Neurophysiology

Hodgkin-Huxley Model

1. How do Neurons Work?
   - Neurophysiology
   - Reversal Potentials
   - Ion Channels
2. ODE Models
   - Example from Physics
3. Modeling the Membrane Potential
4. Modeling Voltage Activated Channels
   - Channel Dynamics
   - Actual Channels

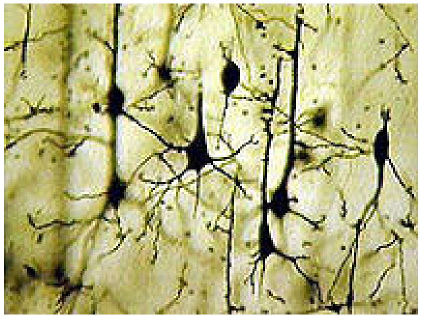
1. How do Neurons Work?
   - Neurophysiology
   - Reversal Potentials
   - Ion Channels
2. ODE Models
   - Example from Physics
3. Modeling the Membrane Potential
4. Modeling Voltage Activated Channels
   - Channel Dynamics
   - Actual Channels

All cells have:

- Thin membrane
- Intracellular and extracellular fluid full of ions
- Ion pumps maintain ion concentrations
- Sodium (Na^+): low inside
- Potassium (K^+): high inside
- Calcium (Ca^{2+}): very low inside

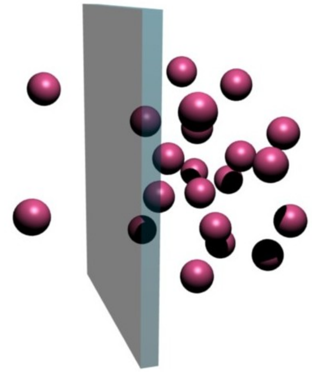
Special for neurons:

- Long, thin branches
- Can fire *action potentials*

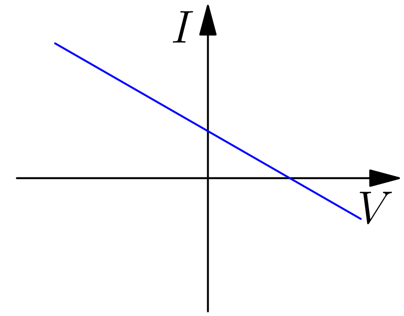


- Neurons have *excitable membranes*: *action potentials* (aka *spikes*) are rapid, dramatic, changes in the internal potential.
- By opening *ion channels* through their membrane neurons control their internal potential.

Reversal Potential

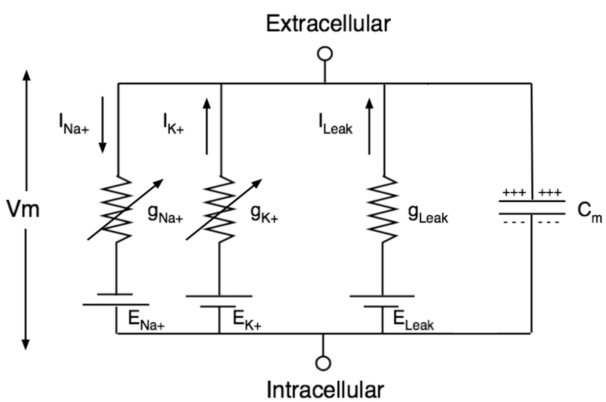


Concentration difference gives rise to a current through the channels.

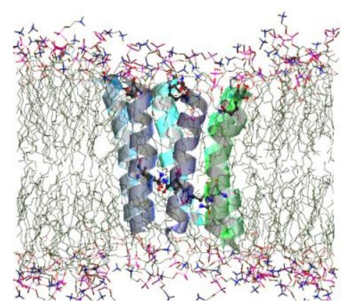


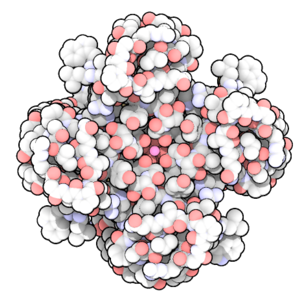
Reversal potential is the membrane potential required to balance this current.

Reversal Potential



Channels are complex protein molecules “floating” in the membrane.

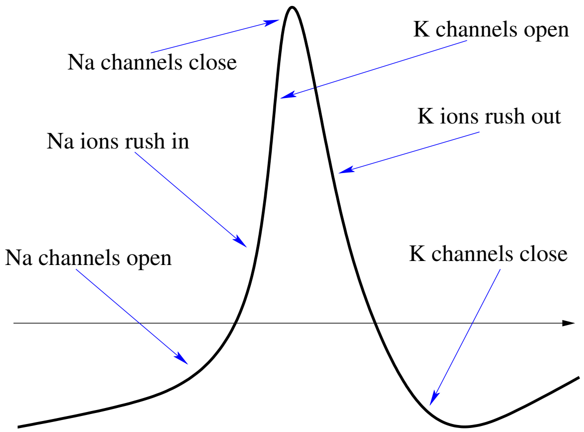




Many are highly selective, and only let one kind of ions pass.

Voltage Activated Channels

Ion channels that open or close depending on the potential over the membrane.



1. How do Neurons Work?
   - Neurophysiology
   - Reversal Potentials
   - Ion Channels
2. ODE Models
   - Example from Physics
3. Modeling the Membrane Potential
4. Modeling Voltage Activated Channels
   - Channel Dynamics
   - Actual Channels

ODE Models

Ordinary Differential Equation (ODE)

$$\frac{du}{dt} = f(u, t)$$

t: time

u: *State variable*

$f(u, t)$: rate of change

ODE Models

Example:

Serving Ale

- *State variable*
  Amount of ale in your glass
- *Rate of change*
  How fast you pour

ODE Models

$$\frac{du}{dt} = f(u,t)$$

Even better:

$$A \cdot \frac{du}{dt} = f(u,t)$$

where A is the cross section area of the glass.

1. How do Neurons Work?
   - Neurophysiology
   - Reversal Potentials
   - Ion Channels
2. ODE Models
   - Example from Physics
3. Modeling the Membrane Potential
4. Modeling Voltage Activated Channels
   - Channel Dynamics
   - Actual Channels

- t: Time
- V: Membrane potential (*state variable*)
- I: Current entering cell
- C: Capacitance
- E: Reversal potential
- G: Channel conductance

$$C \cdot \frac{dV}{dt} = I$$

$$I = G \cdot (E - V)$$

In real neurons we have more kinds of ions involved

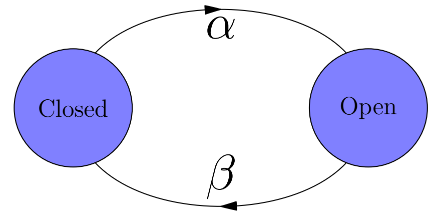
$$C \cdot \frac{dV}{dt} = I_L + I_{Na} + I_K$$

$$I_L = G_L \cdot (E_L - V)$$

$$I_{Na} = g_{Na} G_{Na} \cdot (E_{Na} - V)$$

$$I_K = g_K G_K \cdot (E_K - V)$$

1. How do Neurons Work?
   - Neurophysiology
   - Reversal Potentials
   - Ion Channels
2. ODE Models
   - Example from Physics
3. Modeling the Membrane Potential
4. Modeling Voltage Activated Channels
   - Channel Dynamics
   - Actual Channels



α Opening rate

β Closing rate

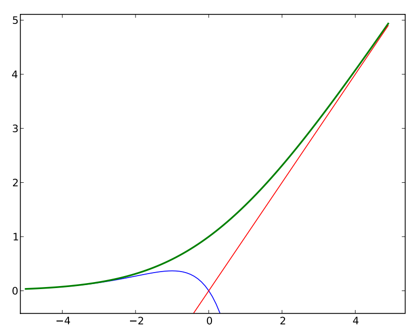
Fraction of channels open: g

$$\frac{dg}{dt} = \alpha \cdot (1 - g) - \beta \cdot g$$

$$\alpha = \frac{A(v - B)}{1 - e^{-\frac{v-B}{C}}} \qquad \beta = \frac{-A(v - B)}{1 - e^{\frac{v-B}{C}}}$$

With $A = 1$, $B = 0$, $C = 1$

$$\alpha = \frac{v}{1 - e^{-v}}$$



The Sodium Channel

$$g_{Na} = m^3 h$$

$$\frac{dm}{dt} = \alpha_m \cdot (1 - m) - \beta_m \cdot m$$

$$\frac{dh}{dt} = \alpha_h \cdot (1 - h) - \beta_h \cdot h$$

$$\alpha_m = \frac{A(v - B)}{1 - e^{-\frac{v-B}{C}}} \qquad \beta_m = \frac{-A(v - B)}{1 - e^{\frac{v-B}{C}}}$$

$$\alpha_h = \frac{-A(v - B)}{1 - e^{\frac{v-B}{C}}} \qquad \beta_h = \frac{A}{1 + e^{-\frac{v-B}{C}}}$$

The Potassium Channel

$$g_{Na} = n^4$$

$$\frac{dn}{dt} = \alpha_n \cdot (1 - n) - \beta_n \cdot n$$

$$\alpha_n = \frac{A(v - B)}{1 - e^{-\frac{v-B}{C}}} \qquad \beta_n = \frac{-A(v - B)}{1 - e^{\frac{v-B}{C}}}$$