Neurophysiology Hodgkin-Huxley Model

1 How do Neurons Work?

- Neurophysiology
- Reversal Potentials
- Ion Channels

2 ODE Models

- Example from Physics
- 3 Modeling the Membrane Potential

4 Modeling Voltage Activated Channels

How do Neurons Work

Modeling Voltage Activated Char

ODE Mode

- Channel Dynamics
- Actual Channels

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Neurophysiology

Reversal Potentials Ion Channels

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Neurophysiology Reversal Potentia Ion Channels

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All cells have:

- Thin membrane
- Intracellular and extracellular fluid full of ions
- Ion pumps maintain ion concentrations
- Sodium (Na⁺): low inside
- Potassium (K⁺): high inside
- Calcium (Ca²⁺): very low inside

Special for neurons:

- Long, thin branches
- Can fire action potentials

How do Neurons Work? ODE Models Modeling the Membrane Potential Modeling Voltage Activated Channels **Neurophysiology** Reversal Potentials Ion Channels



- Neurons have *excitable membranes*: *action potentials* (aka *spikes*) are rapid, dramatic, changes in the internal potential.
- By opening *ion channels* through their membrane neurons control their internal potential.

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Reversal Potentials

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Modeling the Membrane Potentia

Modeling Voltage Activated Channel

Reversal Potential

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Reversal Potential



Concentration difference gives rise to a current through the channels.

Reversal potential is the membrane potential required to balance this current.

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Neurophysiology Reversal Potentials Ion Channels

Channels are complex protein molecules "floating" in the membrane.



Many are highly selective, and only let one kind of ions pass.



Neurophysiology Reversal Potentials Ion Channels

Voltage Activated Channels

lon channels that open or close depending on the potential over the membrane.



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ODE Models

Ordinary Differential Equation (ODE)

$$\frac{du}{dt}=f(u,t)$$

t: time

u: State variable

f(u, t): rate of change

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Example from Physics

ODE Models

Example:

Serving Ale

- *State variable* Amount of ale in your glass
- *Rate of change* How fast you pour



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Example from Physics

ODE Models





Even better:

$$A \cdot \frac{du}{dt} = f(u, t)$$

where A is the cross section area of the glass.

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t: Time

- V: Membrane potential (*state variable*)
- *I*: Current entering cell
- C: Capacitance
- E: Reversal potential
- G: Channel conductance

 $C \cdot \frac{dV}{dt} = I$

 $I = G \cdot (E - V)$

In real neurons we have more kinds of ions involved

$$C \cdot \frac{dV}{dt} = I_L + I_{Na} + I_K$$
$$I_I = G_I \cdot (E_I - V)$$

$$I_{Na} = g_{Na}G_{Na} \cdot (E_{Na} - V)$$
$$I_{K} = g_{K}G_{K} \cdot (E_{K} - V)$$

ODE Models Modeling the Membrane Potential Modeling Voltage Activated Channels

Channel Dynamics Actual Channels Channel Dynamics Actual Channels

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Fraction of channels open: g

$$\frac{dg}{dt} = \alpha \cdot (1 - g) - \beta \cdot g$$

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Channel Dynamics Actual Channels

The Potassium Channel

$$g_{Na} = n^{4}$$

$$\frac{dn}{dt} = \alpha_{n} \cdot (1 - n) - \beta_{n} \cdot n$$

$$\alpha_{n} = \frac{A(v - B)}{1 - e^{-\frac{v - B}{C}}} \qquad \beta_{n} = \frac{-A(v - B)}{1 - e^{\frac{v - B}{C}}}$$

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