CMPU 101 § 3 · Computer Science I

Further Recursion

12 October 2022

Warm up

Exercise

Define a *recursive* function **`add-prev`** that goes through a list of numbers and adds the previous number to the current one, e.g.,

```
››› add-prev([list: 1, 2, 3])
[list: 1, 3, 5]
```

```
fun add-prev(lst :: List<Number>) -> List<Number>:
  doc: "Make a list where each number is the number in the original list plus
the previous number in the original list"

  fun add-prev-helper(l :: List<Number>, prev :: Number) -> List<Number>:
    doc: "Make a list where prev is added to the first number, which is then
added to the next number, and so on"
    cases (List) l:
      | empty => empty
      | link(f, r) => link(f + prev, add-prev-helper(r, f))
    end
  end

  add-prev-helper(lst, 0)

where:
  add-prev([list: ]) is [list: ]
  add-prev([list: 1]) is [list: 1]
  add-prev([list: 1, 2]) is [list: 1, 3]
  add-prev([list: 1, 2, 3]) is [list: 1, 3, 5]
end
```

```
fun add-prev(lst :: List<Number>) -> List<Number>:
  doc: "Make a list where each number is the number in the original list plus
the previous number in the original list"

  fun add-prev-helper(l :: List<Number>, prev :: Number) -> List<Number>:
    doc: "Make a list where prev is added to the first number, which is then
added to the next number, and so on"
    cases (List) l:
      | empty => empty
      | link(f, r) => link(f + prev, add-prev-helper(r, f))
    end
  end

  add-prev-helper(lst, 0)

where:
  add-prev([list: ]) is [list: ]
  add-prev([list: 1]) is [list: 1]
  add-prev([list: 1, 2]) is [list: 1, 3]
  add-prev([list: 1, 2, 3]) is [list: 1, 3, 5]
end
```

This is a wrapper function. It only exists to pass the starting "previous number" value to the function that does the work.

```
fun add-prev(lst :: List<Number>) -> List<Number>:
  doc: "Make a list where each number is the number in the original list plus
the previous number in the original list"

  fun add-prev-helper(l :: List<Number>, prev :: Number) -> List<Number>:
    doc: "Make a list where prev is added to the first number, which is then
added to the next number, and so on"
    cases (List) l:
      | empty => empty
      | link(f, r) => link(f + prev, add-prev-helper(r, f))
    end
  end

  add-prev-helper(lst, 0)

where:
  add-prev([list: ]) is [list: ]
  add-prev([list: 1]) is [list: 1]
  add-prev([list: 1, 2]) is [list: 1, 3]
  add-prev([list: 1, 2, 3]) is [list: 1, 3, 5]
end
```

This helper function recursively goes through the list and it has a second argument, which is whatever the last number it saw was.


SHELDON COOPER
presents
FUN WITH FLAGS
...and lists!

Flags that are just stripes can be represented as lists of colors, e.g.,

```
austria = [list: "red", "white", "red"]
germany = [list: "black", "red", "yellow"]
yemen = [list: "red", "white", "black"]
```

```
fun striped-flag(colors :: List<String>) -> Image:
  doc: "Produce a flag with horizontal stripes"

  cases (List) colors:
    | empty => empty-image
    | link(color, rest) =>
      stripe = rectangle(120, 30, "solid", color)
      above(stripe, striped-flag(rest))
  end
end
```

```
>>> countries = [list: austria, germany, yemen]

>>> map(striped-flag, countries)
[list:  ,  ,  ]
```

A complication

What if we have a different number of stripes?

Consider Ukraine:

```
>>> ukraine = [list: "blue", "yellow"]
>>> striped-flag(ukraine)
```

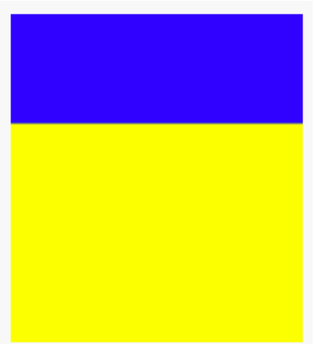
Wrong dimensions!

```
FLAG-WIDTH = 120
FLAG-HEIGHT = 90
```

```
fun striped-flag(colors :: List<String>) -> Image:
  doc: "Produce a flag with horizontal stripes"

  cases (List) colors:
    | empty => empty-image
    | link(color, rest) =>
      height = FLAG-HEIGHT / length(colors)
      stripe = rectangle(FLAG-WIDTH, height, "solid", color)
      above(stripe, striped-flag(rest))
  end
end
```

```
>>> ukraine = [list: "blue", "yellow"]
>>> striped-flag(ukraine)
```



```
>>> germany = [list: "black", "red", "yellow"]
>>> striped-flag(germany)
```

```
FLAG-WIDTH = 120
FLAG-HEIGHT = 90


fun striped-flag(colors :: List<String>) -> Image:
  doc: "Produce a flag with horizontal stripes"

  cases (List) colors:
    | empty => empty-image
    | link(color, rest) =>
      height = FLAG-HEIGHT / length(colors)
      stripe = rectangle(FLAG-WIDTH, height, "solid", color)
      above(stripe, striped-flag(rest))
  end
end
```

What's wrong with this code?

We'll fix this – but not yet!

First, we're going deeper into how we can design functions using lists and recursion.

Going further

Alternating elements

What if we want to select every other element of a list?

```
››› alternating([list: "a", "b", "c", "d"])
[list: "a", "c"]
```

Usually when we want to get just some of the elements of a list, we use **`filter`**, but it's hard to think how we could do that for this problem.

In this case, it's easier to use explicit recursion – though we'll see there's an interesting difference from the recursive functions we've written so far.

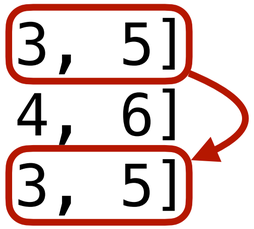
```
fun alternating(lst :: List<Number>) -> List<Number>:
  doc: "Select every other element of the list"
  ...

where:
  ...

end
```

```
fun alternating(lst :: List<Number>) -> List<Number>:
  doc: "Select every other element of the list"
  ...
```

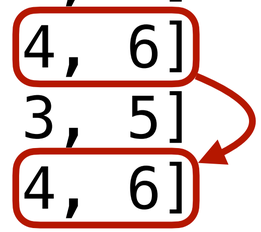
```
where:
  alternating([list: 1, 2, 3, 4, 5, 6]) is [list: 1, 3, 5]
  alternating([list:    2, 3, 4, 5, 6]) is [list: 2, 4, 6]
  alternating([list:       3, 4, 5, 6]) is [list:    3, 5]
  alternating([list:          4, 5, 6]) is [list:    4, 6]
end
```



```
fun alternating(lst :: List<Number>) -> List<Number>:
  doc: "Select every other element of the list"
  ...
```

The result doesn't depend on the next smallest case – it depends on the one after that!

```
where:
  alternating([list: 1, 2, 3, 4, 5, 6]) is [list: 1, 3, 5]
  alternating([list:    2, 3, 4, 5, 6]) is [list: 2, 4, 6]
  alternating([list:       3, 4, 5, 6]) is [list:    3, 5]
  alternating([list:          4, 5, 6]) is [list:    4, 6]
end
```



```
fun alternating(lst :: List<Number>) -> List<Number>:
  doc: "Select every other element of the list"
  cases (List) lst:
    | empty => ...
    | link(f, r) => ...

  end
where:
  alternating([list: 1, 2, 3, 4, 5, 6]) is [list: 1, 3, 5]
  alternating([list:    2, 3, 4, 5, 6]) is [list: 2, 4, 6]
  alternating([list:       3, 4, 5, 6]) is [list:    3, 5]
  alternating([list:          4, 5, 6]) is [list:    4, 6]
end
```

```
fun alternating(lst :: List<Number>) -> List<Number>:
  doc: "Select every other element of the list"
  cases (List) lst:
    | empty => empty
    | link(f, r) => ...
  end
where:
  alternating([list: 1, 2, 3, 4, 5, 6]) is [list: 1, 3, 5]
  alternating([list:    2, 3, 4, 5, 6]) is [list: 2, 4, 6]
  alternating([list:       3, 4, 5, 6]) is [list:    3, 5]
  alternating([list:          4, 5, 6]) is [list:    4, 6]
end
```

```
fun alternating(lst :: List<Number>) -> List<Number>:
  doc: "Select every other element of the list"
  cases (List) lst:
    | empty => empty
    | link(f, r) =>
      cases (List) r:
        | empty => ...

        | link(fr, rr) => ...

      end
  end
where:
  alternating([list: 1, 2, 3, 4, 5, 6]) is [list: 1, 3, 5]
  alternating([list:    2, 3, 4, 5, 6]) is [list: 2, 4, 6]
  alternating([list:       3, 4, 5, 6]) is [list:    3, 5]
  alternating([list:          4, 5, 6]) is [list:    4, 6]
end
```

```
fun alternating(lst :: List<Number>) -> List<Number>:
  doc: "Select every other element of the list"
  cases (List) lst:
    | empty => empty
    | link(f, r) =>
      cases (List) r:
        | empty =>
          [list: f]
        | link(fr, rr) => ...

      end
  end
where:
  alternating([list: 1, 2, 3, 4, 5, 6]) is [list: 1, 3, 5]
  alternating([list:    2, 3, 4, 5, 6]) is [list: 2, 4, 6]
  alternating([list:       3, 4, 5, 6]) is [list:    3, 5]
  alternating([list:          4, 5, 6]) is [list:    4, 6]
end
```

In this case, the list has an odd number of elements!

```
fun alternating(lst :: List<Number>) -> List<Number>:
  doc: "Select every other element of the list"
  cases (List) lst:
    | empty => empty
    | link(f, r) =>
      cases (List) r:
        | empty =>
          [list: f]
        | link(fr, rr) => ...

      end
  end
where:
  alternating([list: 1, 2, 3, 4, 5, 6]) is [list: 1, 3, 5]
  alternating([list:    2, 3, 4, 5, 6]) is [list: 2, 4, 6]
  alternating([list:       3, 4, 5, 6]) is [list:    3, 5]
  alternating([list:          4, 5, 6]) is [list:    4, 6]
end
```

`fr` = *first of the rest. Skip this!*

```
fun alternating(lst :: List<Number>) -> List<Number>:
  doc: "Select every other element of the list"
  cases (List) lst:
    | empty => empty
    | link(f, r) =>
      cases (List) r:
        | empty =>
          [list: f]
        | link(fr, rr) => ...
      end
  end
where:
  alternating([list: 1, 2, 3, 4, 5, 6]) is [list: 1, 3, 5]
  alternating([list:    2, 3, 4, 5, 6]) is [list: 2, 4, 6]
  alternating([list:       3, 4, 5, 6]) is [list:    3, 5]
  alternating([list:          4, 5, 6]) is [list:    4, 6]
end
```

rr = *rest of the rest. This is where we keep going!*

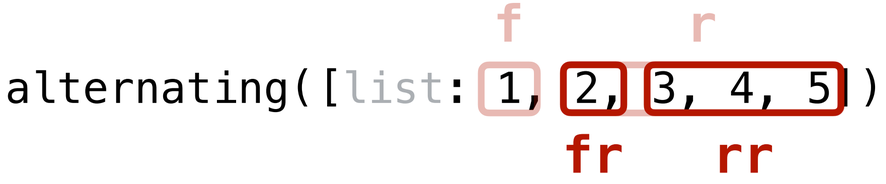
```
fun alternating(lst :: List<Number>) -> List<Number>:
  doc: "Select every other element of the list"
  cases (List) lst:
    | empty => empty
    | link(f, r) =>
      cases (List) r:
        | empty =>
          [list: f]
        | link(fr, rr) =>
          link(f, alternating(rr))
      end
  end
where:
  alternating([list: 1, 2, 3, 4, 5, 6]) is [list: 1, 3, 5]
  alternating([list:    2, 3, 4, 5, 6]) is [list: 2, 4, 6]
  alternating([list:       3, 4, 5, 6]) is [list:    3, 5]
  alternating([list:          4, 5, 6]) is [list:    4, 6]
end
```

```
fun alternating(lst :: List<Number>) -> List<Number>:
  cases (List) lst:
    | empty => empty
    | link(f, r) =>
      cases (List) r:
        | empty =>
          [list: f]
        | link(fr, rr) =>
          link(f, alternating(rr))
      end
  end
end
```

```
alternating([list: 1, 2, 3, 4, 5])
```

```
fun alternating(lst :: List<Number>) -> List<Number>:
  cases (List) lst:
    | empty => empty
    | link(f, r) =>
      cases (List) r:
        | empty =>
          [list: f]
        | link(fr, rr) =>
          link(f, alternating(rr))
      end
  end
end
```

```
fun alternating(lst :: List<Number>) -> List<Number>:
  cases (List) lst:
    | empty => empty
    | link(f, r) =>
      cases (List) r:
        | empty =>
          [list: f]
        | link(fr, rr) =>
          link(f, alternating(rr))
      end
  end
end
```



```
fun alternating(lst :: List<Number>) -> List<Number>:
  cases (List) lst:
    | empty => empty
    | link(f, r) =>
      cases (List) r:
        | empty =>
          [list: f]
        | link(fr, rr) =>
          link(f, alternating(rr))
      end
  end
end
```

```
  alternating([list: 1, 2, 3, 4, 5])

→ link(1,
    alternating([list: 3, 4, 5]))
```

```
fun alternating(lst :: List<Number>) -> List<Number>:
  cases (List) lst:
    | empty => empty
    | link(f, r) =>
      cases (List) r:
        | empty =>
          [list: f]
        | link(fr, rr) =>
          link(f, alternating(rr))
      end
  end
end
```

```
alternating([list: 1, 2, 3, 4, 5])

→ link(1,
    alternating([list: 3, 4, 5]))

→ link(1,
    link(3,
      alternating([list: 5])))
```

```
fun alternating(lst :: List<Number>) -> List<Number>:
  cases (List) lst:
    | empty => empty
    | link(f, r) =>
      cases (List) r:
        | empty =>
          [list: f]
        | link(fr, rr) =>
          link(f, alternating(rr))
      end
  end
end
```

```
  alternating([list: 1, 2, 3, 4, 5])

→ link(1,
    alternating([list: 3, 4, 5]))

→ link(1,
    link(3,
      alternating([list: 5])))

→ link(1,
    link(3,
      [list: 5 ]))
```

```
fun alternating(lst :: List<Number>) -> List<Number>:
  cases (List) lst:
    | empty => empty
    | link(f, r) =>
      cases (List) r:
        | empty =>
          [list: f]
        | link(fr, rr) =>
          link(f, alternating(rr))
      end
  end
end
```

```
  alternating([list: 1, 2, 3, 4, 5])

→ link(1,
    alternating([list: 3, 4, 5]))

→ link(1,
    link(3,
      alternating([list: 5])))

→ link(1,
    link(3,
      [list: 5 ]))

→ [list: 1, 3, 5]
```

Max

What if we want the biggest number in a list?

```
>>> max([list: -10, 0, 8, 4])
8
```

This function is provided by Pyret:

```
>>> import math as M
>>> M.max([list: -10, 0, 8, 4])
8
```

But let's try writing it ourselves!

```
fun max(lst :: List<Number>) -> Number:
  doc: "Return the max number in the list"
  cases (List) lst:
    | empty => raise("The list is empty")
    | link(f, r) =>
      cases (List) r:
        | empty => f
        | else  => num-max(f, max(r))
      end
  end
where:
  max([list: 1, 2, 3]) is 3
  max([list: 3, 1, 2]) is 3
  max([list: 1, 3, 2]) is 3
  max([list: 1, 2, 3]) is 3
end
```

Recursion is all you need?

```
fun sum-of-squares(lst :: List<Number>) -> Number:
  doc: "Add up the square of each number in the list"
  cases (List) lst:
    | empty => 0
    | link(f, r) =>
      (f * f) + sum-of-squares(r)
  end
where:
  sum-of-squares([list: ]) is 0
  sum-of-squares([list: 1, 2]) is 5
end
```

```
fun sum-of-squares(lst :: List<Number>) -> Number:
  doc: "Add up the square of each number in the list"
  M.sum(map(lam(x): x * x end, lst))
where:
  sum-of-squares([list: ]) is 0
  sum-of-squares([list: 1, 2]) is 5
end
```

Just because lists are structurally recursive data doesn't mean you need to design a recursive function.

```
fun avg(lst :: List<Number>) -> Number:
  doc: "Compute the average of the numbers in lst"
  ...
where:
  avg([list: 1, 2, 3, 4]) is 10/4
  avg([list:    2, 3, 4]) is  9/3
  avg([list:       3, 4]) is  7/2
  avg([list:          4]) is  4/1
end
```

```
fun avg(lst :: List<Number>) -> Number:
  doc: "Compute the average of the numbers in lst"
  M.sum(lst) / length(lst)
where:
  avg([list: 1, 2, 3, 4]) is 10/4
  avg([list:    2, 3, 4]) is  9/3
  avg([list:       3, 4]) is  7/2
  avg([list:          4]) is  4/1
end
```

Resolution

```
FLAG-WIDTH = 120
FLAG-HEIGHT = 90

fun striped-flag(colors :: List<String>) -> Image:
  doc: "Produce a flag with horizontal stripes"

  cases (List) colors:
    | empty => empty-image
    | link(color, rest) =>
      height = FLAG-HEIGHT / length(colors)
      stripe = rectangle(FLAG-WIDTH, height, "solid", color)
      above(stripe, striped-flag(rest))
  end
end
```

This is like the denominator for computing the average!

```
FLAG-WIDTH = 120
FLAG-HEIGHT = 90

fun striped-flag(colors :: List<String>) -> Image:
  doc: "Produce a flag with horizontal stripes"

  height = FLAG-HEIGHT / length(colors)

  fun stripe-helper(lst :: List<String>) -> Image:
    cases (List) colors:
      | empty => empty-image
      | link(color, rest) =>
        stripe = rectangle(FLAG-WIDTH, height, "solid", color)
        above(stripe, stripe-helper(rest))
    end
  end

  stripe-helper(colors)
end
```

```
››› map(striped-flag, [list: germany, ukraine])
[list:  ,  ]
```

Code from class:

https://tinyurl.com/101-2022-10-12

Acknowledgments

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