment 1

Due: Thursday, Feb. 6th, by midnight

NAME: (2 points)____________________________

1. (8 points) Consider sorting n numbers stored in array A by first finding the smallest element of A and exchanging it with the number stored in $A[1]$. Then find the second smallest element of A, and exchange it with $A[2]$. Continue in this manner for the first $n-1$ elements of A. This algorithm, called Selection-Sort, is given below. Assume the input and output are as specified for the sorting problem on page 16 of our textbook.

```
Selection-Sort(A):
1.  n = A.length
2.  for j = 1 to n − 1
3.        smallest = j
4.        for i = j + 1 to n
5.             if A[i] < A[smallest]
6.                  smallest = i
7.        exchange A[j] with A[smallest]
```

   (a) (1 point) Give the line number(s) of the basic operation in the algorithm

   (b) (2 points) Write a nested summation to describe the number of times the basic operation of Selection-Sort is executed on an input of size n and then express this summation as a closed-form solution.

   (c) (2 points) Why does the outer for loop of the algorithm need to run for only the first $n-1$ elements?

   (d) (3 points) Are there best-case and worst-case asymptotic running times for Selection-Sort? If so, give the best-case and worst-case running times of the algorithm in Θ notation. If not, give the worst-case running time in Θ notation. Explain your answer.

2. (4 points) Answer true or false. Use the definitions of O, Θ, and Ω to determine whether the following assertions are true or false. Briefly justify your answers.

   a. $(n^2(n+1))/2 \in O(n^3)$

   b. $n(n+1)/2 \in O(n)$

   c. $n(n+1)/2 \in \Theta(n^3)$

   d. $n(n+1)/2 \in \Omega(n)$

3. (4 points) For each of the following functions, indicate the set $\Theta(g(n))$ the function belongs to. Show your work in simplifying these expressions to arrive at each answer.

   a. $(n^2+1)^{10}$

   b. $\sqrt{10n^2+7n+3)}$

   c. $2^{n+1} + \ 3^{n-1}$

   d. $\lfloor \lg n \rfloor$

4. (2 points) Answer Yes or No and provide a brief justification for each question below:

   a. Is $2^{n+1} \in O(2^n)$?

   b. Is $2^{2n} \in O(2^n)$?