CMPU 241 - Analysis of Algorithms

Spring 2020

Assignment 3

Due: Thursday, February 20th, by 5 pm

NAME: (1 point)____________________

1. (2 points) Use induction to show that the actual number of nodes at level l of a full and complete binary tree is 2^l. Recall that the root is at level 0.

2. (3 points) Use induction to show that the actual number of nodes in a full and complete binary tree of height h is $2^{h+1} - 1$, where h is the height of the tree.

3. (4 points) What are the minimum and maximum numbers of nodes in a heap of height h, for $h > 0$? Explain your answers, possibly using the proofs from problems 1 and 2.

4. (5 points) Assume the following sequence of numbers is arranged in level-order numbering in an array like the one shown in Figure 6.1 of your text.

   Is the sequence $\langle 27, 15, 14, 6, 13, 10, 11, 5, 7, 17 \rangle$ a max-heap? List the numbers in each level of the tree formed by the sequence (or draw a tree) and either describe what makes it a max-heap or point out where it violates the properties of a max-heap. If it is not a max-heap, indicate how many exchanges of values would be necessary to make it a max-heap and where those exchanges would occur. If applicable, list the numbers in each level of the tree formed by the new sequence (or draw the tree again).

5. (5 points) If $A[p]$ was always chosen as the pivot in line 1 of QuickSort's PARTITION function (given on page 171 of our textbook), would it change the worst-case running time of QuickSort? Answer "yes" or "no" and *explain your answer*.

6. (10 points) This Question is similar to Problem 7-1, on page 185 of our textbook. Suppose the array A is of length n and this algorithm takes $(A, 1, n)$ as initial input.

```
HOARE-PARTITION(A,p,r):
 1.  x = A[p]
 2.  i = p - 1
 3.  j = r + 1
 4.  while TRUE
 5.      repeat
 6.          j = j - 1
 7.      until A[j] ≤ x
 8.      repeat
 9.          i = i + 1
10.      until A[i] ≥ x
11.      if (i < j)
12.          exchange A[i] with A[j]
13.      else
14.          return j
```

Demonstrate the operation of HOARE-PARTITION on the array

$$A = \langle 13,\ 19,\ 9,\ 5,\ 12,\ 8,\ 7,\ 4,\ 11,\ 2,\ 6,\ 21 \rangle$$

showing the values of the array and auxiliary values i and j after each iteration of the **while** loop in lines 4–14 and when the procedure returns. Your depiction of the algorithm operation should be like that shown on page 172 of our textbook for the operation of PARTITION, an algorithm given in our textbook on page 171.