CMPU 241 - Analysis of Algorithms

Spring 2020

Assignment 2

Due: Friday, February 14th at 5pm

NAME: (1 point) ______________________________

1. (12 points) Give tight asymptotic bounds for $T(n)$ in each of the following recurrences. Use backward substitution, either version of the Master Theorem, Recursion Tree, or make a good guess and show that it holds. Assume that $T(n)$ is constant for $n \leq 2$. State your method of solution and *if you use either version of the Master Theorem, state version of the Master Theorem you are using, and the case the recurrence falls into (Case 1, 2, or 3)*. Show all your work for full credit.

   a. $T(n) = T(\frac{9n}{10}) + n$

   b. $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

   c. $T(n) = T(n-1) + n$

   d. $T(n) = 3T(\frac{n}{2}) + n$

   e. $T(n) = 6T(\frac{n}{4}) + n^2$

   f. $T(n) = 3T(\frac{n}{4}) + nlgn$

2. (10 points) Consider the Selection-Sort algorithm, given below. Assume the input and output are as specified for the sorting problem on page 16 of our textbook.

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Selection-Sort(A):
1.  n = A.length
2.  for j = 1 to n − 1
3.      smallest = j
4.      for i = j + 1 to n
5.          if A[i] < A[smallest]
6.              smallest = i
7.      exchange A[j] with A[smallest]
```

Consider the following loop invariant of the outer for loop (lines 2 through 7).

> The algorithm maintains the loop invariant that at the start of each iteration, the subarray $A[1..j-1]$ consists of the $j-1$ smallest elements in the array $A[1..n]$ and the subarray $A[1..j-1]$ is in sorted order.

Show that the loop invariant is true using a formal proof like that shown in the textbook and the course notes for Insertion-Sort (prove the invariant holds initially, at every subsequent iteration, and at termination, according to the value of j, the loop counter).

3. (8 points) Order the following functions in terms of their order of growth (i.e., growth rate), from lowest to highest:

$$n\lg n,\ n + \lg n,\ n!,\ n,\ n^2,\ 2^n,\ n^{\frac{1}{2}},\ 4^n$$

Provide a justification for your answers and indicate which of the functions, if any, grow at the same rate.