CMPU 101 § 2 · Computer Science I

Trees

26 October 2022

Where are we?

Now we can see how lists are defined:

```
data MyList:
  | my-empty
  | my-link(first, rest :: MyList)
end
```

Self-reference

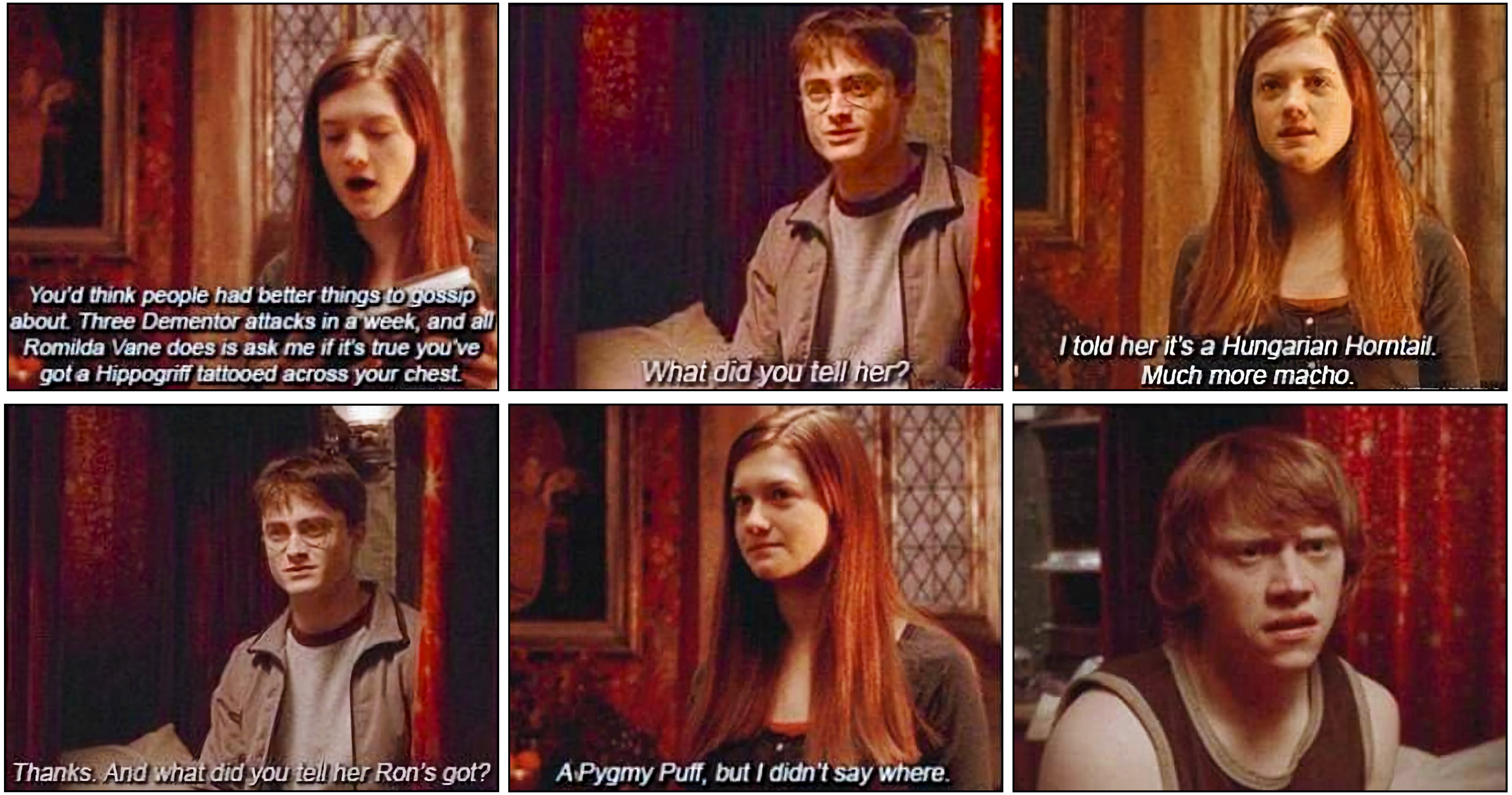
And just like we did for a List, we use this template to write a function that recursively processes the data:

```
fun my-list-fun(ml :: MyList) -> ...:
  doc: "Template for a fn that takes a MyList"
  cases (MyList) ml:
    | my-empty => ...
    | my-link(f, r) =>
      ... f ...
      ... my-list-fun(r) …
  end
where:
   my-list-fun(...) is ...
end
```

Every data definition has a corresponding template.

The more complex the data definition is – lots of variants, recursion, etc. – the more helpful it is to use the template!

Rumor mills


You'd think people had better things to gossip about. Three Dementor attacks in a week, and all Romilda Vane does is ask me if it's true you've got a Hippogriff tattooed across your chest.
What did you tell her?
I told her it's a Hungarian Horntail. Much more macho.
Thanks. And what did you tell her Ron's got?
A Pygmy Puff, but I didn't say where.

Ginny controls the rumor mill

Tracking rumors

Suppose we want to track gossip in a rumor mill.

Tracking rumors

Suppose we want to track gossip in a rumor mill.

Pansy

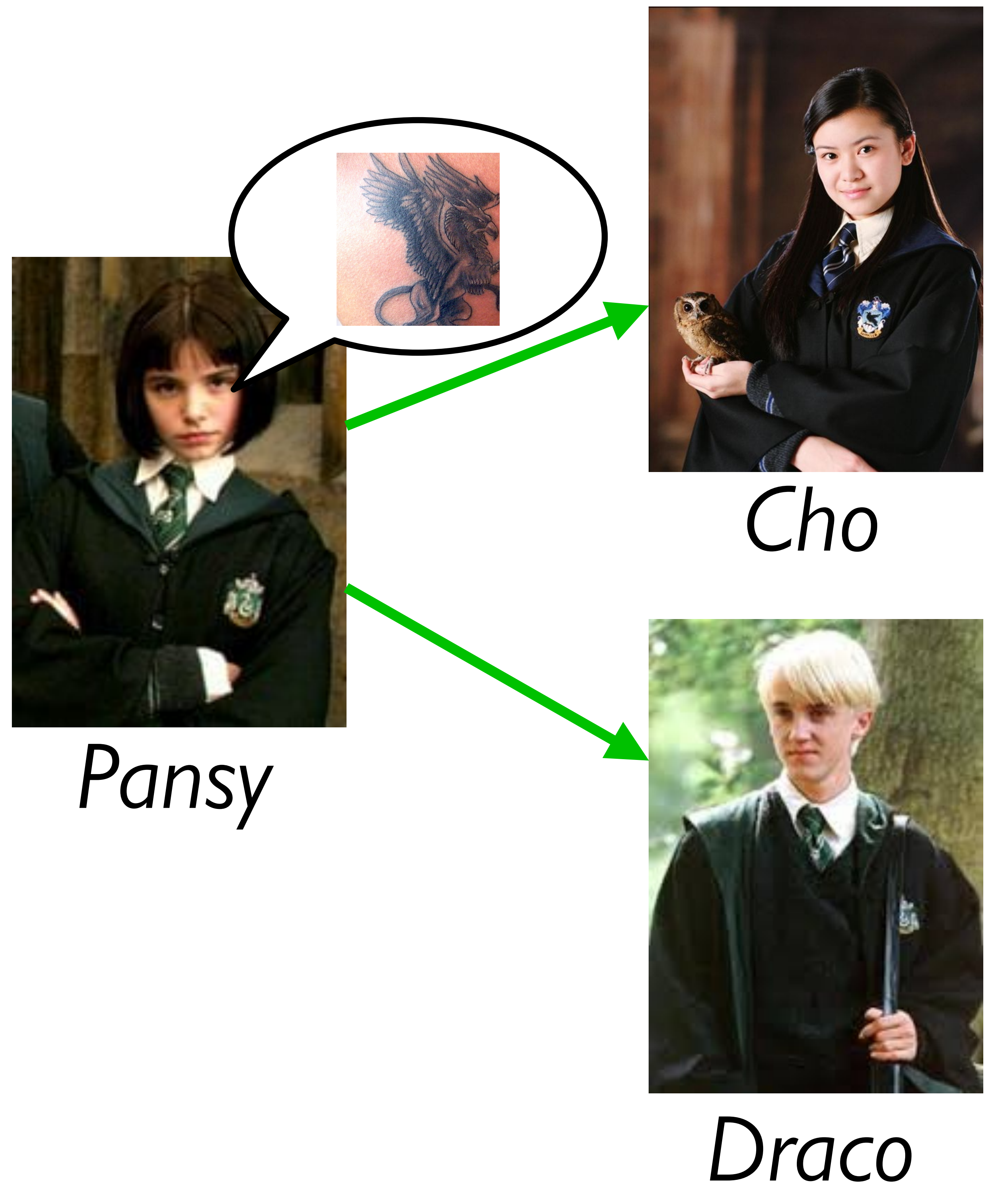
Tracking rumors

Suppose we want to track gossip in a rumor mill.

Pansy

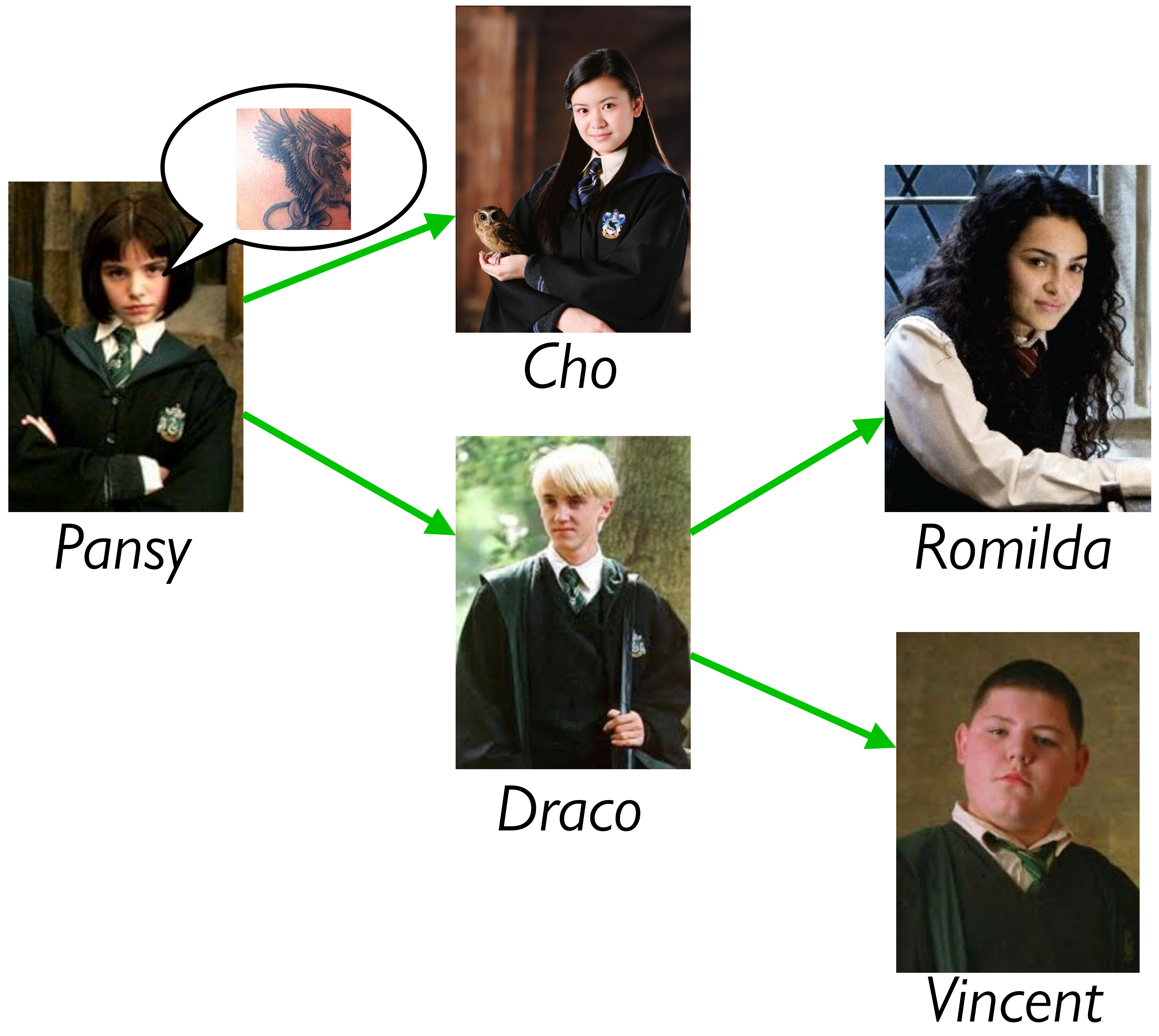
Tracking rumors

Suppose we want to track gossip in a rumor mill.



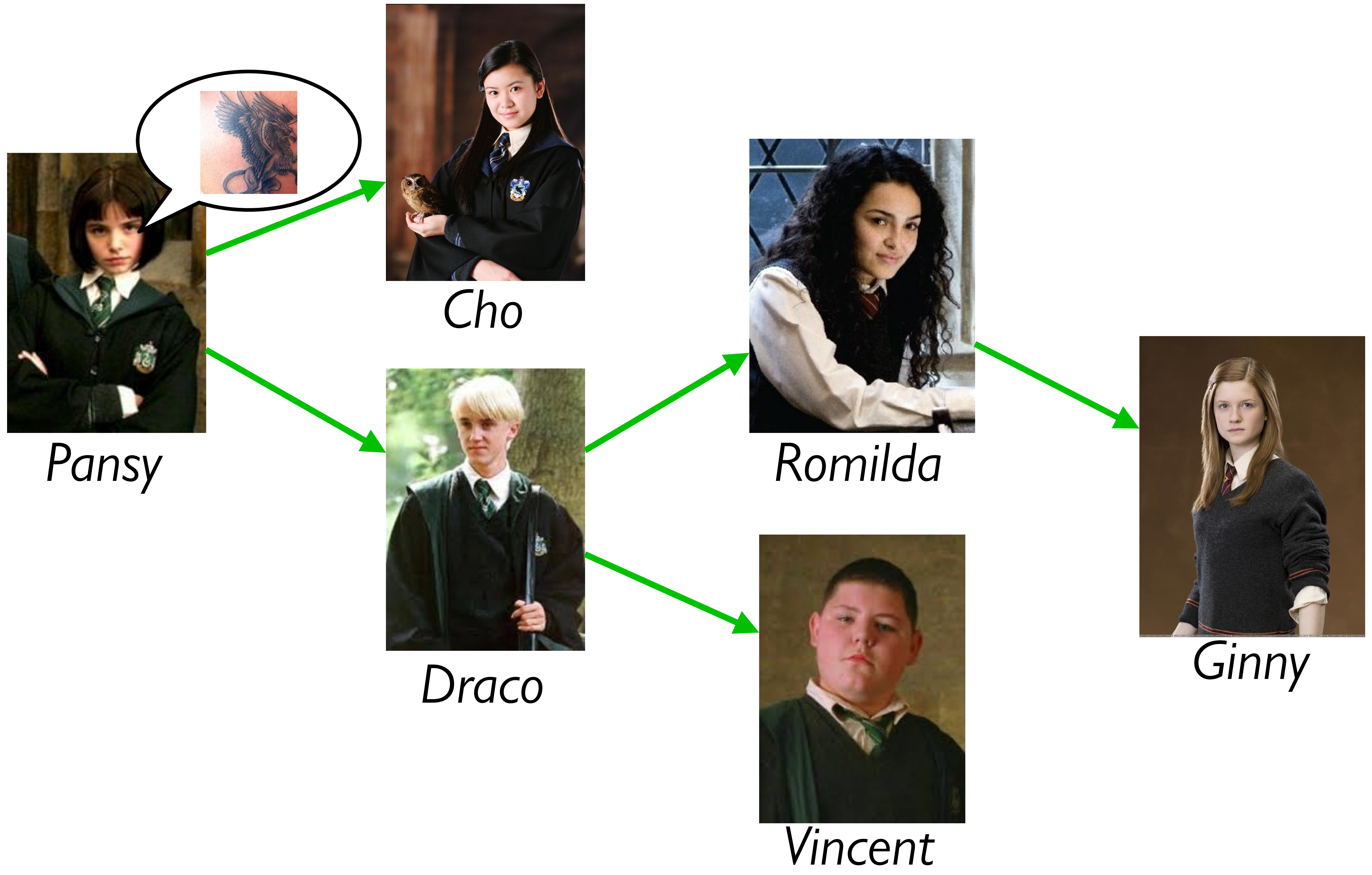
Tracking rumors

Suppose we want to track gossip in a rumor mill.



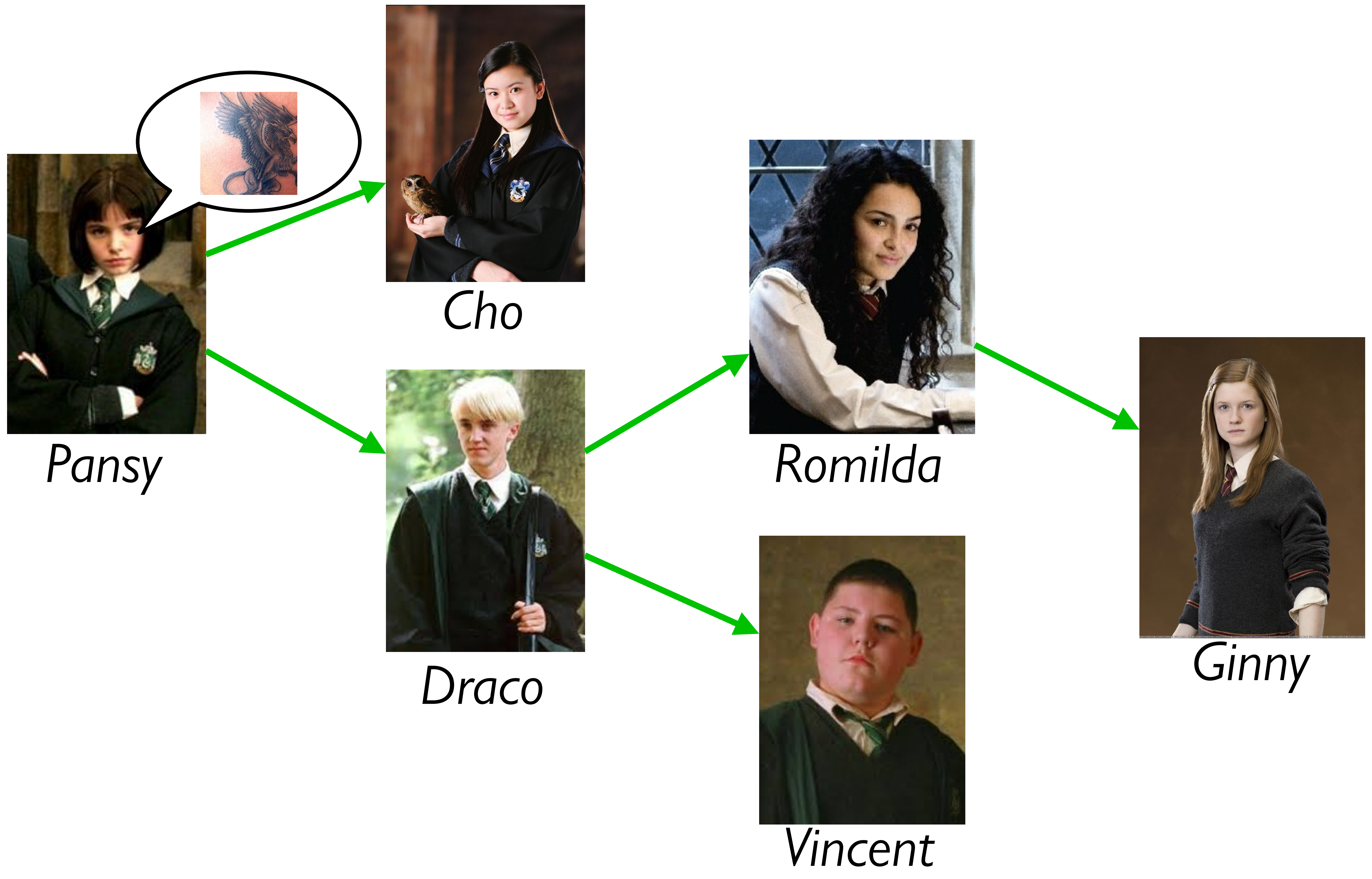
Tracking rumors

Suppose we want to track gossip in a rumor mill.



Tracking rumors

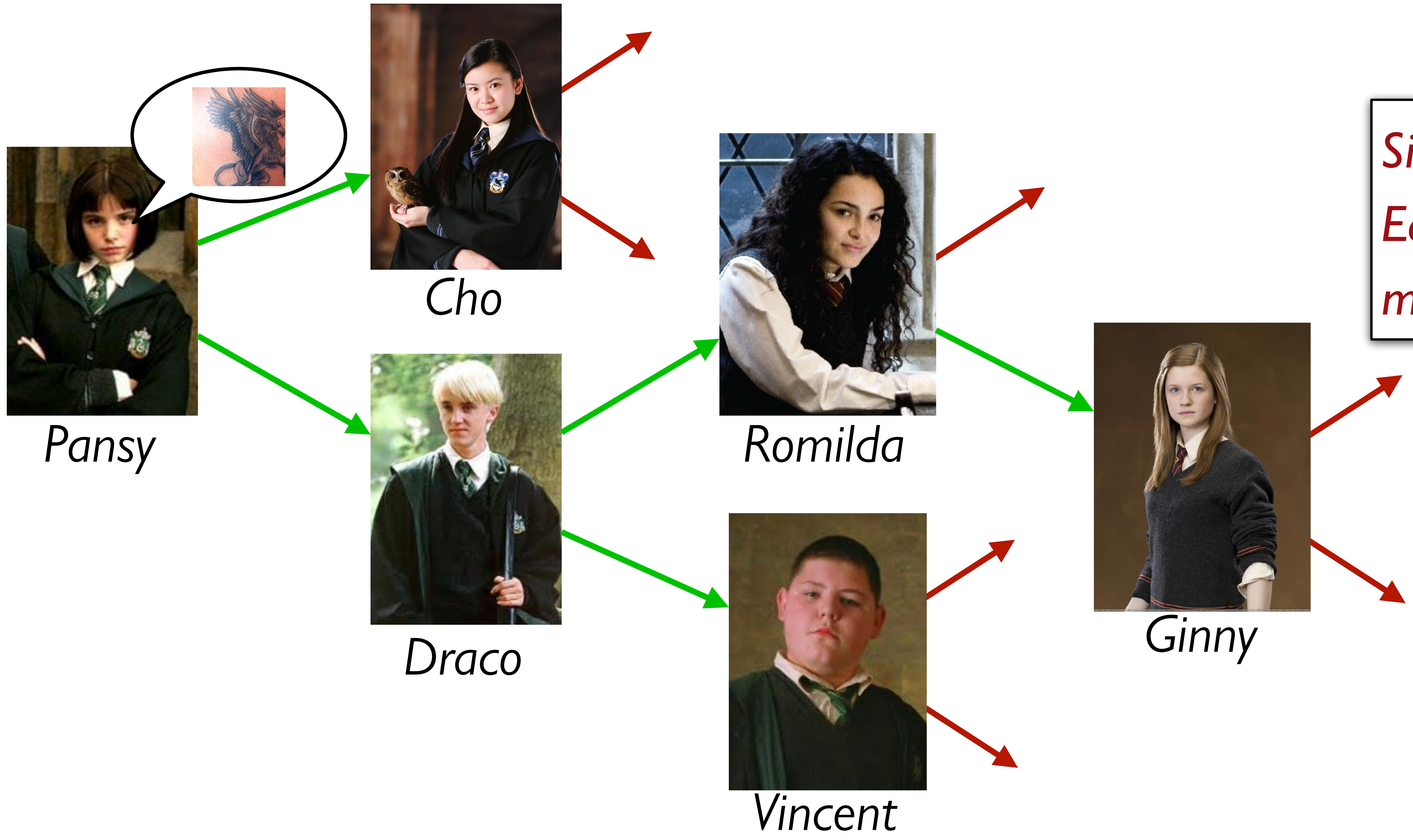
Suppose we want to track gossip in a rumor mill.



Simplifying assumption: Each person tells at most two others

Tracking rumors

Suppose we want to track gossip in a rumor mill.

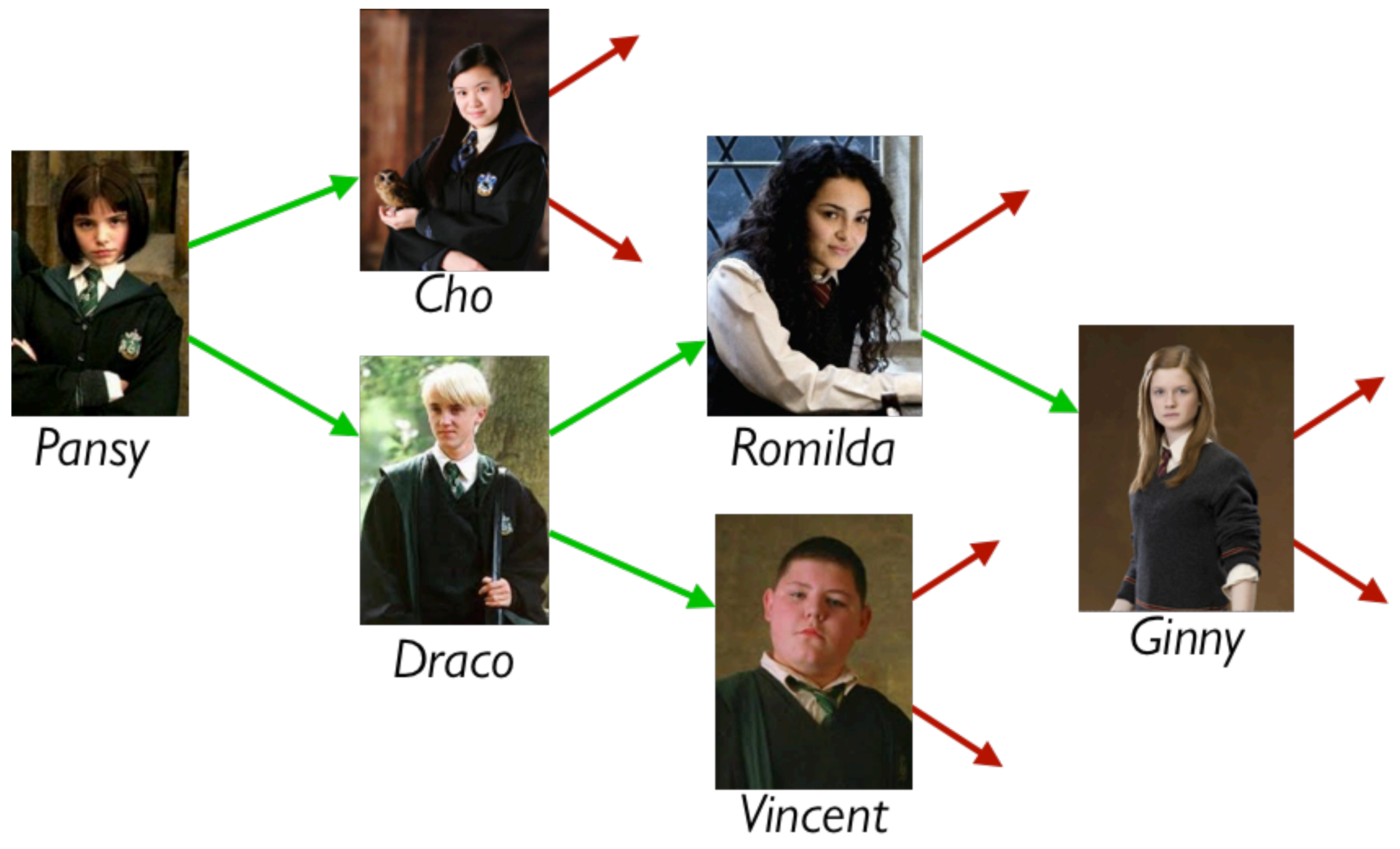


Simplifying assumption: Each person tells at most two others

If you ignore my silly Harry Potter example, this is a pretty serious problem.

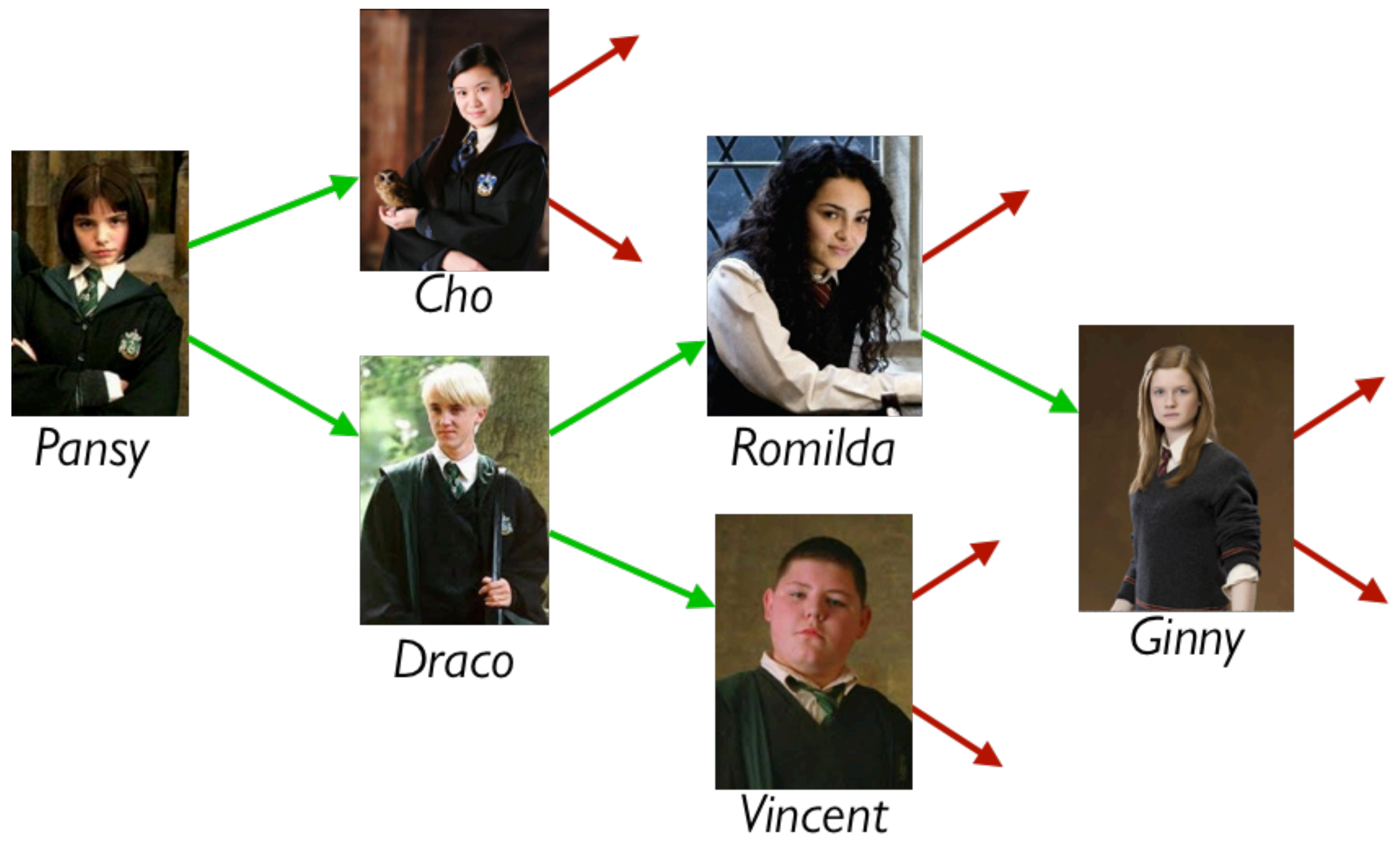
A lot of research right now is focused on building models of how information – and misinformation! – spreads through social networks, both in person and online.

Representing rumor mills



Is a rumor mill simply a list of people?

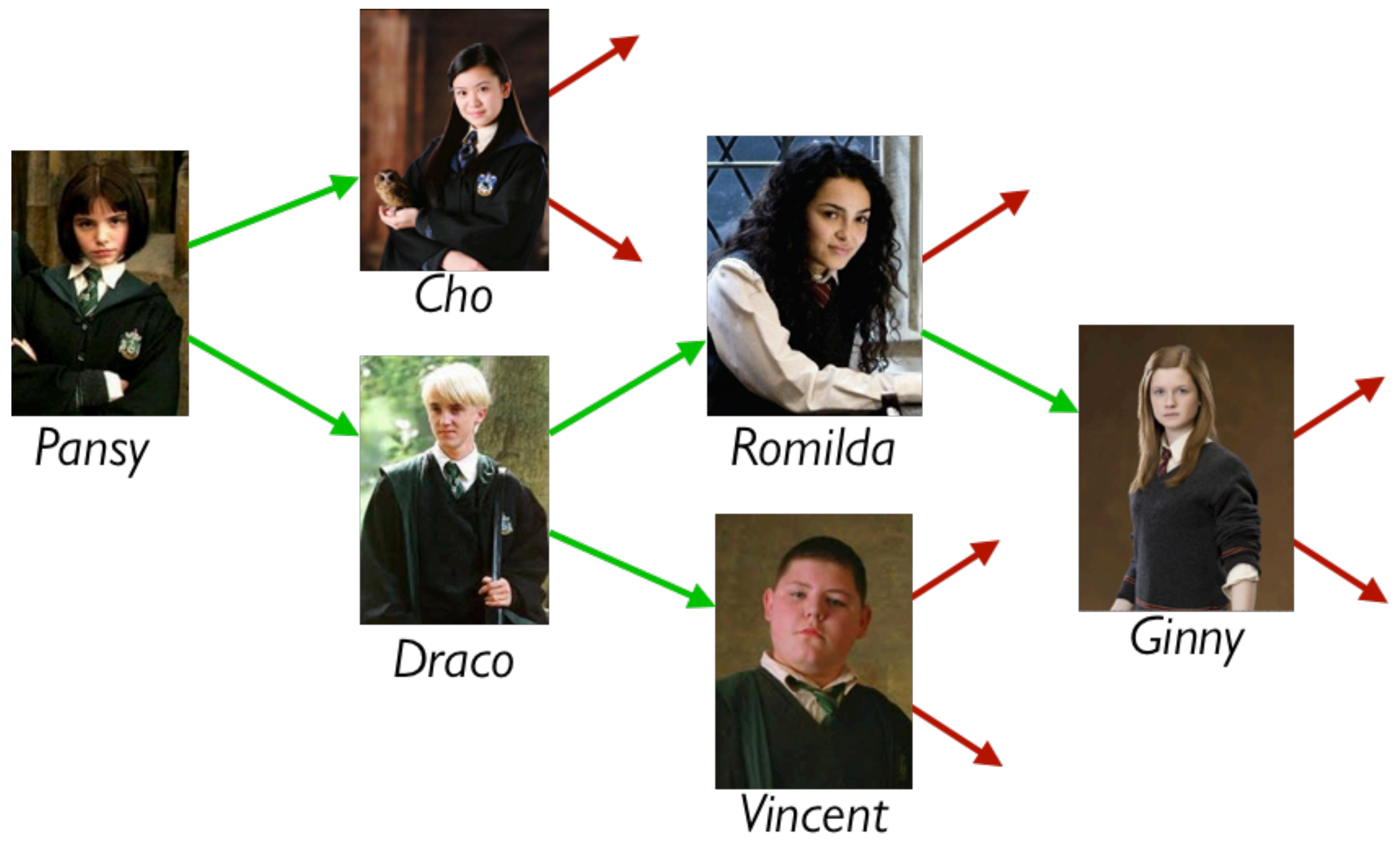
Representing rumor mills



Is a rumor mill simply a list of people?

No, because there are relationships among the people.

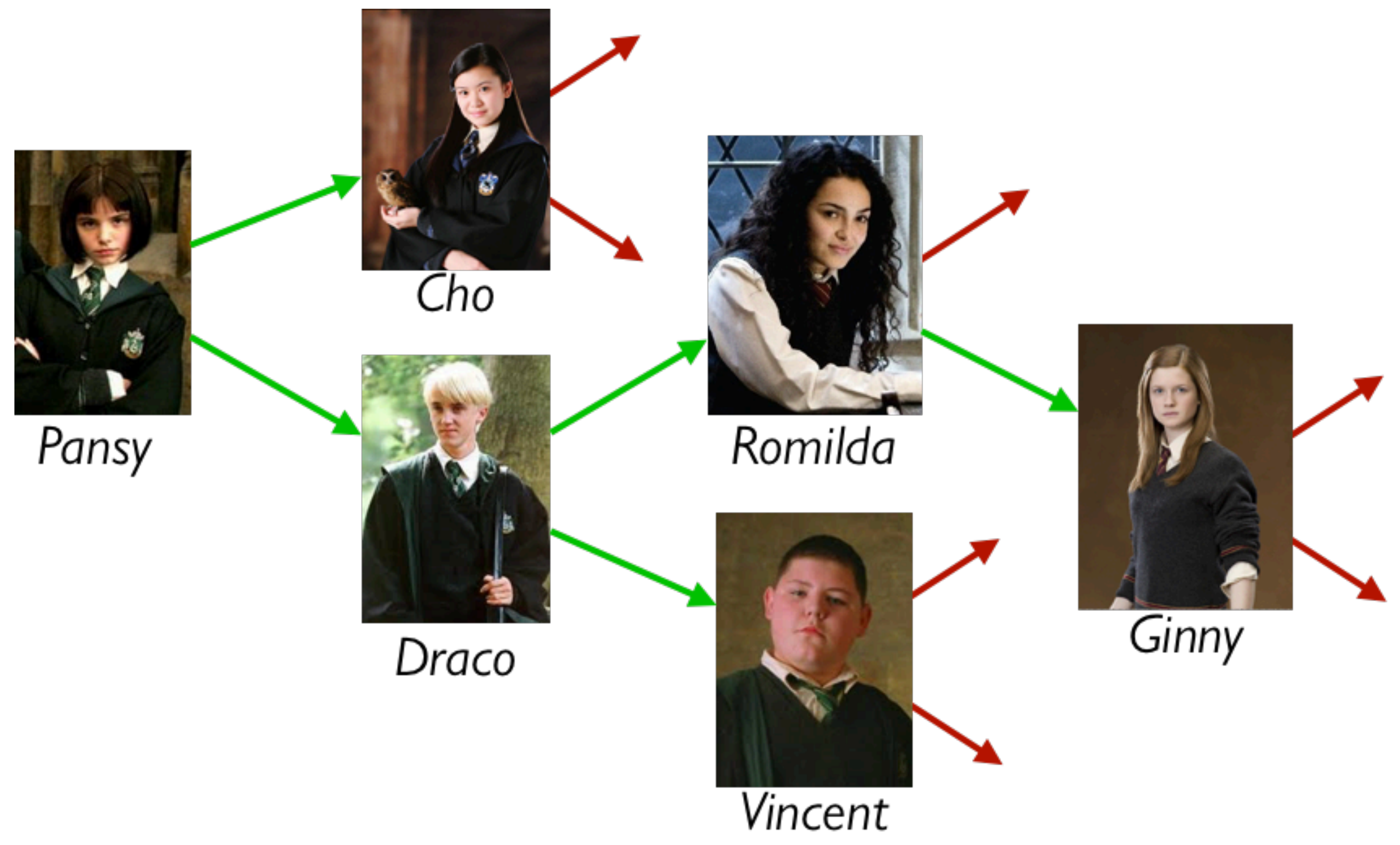
Representing rumor mills



We could represent these relations with a table, e.g.,

name :: String	***next1 :: String***	***next2 :: String***
`"Pansy"`	`"Cho"`	`"Draco"`
`"Cho"`		
…	…	…

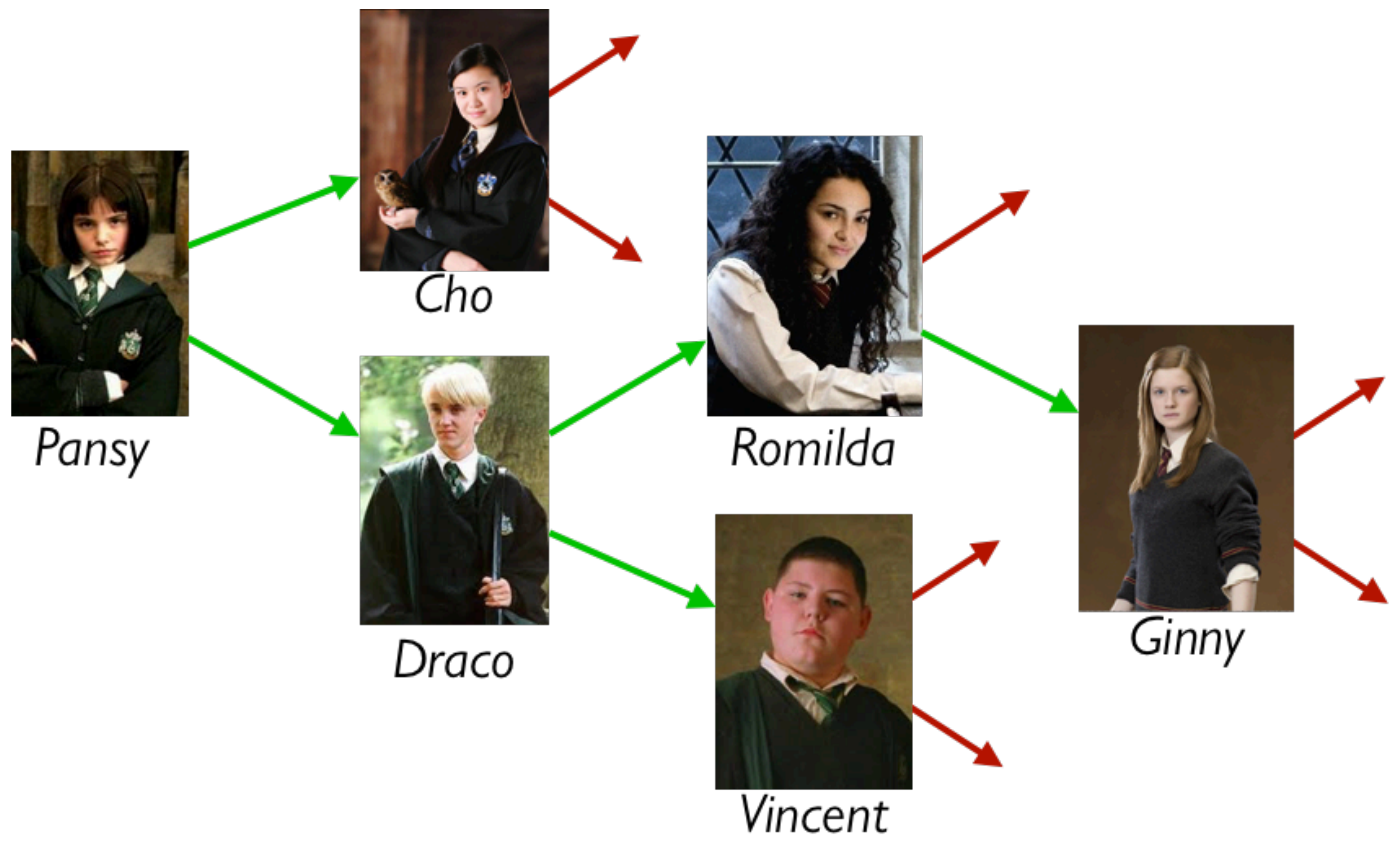
Representing rumor mills



Using a table doesn’t give us any straightforward way to process the rumor mill.

Could we use something *like* a list but representing the relations?

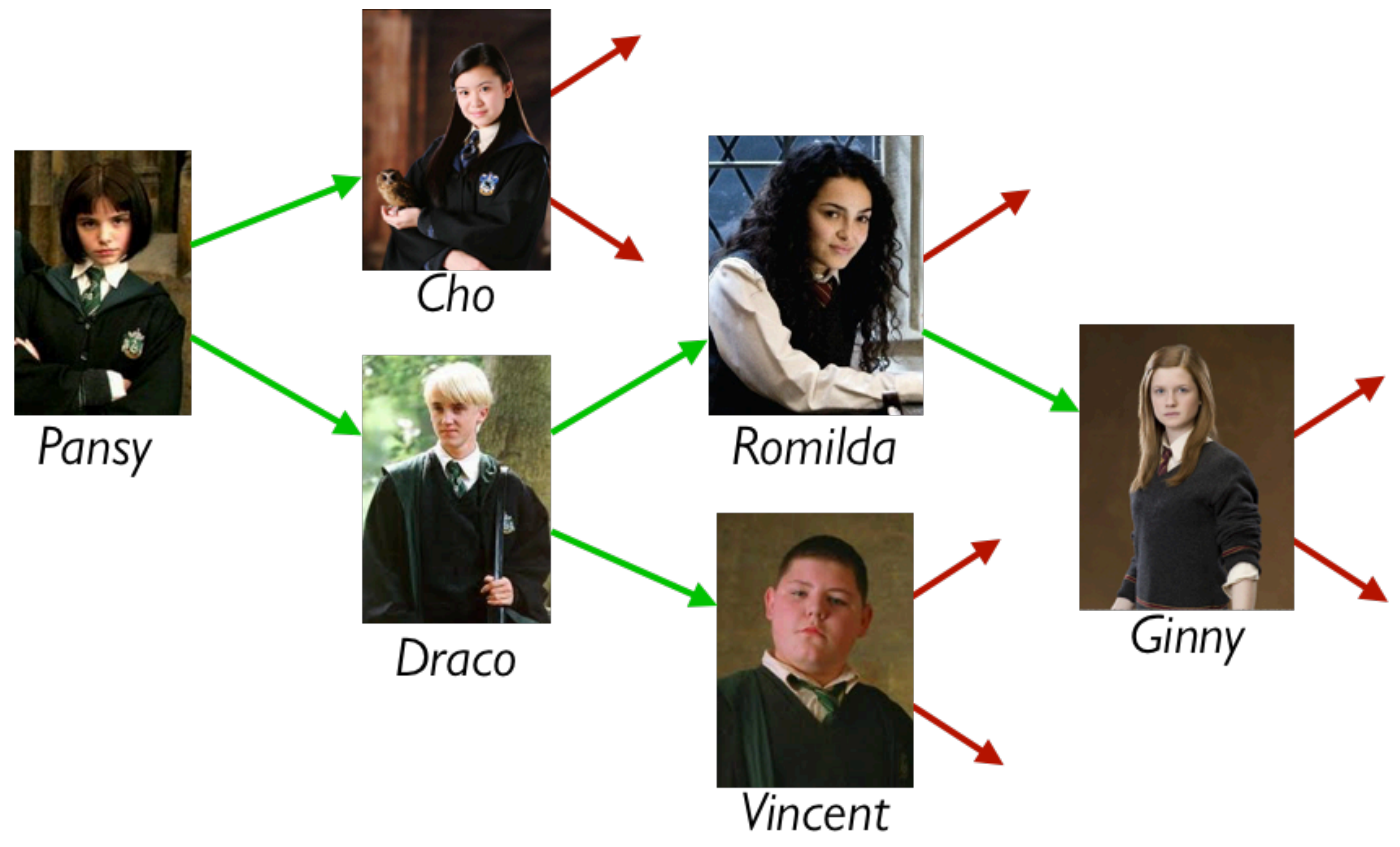
Representing rumor mills



```
data Person:
  | person(name :: String, next1 :: Person, next2 :: Person)
end
```

How about this?

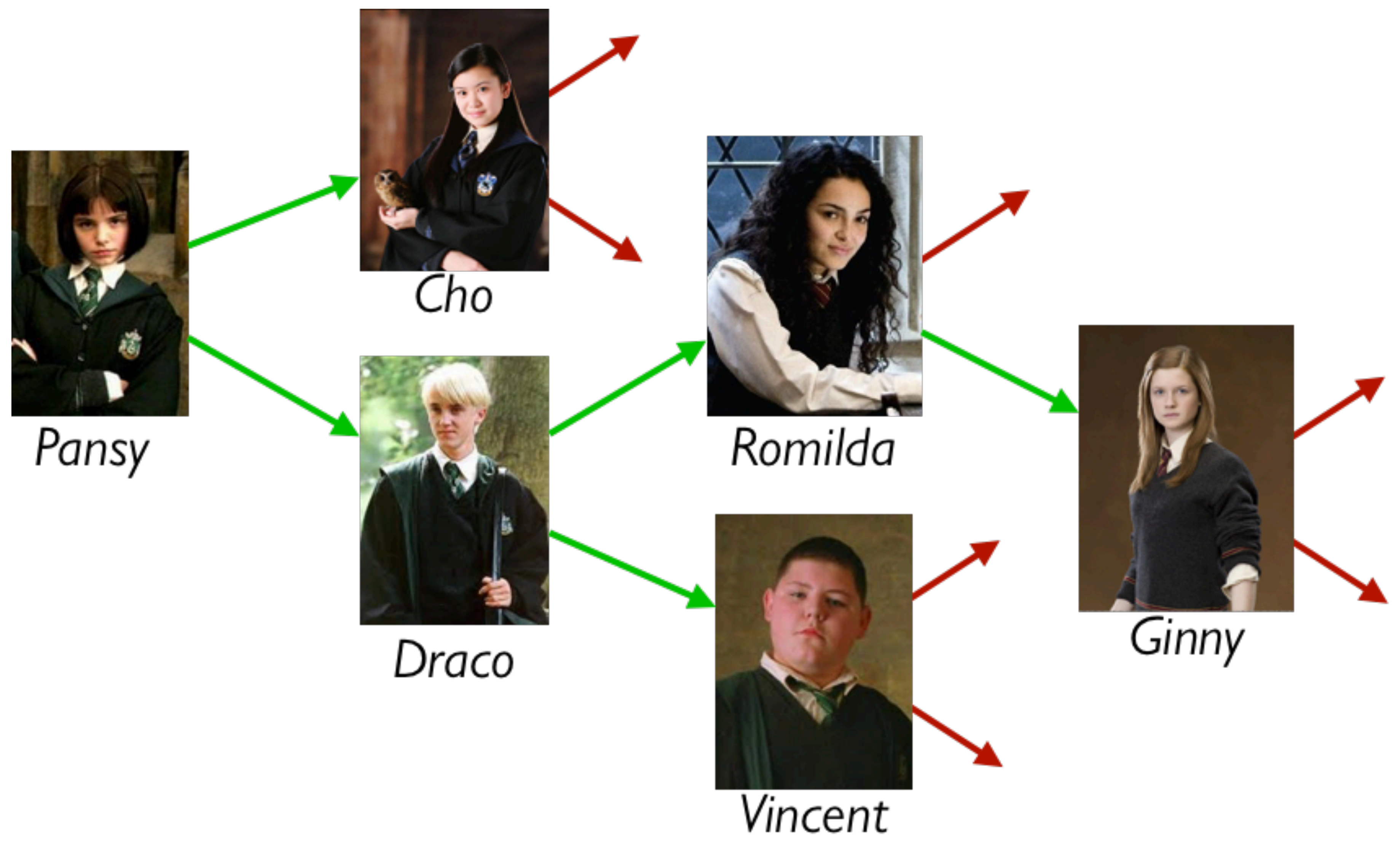
Representing rumor mills



```
data Person:
  | person(name :: String, next1 :: Person, next2 :: Person)
end
```

Some people don't gossip to anyone else – the red arrows above.

Representing rumor mills



```
data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
```

How about this?

Example rumor mills

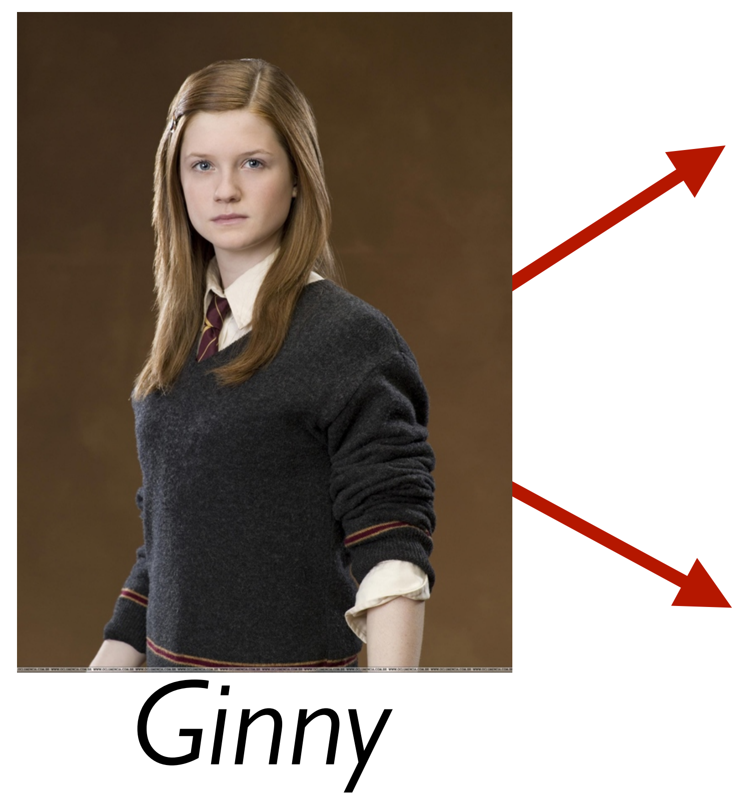
```
data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
```

```
no-one
```

Example rumor mills

```
data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
```

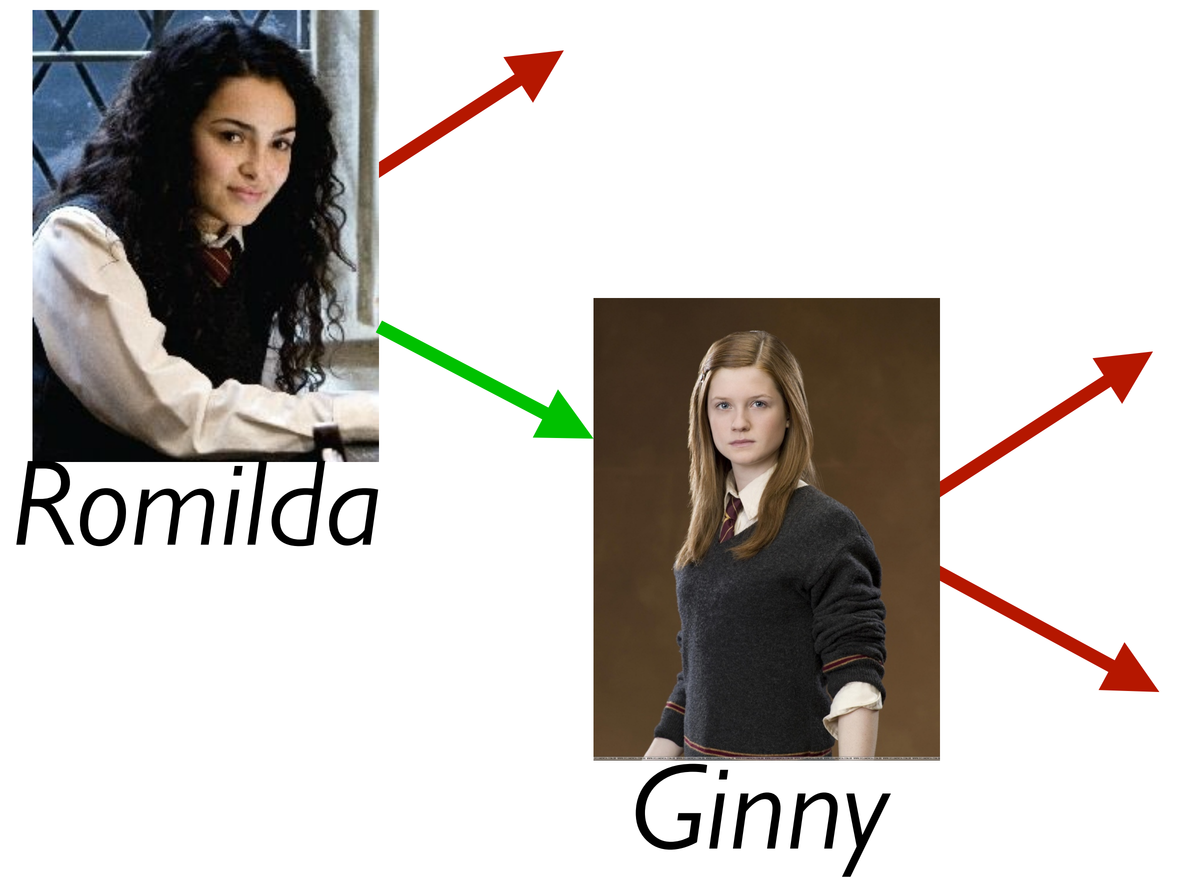
```
gossip("Ginny", no-one, no-one)
```



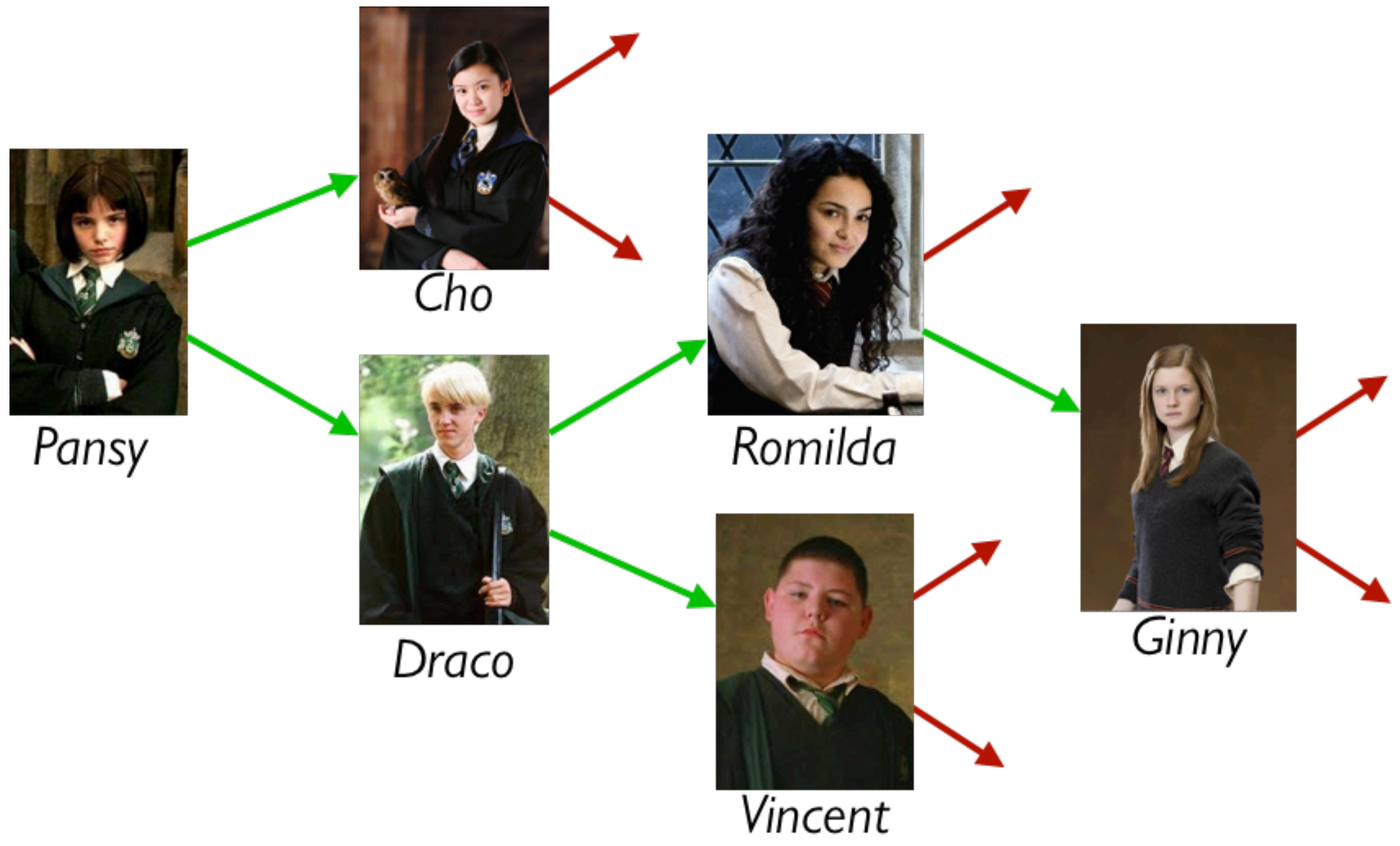
Example rumor mills

```
data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
```

```
gossip("Romilda",
  no-one,
  gossip("Ginny", no-one, no-one))
```



```
gossip("Pansy",
  gossip("Cho", no-one, no-one)
  gossip("Draco",
    gossip("Romilda",
      no-one
      gossip("Ginny", no-one, no-one))
    gossip("Vincent", no-one, no-one)))
```



Example using names for parts:

```
GINNY-MILL =
  gossip("Ginny", no-one, no-one)

ROMILDA-MILL =
  gossip("Romilda", no-one, GINNY-MILL)

VINCENT-MILL =
  gossip("Vincent", no-one, no-one)

DRACO-MILL =
  gossip("Draco", ROMILDA-MILL, VINCENT-MILL)

CHO-MILL =
  gossip("Cho", no-one, no-one)

PANSY-MILL =
  gossip("Pansy", CHO-MILL, DRACO-MILL)
```

A *RumorMill* is a type of structure called a ***tree***.

Each element in the tree is called a ***node***.

The first node in the tree is called the ***root***.

A node with no children is called a *leaf*.

Like a list, a tree is recursive: Every subtree is a tree.

Programming with rumors

```
data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
```

Programming with rumors

```
data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
```

Self-reference × 2

For each element, there's not just one "next" element; there are two!

Programming with rumors

Self-reference × 2

```
data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
#|
fun rumor-mill-template(rm :: RumorMill) -> ...:
  doc: "Template for a function with a RumorMill as input"
  cases (RumorMill) rm:
    | no-one => ...
    | gossip(name, n1, n2) =>
      ... name
      ... rumor-mill-template(n1)
      ... rumor-mill-template(n2)
  end
end
|#
```

Programming with rumors

Self-reference × 2

```
data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
#|
fun rumor-mill-template(rm :: RumorMill) -> ...:
  doc: "Template for a function with a RumorMill as input"
  cases (RumorMill) rm:
    | no-one => ...
    | gossip(name, n1, n2) =>
      ... name
      ... rumor-mill-template(n1)
      ... rumor-mill-template(n2)
  end
end
|#
```

Natural recursion × 2

Starter file:

`https://code.pyret.org/editor#share=1H80RHPhzm15GW__9yAP-DVJiRE_7wHas&v=22f3b65`

Rumor program examples

Design the function **`is-informed`** that takes a person’s name and a rumor mill and determines whether the person is part of the rumor mill.

Rumor program examples

Design the function **`rumor-delay`** that takes a rumor mill and determines the maximum number of days required for a rumor to reach everyone, assuming that each person waits a day before passing on a rumor.

Solutions:

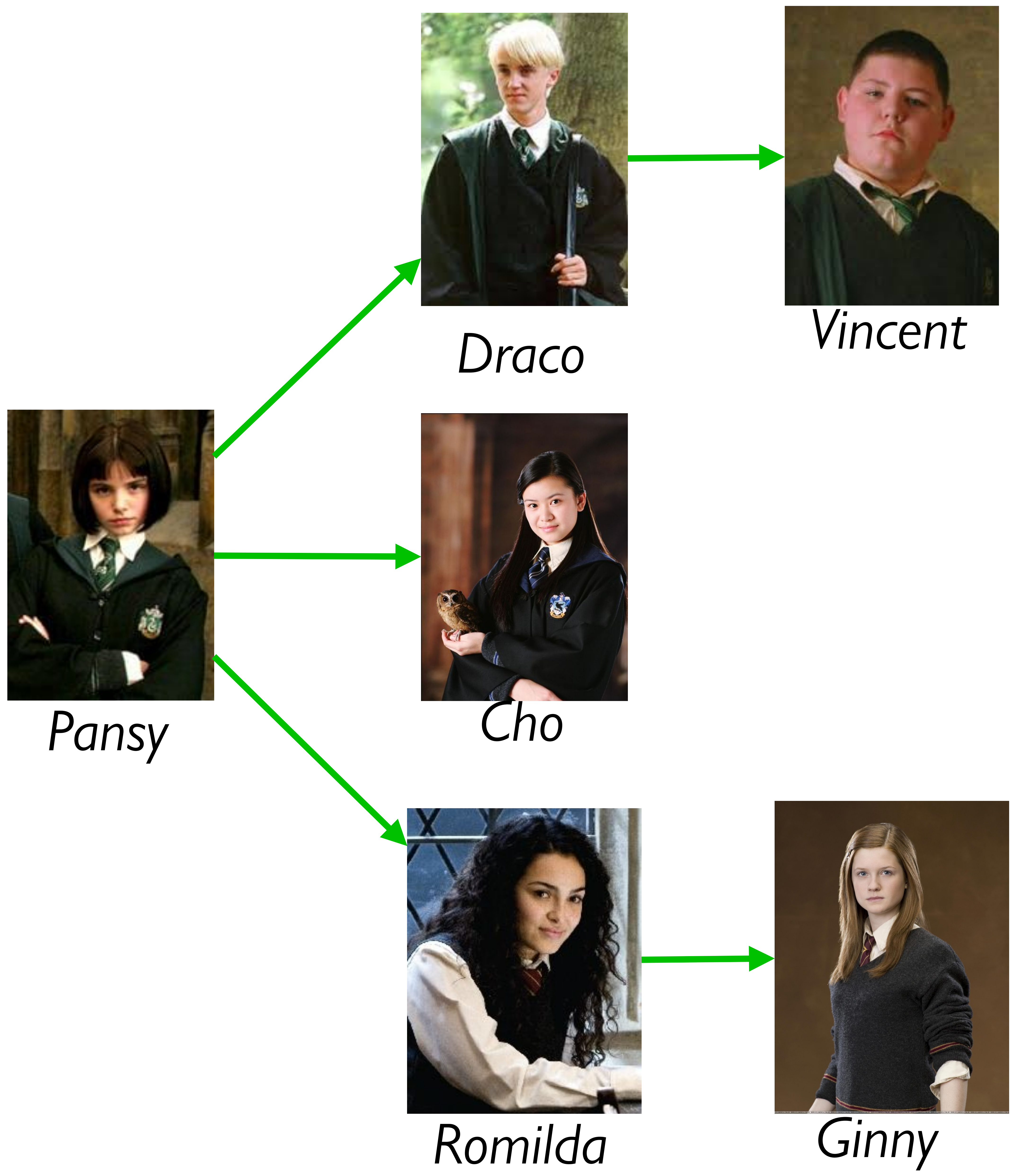
```
https://code.pyret.org/
editor#share=1hFXf0kyaVx9akJlL3Gr19bWKFhCe9rRQ&v=22f3b65
```

A more realistic rumor mill

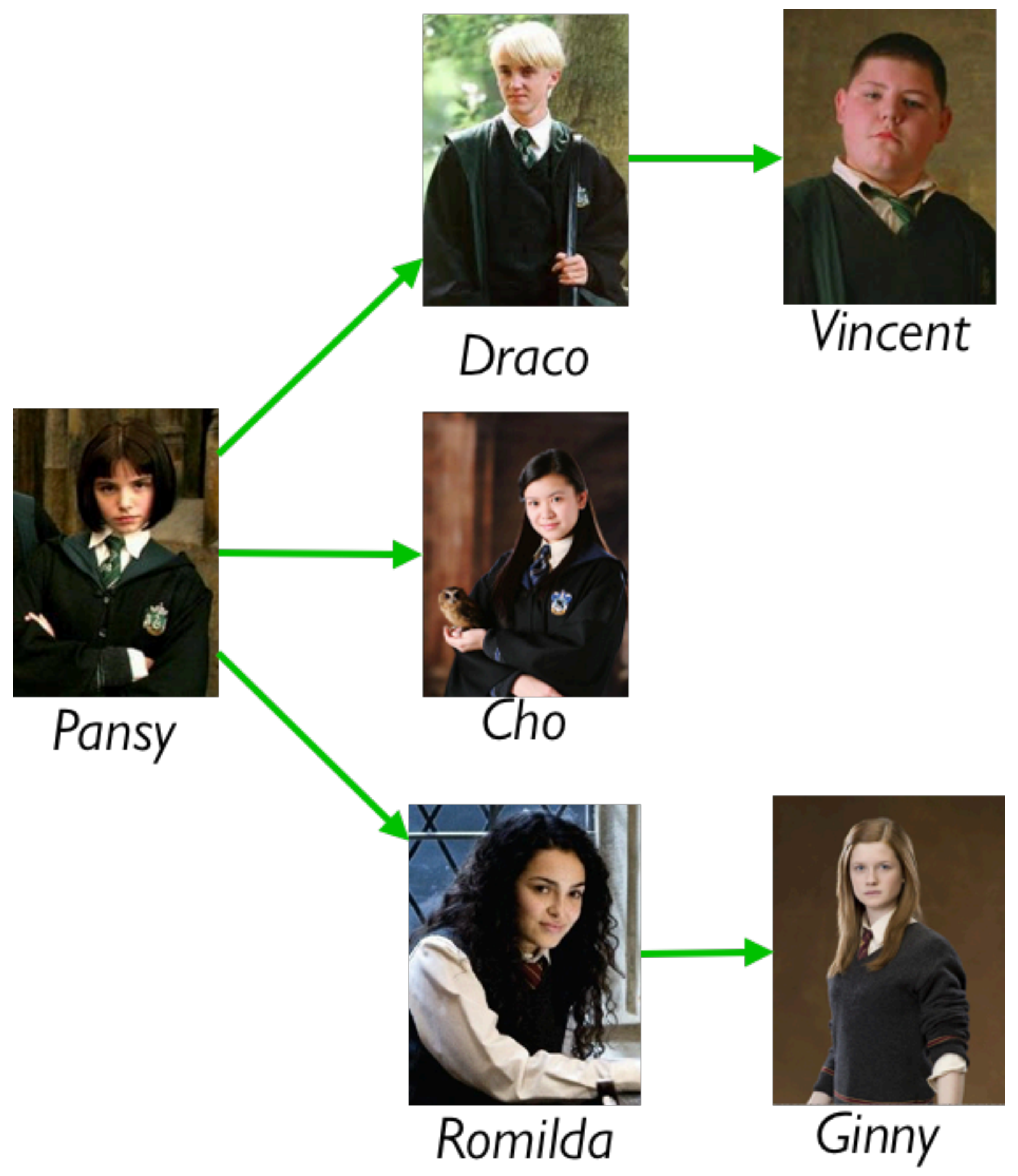
In our rumor mill, we restricted each person to spread gossip to at most two other people.

This isn’t very realistic; some gossips talk to lots of people!

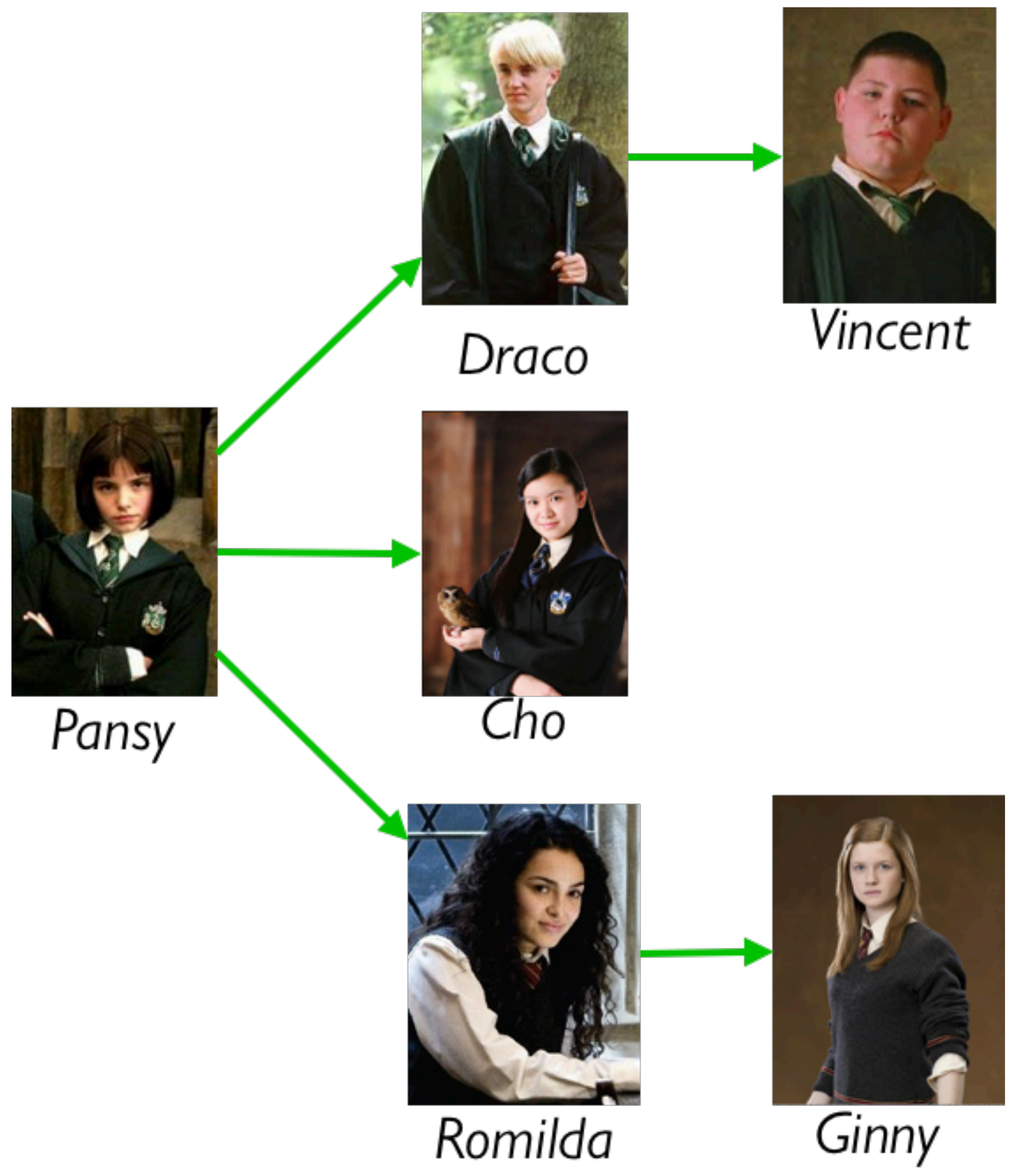
Let each gossip talk to any number of people:



How do we represent an arbitrary number of gossip connections?



How do we represent an arbitrary number of gossip connections?



```
data Gossip:
  | gossip(name :: String, next :: List<Gossip>)
end
```

```
data Gossip:
  | gossip(name :: String, next :: List<Gossip>)
end

#|
fun gossip-template(g :: Gossip) -> ...:
  ... gossip.name
  ... log-template(g.next)
end

fun log-template(l :: List<Gossip>) -> ...:
  cases (List) l:
    | empty => ...
    | link(f, r) =>
      ... gossip-template(f)
      ... log-template(r)
  end
end
|#
```

Starter file:

```
https://code.pyret.org/
editor#share=1gwQ4AVUMHm4vg5JJ_1aIQrpkx0kytxdi&v=22f3b65
```

Design **`count-gossips`** which takes a gossip and returns the number of people informed by the gossip (including the starting person).

Solutions:

```
https://code.pyret.org/
editor#share=1wfB4lTc5b7dMUV4f1QxzwMaMU9-fMn9L&v=22f3b65
```

Acknowledgments

This lecture incorporates material from:

J. K. Rowling, *Harry Potter and the Half-Blood Prince*

Marc Smith, Vassar College

Jonathan Gordon, Vassar College