

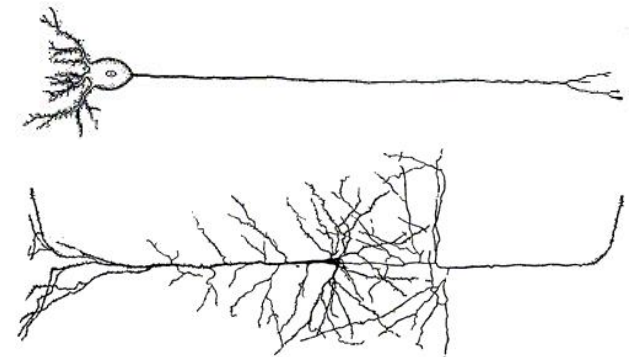
Integrate-and-Fire Models

Abstract Models

- 1 Model Abstraction
- 2 Integrate-and-Fire Models
 - Pure Integrate-and Fire
 - Leaky Integrate-and Fire
 - Absolute Refractory Period
- 3 Non-Spiking Models
 - Integrating
 - Non-Integrating

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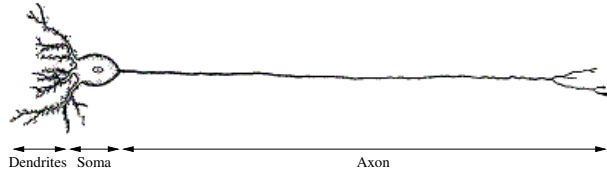
There are too many things going on in real neurons



Model Abstraction

Remove irrelevant details while preserving relevant properties

What properties are important to preserve?



- **Dendrites**
Passive reception of signals
- **Soma (Cell Body)**
Summing, Thresholding
- **Axon**
Aktive pulses are transmitted to other cells

1 Model Abstraction

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Early attempt: McCulloch and Pitts model (1943)

- Two output states: spike or no spike
- No time dynamics, immediate response
- Sufficient number of excitatory input spikes required
- Inhibitory input completely blocks output

Note similarity with digital electronics (gates)

Intrgrate-and-Fire Models

Point neuron model with simplified spikes

- Ignore the precise action potential mechanisms
- Assume linear summation of inputs
- Assume that each incoming spike contributes a constant amount of charge (current pulse)

Reasonable objective

Produce the right spiking frequency for different input levels

Idea

Replace Na/K mechanism with a simple reset mechanism

- Point neuron style integration of input

$$C \frac{dV}{dt} = I$$

- Reset Mechanism
When V reaches a threshold, Θ , the cell fires and V is reset to zero.

Shortcomings of the pure Integrate-and-Fire model

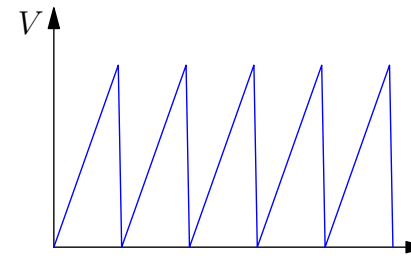
- Infinite “memory” of old inputs
- No silence for low input levels

More realistic alternative: [Leaky Integrate-and-Fire Model](#)

What spiking frequency will this model produce?

Constant input current

$$C \frac{dV}{dt} = I$$



- Time to reach threshold

$$T = \frac{\Theta C}{I}$$

- Firing frequency for constant current

$$f = \frac{1}{T} = \frac{I}{\Theta C}$$

- Re-introduce the *leak* current

$$C \frac{dV}{dt} = -GV + I$$

- Standard form

$$\tau \frac{dV}{dt} = -V + RI$$

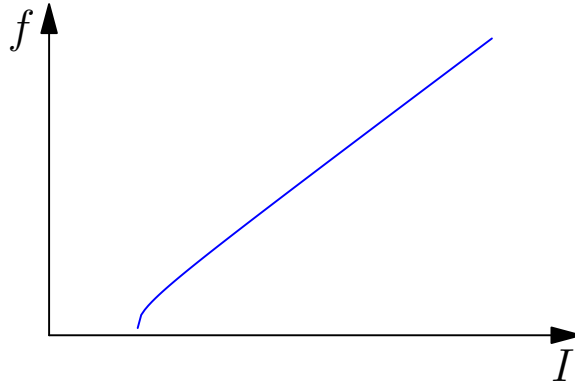
- Time to reach threshold

$$T = \tau \ln \frac{RI}{RI - \Theta}$$

- Firing frequency

$$f = \frac{1}{\tau \ln \frac{RI}{RI - \Theta}}$$

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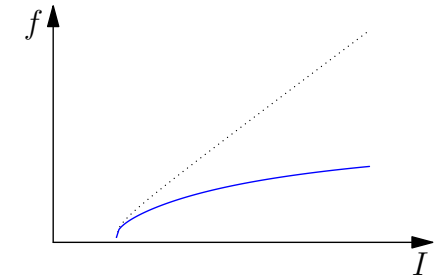


Absolute Refractory Period

Time after a spike when no new spike can be generated, regardless of input

- Model version: Absolute delay, Δ , before integration starts after a reset
- Firing frequency

$$f = \frac{1}{\Delta + T} = \frac{1}{\Delta + \tau \ln \frac{RI}{RI - \Theta}}$$



1 Model Abstraction

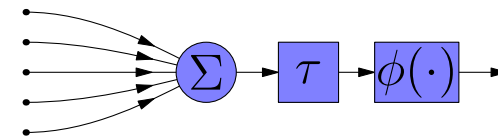
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If actual spikes are irrelevant, can't we simply use a scalar output value representing the frequency?



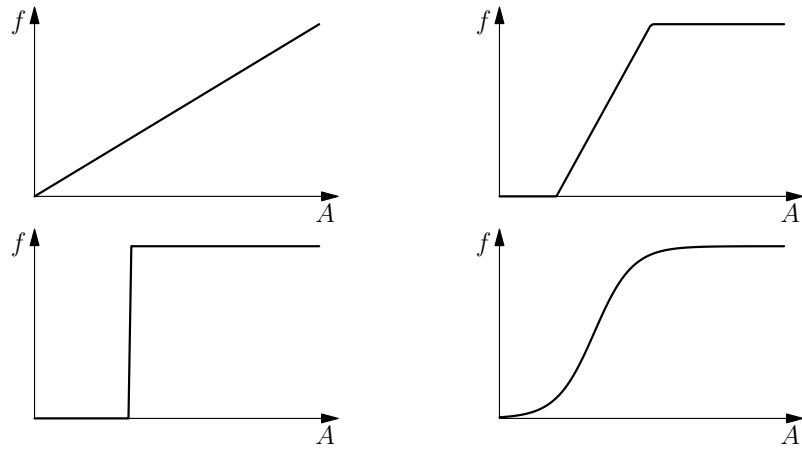
- Leaky integrator

$$\tau \frac{dA}{dt} = I - A$$

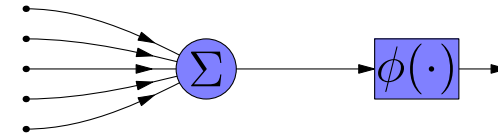
- Squashing function

$$f = \phi(A)$$

Common squashing functions



When dynamics is not important, the integrator can be removed



Standard model used in most Artificial Neural Networks

