Project: Predator / Prey Interactions!

- This assignment is about modeling system dynamics with interactions, focusing on predator / prey interactions. (See Chapter 4.2 of your textbook for additional details!) All code is to be written in Matlab; as usual, looking at a classmate’s code or sharing specifics about code on an assignment is a violation of course policy. Please note that some of this assignment is specifically intended to be completed without any collaboration (although you may always ask your Prof. any questions you might have!), even about general ideas; please see the exercises below for details.

- Some exercises ask you to develop models. When an exercise asks you to develop a model, your write-up should include not only the model itself—e.g., the differential equations (or finite difference equations), constants, and initial conditions that may comprise the model—but also an explanation of how the model was developed, including what each variable / parameter in the model stands for (“G is the growth constant . . . ”), what simplifying assumptions were made, and what the reasons were for your decisions (“We assume H is constant because we are modeling on the order of months, not years . . . ”). If other information is pertinent (e.g., why a certain variable / parameter is not present in a model), explain that in the write-up as well.

This information can be provided either as part of commenting in your code or as part of a separate write-up. Either way, readability is an essential part of the assignment: Make sure both your code and your write-up are easy to read and understand.

- Some exercises ask you to run simulations. When an exercise asks you to run a simulation, your write-up should include the values of constants / parameters employed for each run of the simulation (e.g., in simulation 0, \( G = 0.01, dt = 0.1, \ldots \); in simulation 1, \( G = 0.01, dt = 0.01, \ldots \)) and a very brief explanation of why you chose to run those particular values for simulations (“We tested five values between 0 and 1 to illustrate how the system behaves with short time parameters, which is consistent with our assumption that . . . ”), as well as a description of the results of the simulation (“. . . observed that . . . , and in particular, at time \( t = 3, \ldots \)”). Descriptions of results should be concise and information-heavy; feel free to include figures (e.g., Matlab plots) in write-ups to illustrate your observations.

As always, readability is an essential part of the assignment: Make sure both your code and your write-up are easy to read and understand.
In all exercises, for full credit, your work must be sufficiently documented to demonstrate an understanding of the relevant concepts and questions for each exercise. In general, always explain your answers and document the process you used to arrive at those answers. If there are questions about this or anything else regarding these exercises, please feel free to ask your Prof.!

For each exercise, please electronically submit your code in .m files to the course dropbox, using the submit250 script

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submit250 hw2 <your-directory-name>.
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In addition, on or before the due date, please turn in printouts of your code and answers to non-programming questions on paper.

The non-programming portions of this homework, including printouts of all code, are due at the beginning of class on October 29.

The programming portions of this assignment are to be electronically submitted to our course dropbox by 11:59pm on October 28.

Exercises

1. **Up The Food Chain!** These exercises are based on Project 3, Chapter 4.2 in your textbook—here, you’ll be modeling predator / prey interactions with three species involved! There’s a prey species $Y$ (e.g., tuna) and a predator species $P$ (e.g., sharks) that consumes $Y$, and in addition, there’s a third species $H$ (e.g., humans) that hunts both $P$ and $Y$.

   (a) Develop a predator-prey model for this scenario, under the assumption that $H$ hunts $P$ and $Y$ equally—e.g., in the specific case mentioned above, that humans have equal fishing rates for tuna and sharks. Moreover, assume that the timeframe to be modeled will be on the order of months (rather than, e.g., decades), so we can assume the population of $H$ stays constant over the time being simulated.

   Write the differential equations for this model. As always, be sure to document your process, which includes listing the assumptions and sources used in developing the model!

   (b) Analytically solve (using mathematics, not computational tools such as Matlab!) for the equilibria solutions for the equations you developed. As always, explain your answers!

   (c) To illustrate the predator-prey interactions between $P$ and $Y$, without $H$, use your model to run several simulations with no fishing (but with different values of other relevant factors for predator-prey interaction).
Then, run additional simulations with fishing, gradually increasing the fishing rate (i.e., the rate constant for \( H \) hunting \( P \) and \( Y \)), but keep it lower than the birth rate of \( Y \). As always, document your process (including what values you used in simulations) and report your results.

For this exercise, also answer the following questions in your write-up: As the fishing rate changes, what happens to the number of prey and predators? How do you explain the impact of predator \( H \) given your model of \( P \), \( Y \), and \( H \)?

(d) Run more simulations as in Exercise 1c, increasing the fishing rate. At what fishing rate do the predators \( P \) all die? How does this relate to the birth rate of \( Y \)? Explain the reasons for this, using the equilibrium solutions from Exercise 1b to inform your answer.

(e) In Project 1 of Chapter 4.2 in your textbook, a framework for periodic variation of a parameter in a model (there, it was prey birth fraction) is presented. Adapt that idea to implement periodic seasonal fishing in your model of \( H \), \( P \), and \( Y \). In particular, the period of the fishing cycle should be 12 months: for the 6-month off-season span in the cycle, the fishing rate is 0; for the 6-month fishing season span in the cycle, the fishing rate starts at 0, rises to its maximal value 3 months in to the fishing season, and falls again to 0 at the end of the 6-month season. Use the cosine function form given in Project 1, Chapter 4.2, with parameters \( f \), \( a \), and \( p \), to implement seasonal fishing. (As always, explain your work in developing your model!)

Run simulations of your model with several different values of model parameters. Describe and discuss your results. Are there any values of the relevant constants / parameters that lead to particularly interesting results?

2. **Change It Up!** In Exercise 1, the population of species \( H \) stayed constant in the model. In this exercise, that restriction is removed.

   (a) Develop a model similar to Exercise 1a, with two differences: species \( H \) hunts only species \( P \), not species \( Y \); and the population of \( H \) should also change over time. (As a result, this model will have a differential equation for \( \frac{dH}{dt} \) that was not present in the previous model!) In this model, you can treat species \( P \) as prey with respect to how its deaths are modeled, and as predator with respect to how its births are modeled.

   Write the differential equations for this model and document your process for developing the model, as in Exercise 1.

   (b) Develop a model similar to Exercise 2a, except that in this model, species \( H \) hunts both species \( P \) and species \( Y \). As in Exercise 2a, write the differential equations and document your process for developing the model.

   (c) **Collaboration is not permitted on this exercise.** Do hypothesis testing with your two models! Come up with a question you’d like to see answered involving the systems being modeled and the specific values of constants in those
models, then formulate (and write down, in your write-up) a hypothesis and predictions to be tested. For example, you might wonder if species $H$ and $P$ could survive if there were never any of species $Y$ in the system, and your hypothesis could be “In both models, all of species $H$ and $P$ will die out if the initial condition for the population of $Y$ is $Y = 0$.” (Please don’t use this hypothesis for your own work—please come up with something much more interesting!)

Then, run simulations of both models as needed to test your hypothesis. Do the data support or contradict your hypothesis? Be sure to fully document your process, describe the results of simulations, and give an explicit answer to whether or not the data support your hypothesis!

(Note: This exercise is conceptually about the process of coming up with hypotheses and testing them with models and simulations. Whether or not a hypothesis is actually supported by data is not directly relevant to grading your answers for this exercise; the content and depth of the process of coming up with a hypothesis and testing it will determine the grades given to answers. As always, please ask your Prof. if you have any questions about this exercise!)