

# EXAM

*Please read all instructions, including these, carefully*

- There are 9 questions on the exam, with multiple parts. You have 3 hours to work on the exam.
- The exam is open book, open notes.
- Please write your answers in the space provided on the exam and clearly mark your solutions.
- You may use the backs of the exam pages as scratch paper, or use additional pages (available at the front of the room).
- Each problem has a straightforward solution. Solutions will be graded on correctness and clarity. Partial solutions will be given partial credit.

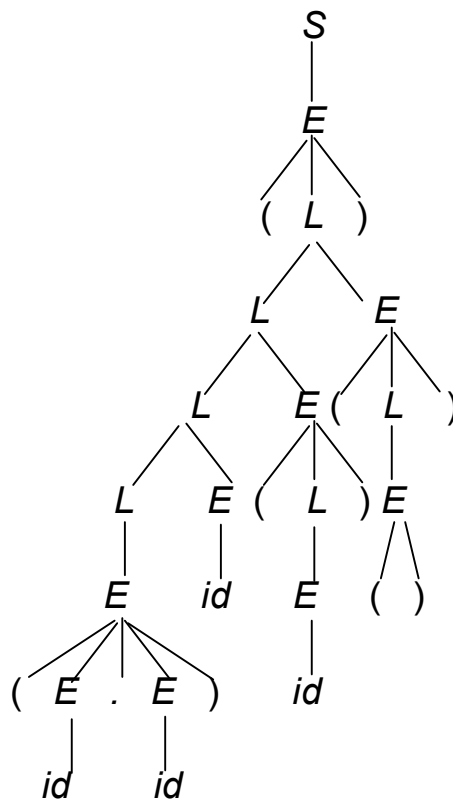
NAME : \_\_\_\_\_

Problem	Max points	Points
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	15	
TOTAL	100	

1. (a) Using the grammar below, draw a derivation tree for the following string:

**(( id . id ) id ( id ) ( ( ) ) )**

$S \rightarrow E$   
 $E \rightarrow id$   
 $\quad | ( E . E )$   
 $\quad | ( L )$   
 $\quad | ( )$   
 $L \rightarrow L E$   
 $\quad | E$



(b) Give a rightmost canonical derivation for the string given in part (a).

S  
E  
( L )  
( L E )  
( L ( L ) )  
( L ( E ) )  
( L ( ( ) ) )  
( L E ( ( ) ) )  
( L ( L ) ( ( ) ) )  
( L ( E ) ( ( ) ) )  
( L ( id ) ( ( ) ) )  
( L E ( id ) ( ( ) ) )  
( L id ( id ) ( ( ) ) )  
( E id ( id ) ( ( ) ) )  
( ( E . E ) id ( id ) ( ( ) ) )  
( ( E . id ) id ( id ) ( ( ) ) )  
( ( id . id ) id ( id ) ( ( ) ) )

2. Consider the following grammar:

- 1  $S \rightarrow ABA$
- 2  $A \rightarrow Bc$
- 3       |  $dA$
- 4       |  $\epsilon$
- 5  $B \rightarrow eA$

(a) Fill in the table below with the FIRST and FOLLOW sets for the non-terminals in this grammar:

	<i>First</i>	<i>Follow</i>
S	<b><math>d, e</math></b>	<b><math>\\$</math></b>
A	<b><math>d, e, \epsilon</math></b>	<b><math>c, d, e, \\$</math></b>
B	<b><math>e</math></b>	<b><math>c, d, e</math></b>

(b) Fill in the LL (1) parse table for this grammar.

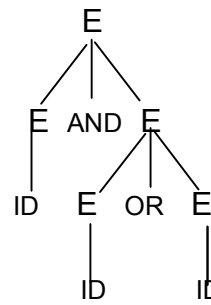
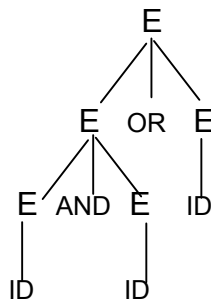
	<i>c</i>	<i>d</i>	<i>e</i>	<i>\$</i>
<i>S</i>		<b>1</b>	<b>1</b>	
<i>A</i>	<b>4</b>	<b>3,4</b>	<b>2,4</b>	<b>4</b>
<i>B</i>			<b>5</b>	

3. Consider the following grammar for boolean expressions (words in capitals are terminals).

- $$E \rightarrow E \text{ OR } E$$
- $$E \rightarrow E \text{ AND } E$$
- $$E \rightarrow \text{NOT } E$$
- $$E \rightarrow ( E )$$
- $$E \rightarrow \text{TRUE}$$
- $$E \rightarrow \text{FALSE}$$
- $$E \rightarrow \text{ID}$$

(a) Show that this grammar is ambiguous.

There are two parse trees for several strings generated by this grammar. For example, the string *ID AND ID OR ID* has two parse trees:



- (b) Rewrite the grammar to remove the ambiguity and enforce the intended precedence order by introducing new non-terminals. Make sure that your revised grammar accepts the same language as the original.

$E \rightarrow \text{NOT } E$   
     $| E'$   
 $E' \rightarrow E' \text{ OR } T$   
     $| T$   
 $T \rightarrow T \text{ AND } F$   
     $| F$   
 $F \rightarrow \text{TRUE}$   
     $| \text{FALSE}$   
     $| \text{ID}$   
     $| ( E )$

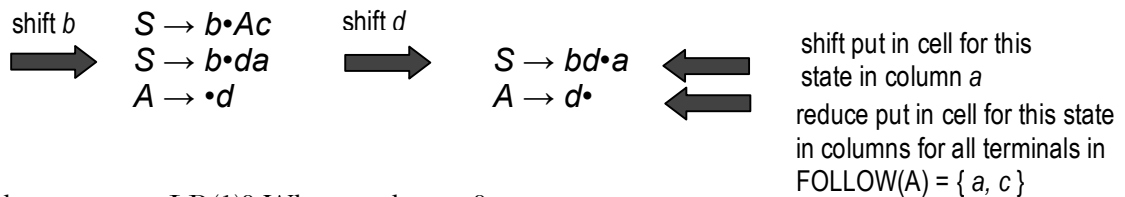
4. Consider the following context free grammar:

$$\begin{array}{l}
 S \rightarrow Aa \\
 \quad | bAc \\
 \quad | dc \\
 \quad | bda \\
 A \rightarrow d
 \end{array}$$

(a) Is the grammar SLR(1)? Why or why not? You don't need to construct the parse table to answer.

No, because when the stack contains  $bd$  with  $a$  next in input, there is a shift-reduce conflict. That is, we reach a certain state in the DFA by shifting  $b$  and then  $d$ . Since  $a$  is in  $\text{FOLLOW}(A)$ , reduce by  $A \rightarrow d$  will be indicated in the parse table cell indexed by the current state and the terminal  $a$ . However, because of the production  $S \rightarrow bda$ , shift  $a$  will also be indicated in the same cell.

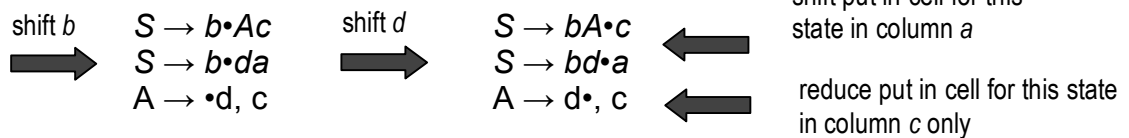
The relevant item sets:



(b) Is the grammar LR(1)? Why or why not?

Yes, because the explicit lookahead will prevent the reduce move from being indicated in the cell.

The relevant item sets:



(c) Consider the following context free grammar for the same language:

$$\begin{array}{l}
 S \rightarrow Aa \\
 \quad | bAc \\
 \quad | Bc \\
 \quad | bBa \\
 A \rightarrow d \\
 B \rightarrow d
 \end{array}$$

Is this grammar LR(1)? Why or why not?

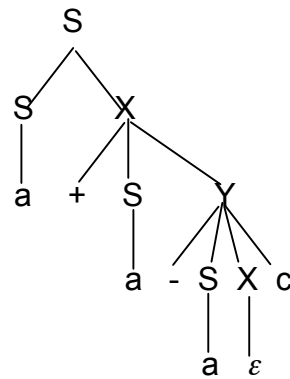
Yes. After shifting  $b$  and  $d$ , lookahead will ensure that we reduce to  $A$  if a  $c$  is in the input, and reduce to  $B$  if an  $a$  is in the input.

5. Let  $G$  be the following grammar:

- 1  $S \rightarrow SX$
- 2     |  $a$
- 3  $X \rightarrow \epsilon$
- 4     |  $+SY$
- 5     |  $Yb$
- 6  $Y \rightarrow \epsilon$
- 7     |  $-SXC$

(a) Show the parse tree for the following string :

**$a + a - ac$**



(b) Complete the table below tracing an LR(1) parse of the same string. The *Stack* column shows the stack (with the top at right), the *Input* column shows the not-yet-processed input, and the *Action* column shows whether the parser performs a shift action, a reduce action, or accepts the input. In the case of a reduce action, indicate (by number) which production is used. (NOTE: you do not need to construct the parse table; just use your knowledge of how the parser works and the parse tree above).

Stack (with top at right)	Input	Action
\$	a+a-ac\$	shift
\$a	+a-ac\$	reduce
\$S	+a-ac\$	shift
\$S+	a-ac\$	shift
\$S+a	-ac\$	reduce
\$S+S	-ac\$	shift
\$S+S-	ac\$	shift
\$S+S-a	c\$	reduce
\$S+S-S	c\$	reduce
\$S+S-SX	c\$	shift
\$S+S-SXc	\$	reduce
\$S+SY	\$	reduce
\$SX	\$	reduce
\$S	\$	accept

6. You may give any context-free grammar in answer to the questions in this part; i.e it doesn't matter if the grammar is LL (1), unambiguous, or any other subclass of all context free grammars.

- (a) Consider the following FIRST and FOLLOW sets:

$$\begin{aligned} \text{FIRST}(S) &= \{b, \varepsilon\} \\ \text{FIRST}(T) &= \{b, \varepsilon\} \\ \text{FOLLOW}(S) &= \{a, \$\} \\ \text{FOLLOW}(T) &= \{a, b, \$\} \end{aligned}$$

Give the simplest grammar (fewest productions and shortest right-hand sides) that produces these sets.  $S$  is the start symbol.

There are several possible correct answers to this question. Here is one of them:

$$\begin{aligned} S &\rightarrow Tb \mid T \\ T &\rightarrow bSa \mid \varepsilon \end{aligned}$$

- (b) Give a grammar with a single non-terminal  $S$  such that  $\text{FIRST}(S) = \emptyset$ .

$$\begin{aligned} S &\rightarrow S \\ \text{A common error is } S &\rightarrow \varepsilon, \text{ which has } \varepsilon \text{ in } \text{FIRST}(S) \end{aligned}$$

- (c) Give a grammar with a single non-terminal  $S$  such that  $\text{FIRST}(S) = (\text{FOLLOW}(S) - \{\$\}) = \emptyset$ .

$$\begin{aligned} S &\rightarrow aSa \\ \text{Common errors include having } \text{FIRST}(S) &\text{ empty or } \text{FOLLOW}(S) \text{ empty.} \end{aligned}$$

- (d) Give a simple example of a grammar with a shift-reduce conflict.

$$\begin{aligned} S &\rightarrow Ta \mid a \\ T &\rightarrow \varepsilon \end{aligned}$$

7. For each grammar explain why, or why not, the grammar is LL (1):

- (a)  $S \rightarrow 0 \mid 12 \mid 345$

Clearly LL(1), as each production rhs begins with a distinct terminal.

- (b)  $S \rightarrow 0 \mid T1$   
 $T \rightarrow 1 \mid S0$

Not LL(1) because it is left-recursive:  $S \Rightarrow T1 \Rightarrow S01$

- (c)  $S \rightarrow 0 \mid 11 \mid 01$

Not LL(1), because two of  $S$ 's right hand sides begin with the same terminal.

8. Let synthesized attribute  $F.val$  give the value of the binary fraction generated by  $F$  in the grammar that follows:

$$\begin{aligned} F &\rightarrow .L \\ L &\rightarrow LB \mid B \\ B &\rightarrow 0 \mid 1 \end{aligned}$$

For instance, on input **.101**,  $F.val = .625$ .

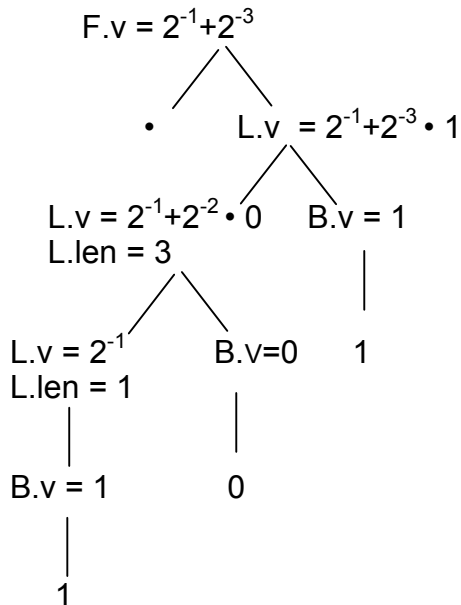
- (a) Using only synthesized attributes, give a translation scheme for the above grammar.

You need two attributes; one to hold the current value ( $.v$ ) and one to hold the length of the string ( $.len$ ). There are several ways to do it:

	Answer 1	Answer 2	Answer 3
$F \rightarrow .L$	$\{F.v=L.v\}$	$\{F.v=L.v \cdot 2^{-L.len}\}$	$\{F.v=L.v/2\}$
$L \rightarrow L_1B$	$\{L.len=L_1.len+1, L.v=L_1.v+2^{-L.len} \cdot B.v\}$	$\{L.len=L_1.len+1, L.v=L_1.v \cdot 2+B.v\}$	$\{L.len=L_1.len+1, L.v=L_1.v+2^{-L.len} \cdot B.v\}$
$L \rightarrow B$	$\{L.len=1, L.v=B.v/2\}$	$\{L.len=1, L.v=B.v\}$	$\{L.len=1, L.v=B.v\}$
$B \rightarrow 0$	$\{B.v=0\}$	$\{B.v=0\}$	$\{B.v=0\}$
$B \rightarrow 1$	$\{B.v=1\}$	$\{B.v=1\}$	$\{B.v=1\}$

- (b) Show the translation of the input string **.101** by decorating its parse tree. Make sure your tree is drawn clearly.

Here is the tree for Answer 1:



9. Consider the following parsing automaton. All that is shown are the states, transitions, and the *Left-hand side* of each item that has the “•” *all the way to right*. Complete the partial items and fill in all of the missing items (be sure to include the “•”). Note that the question requires only LR(0) items.

