CMPU 101 § 53 · Computer Science I

Generating Fractals

27 February 2023

Where are we?

```
data List:
  | empty
  | link(first :: Any, rest :: List)
end
```

Self-reference

Recursive data

```
fun list-fun(lst :: List) -> ...:
 cases (List) lst:
   | empty => ...
   | link(f, r) =>
     ... f ...
     ... list-fun(r) ...
  end
end
```

Natural recursion

Recursive functions

The same idea holds for lists, binary trees, trinary trees, *n*-ary trees, and all kinds of other recursive data types: *The structure of the function follows the structure of the data.*

The recursive functions we've written have used *structural* (or *natural*) *recursion*.

In structural recursion, each recursive call takes some sub-piece of the data.

Going through a list, we keep taking the **rest** of the list.

Going through a tree, we keep looking at the sub-trees.

Generative recursion

In *generative recursion*, the recursive cases are generated based on the problem to be solved.

Generative recursion can be harder because neither the base nor recursive cases follow from a data definition.

Template for generative recursion

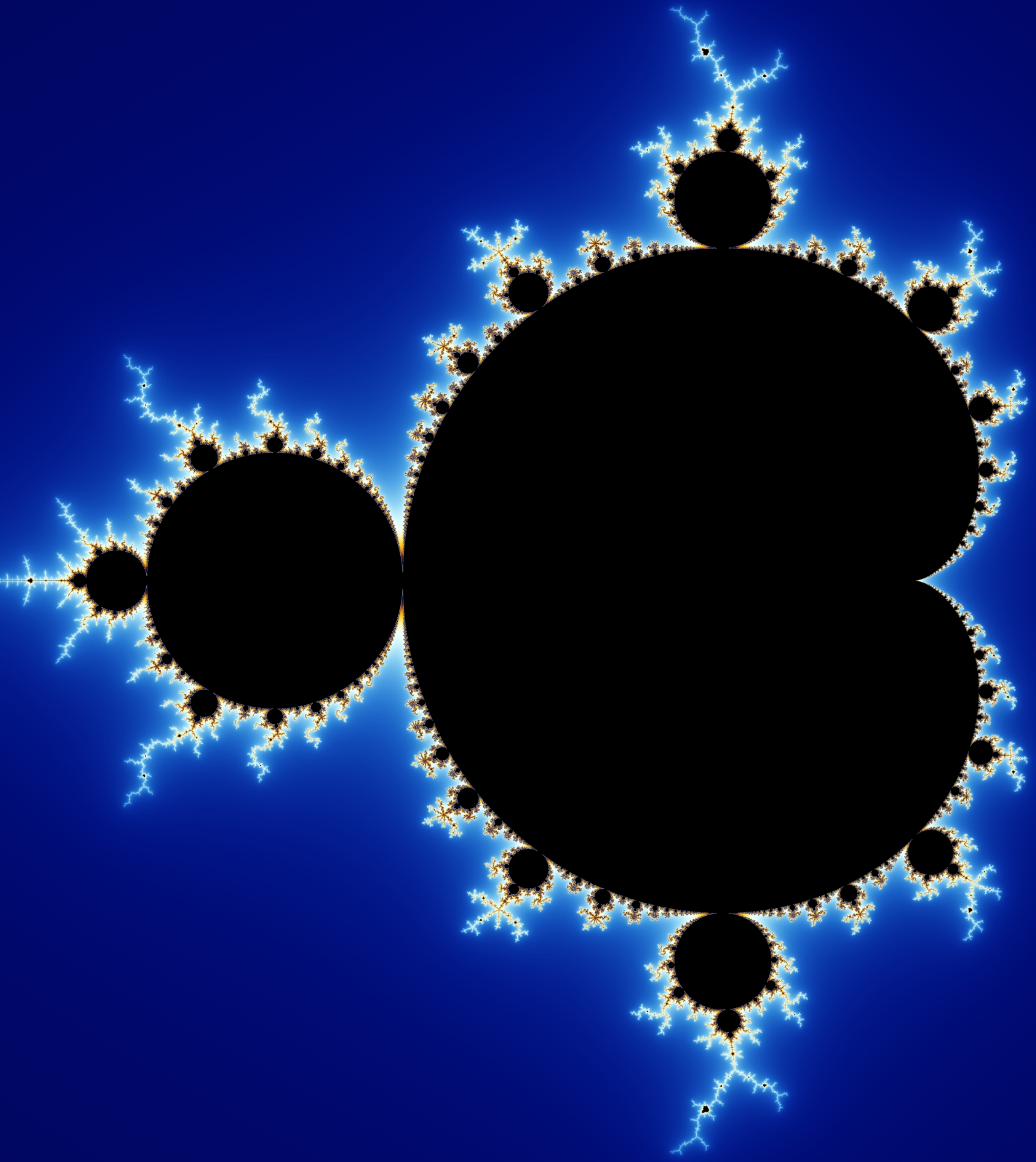
```
fun problem-solver(d) -> ...:
  if is-trivial(d):
    # Base case: The computation is in some way
    #   trivial.
    ... d ...
  else:
    # Recursive case: Transform the data d to generate
    #   new problems.
    combiner(
      ...d...,
      problem-solver(transform(d)),
      ...)
  end
end
```

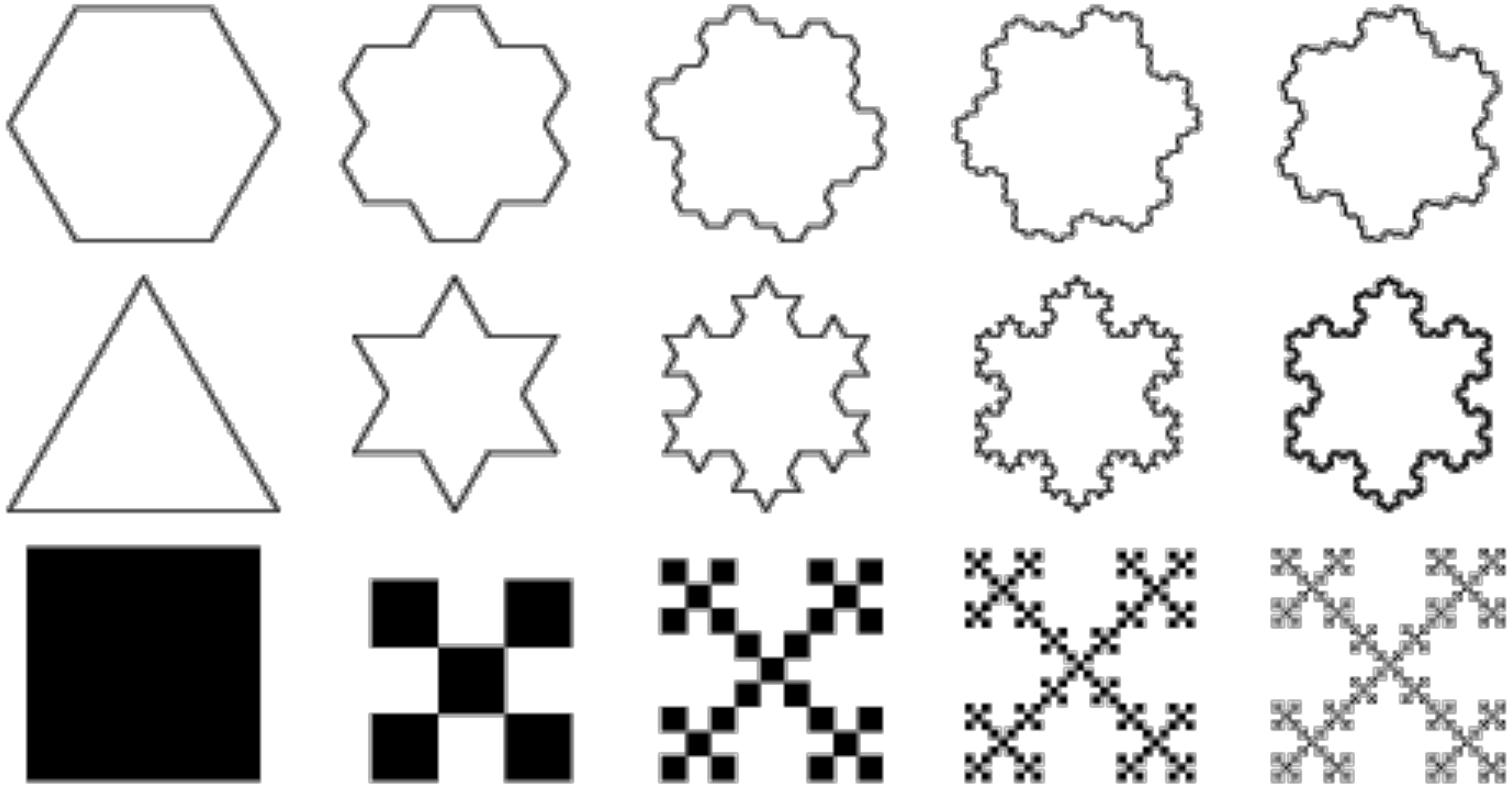
When you write a function with generative recursion you need to be careful about ***termination*** – how do you know you'll ever reach the base case?

Fractals

“A fractal is a way of seeing infinity.”

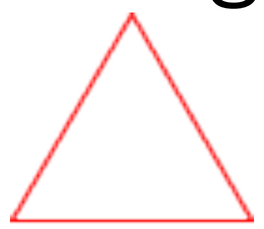
Benoit Mandelbrot



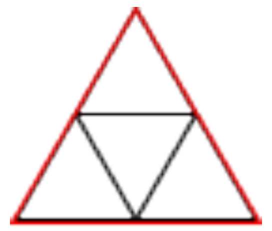


Design a function that consumes a number and produces a *Sierpiński triangle* of that size:

Start with an equilateral triangle with side length *s*:

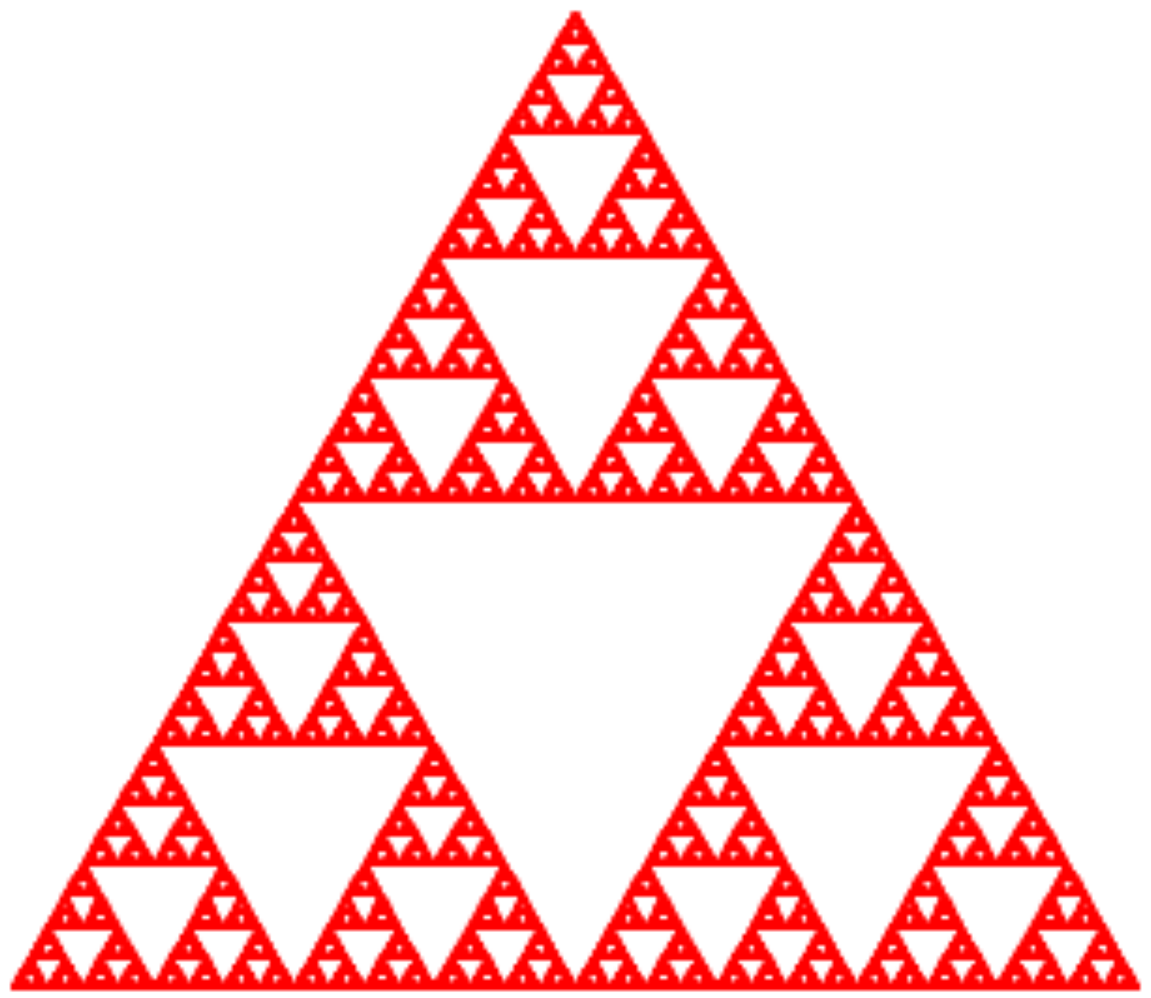


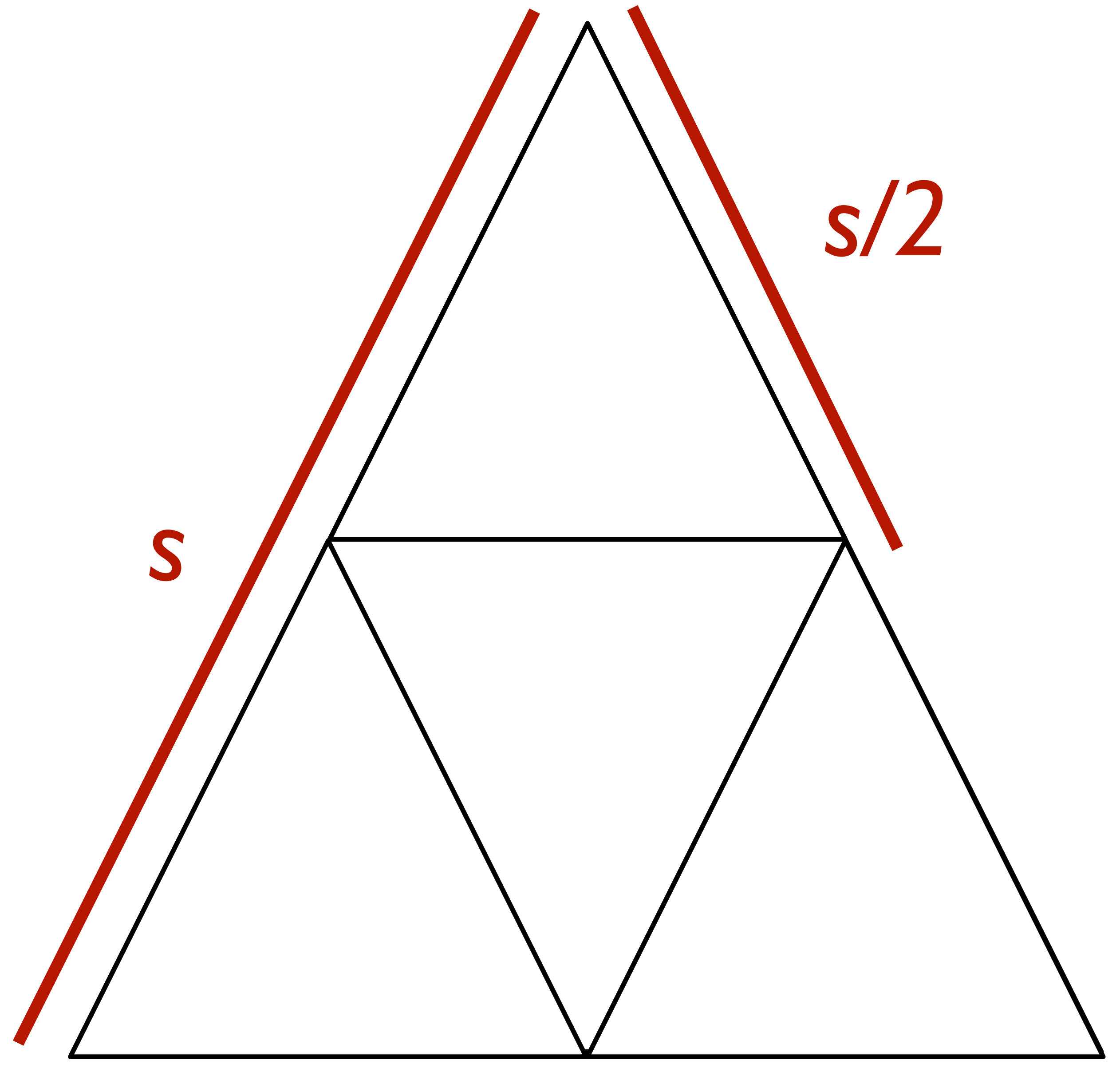
Inside that triangle are three more Sierpiński triangles:



And inside of each of those … and so on.

Producing something that looks like this:





s
s/2

[See class code]

How do we know that this function won't run forever?

Three-part termination argument:

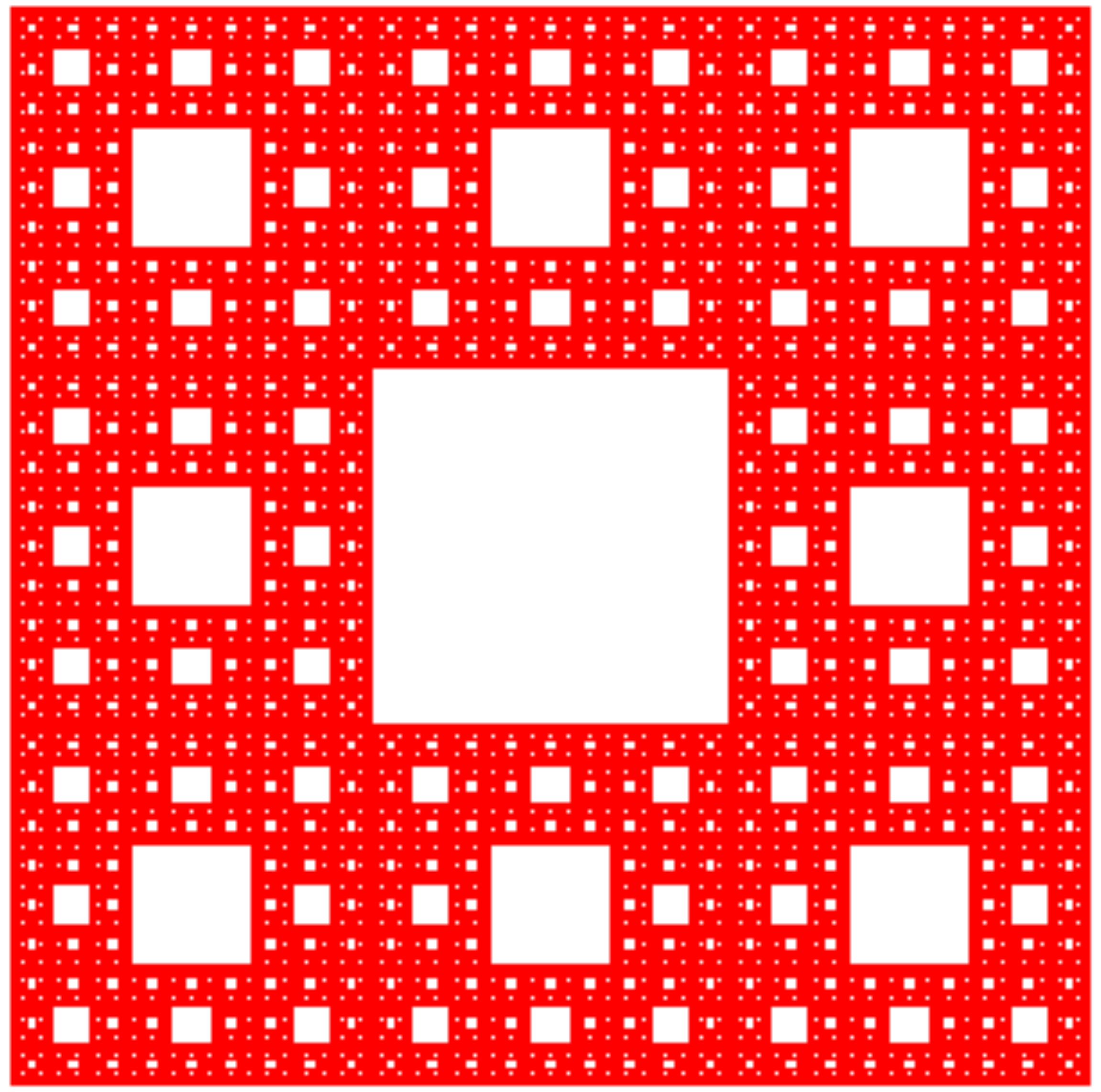
Base case: `s <= CUTOFF`

Reduction step: `s / 2`

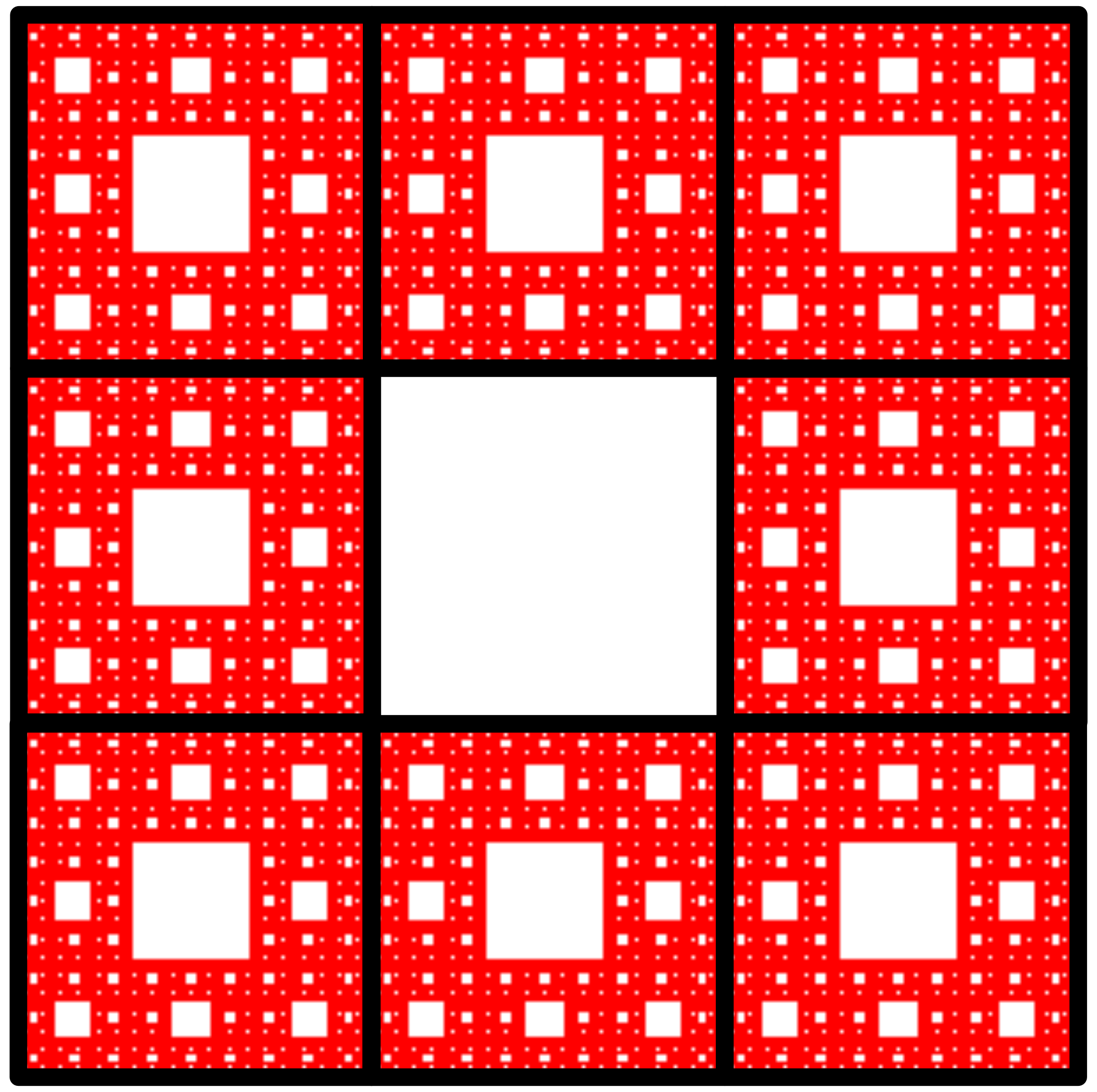
Argument that repeated application of reduction step will eventually reach the base case:

As long as the cutoff is > 0 and s starts >= 0, repeated division by 2 will eventually be less than the cutoff.

Design a function **`s-carpet`** to produce a Sierpiński carpet of size *s*:



Design a function **s-carpet** to produce a Sierpiński carpet of size *s*:



*There are **eight** copies of the recursive call positioned around a blank square*

[See class code]

How do we know that this function won't run forever?

Three-part termination argument:

Base case: s <= CUTOFF

Reduction step: s / 3

Argument that repeated application of reduction step will eventually reach the base case:

As long as the cutoff is > 0 and s starts >= 0, repeated division by 3 will eventually be less than the cutoff.

Animation

Class code:

https://tinyurl.com/101-52-2023-02-27

Acknowledgments

This lecture incorporates material from:

Gregor Kiczales, University of British Columbia

Marc Smith, Vassar College