CMPU 101 § 54 · Computer Science I

Data Definitions

13 February 2023

Where are we?

How was the lab?

We’ve been working with tables for the past few weeks.

Last class we saw a new data type: lists.

››› **grades**

number-grade	letter-grade
98	"A"
100	"A"
74	"C"
84	"B"

[list:

"A",

"A",

"C",

"B"]

››› grades

number-grade	letter-grade
98	"A"
100	"A"
74	"C"
84	"B"

››› grades.get-column("letter-grade")

[list:

"A",

"A",

"C",

"B"]

We used higher-order functions to work with tables, and we can do the same with lists:

Tables	*Lists*
transform-column	map

We used higher-order functions to work with tables, and we can do the same with lists:

Tables	*Lists*
transform-column	map
filter-with	filter

```
››› lst = [list: "a", "b", "c"]
››› filter(
      lam(i): not(i == "a") end,
      lst)
[list: "b", "c"]
```

This is an anonymous (i.e., unnamed) function made using a lambda expression.

Numbers, strings, images, Booleans, tables, and lists let us represent many kinds of real data quite naturally.

But there are times when we’re going to want something a bit different.

Defining structured data

Imagine that we’re doing a study on communication patterns among students.

We don’t have access to the messages the students sent – hopefully they’re encrypted! – but we have *metadata* for each message:

- sender
- recipient
- day of the week
- time (hour and minute)

This kind of metadata might sound uninteresting, but it can tell us a lot!

Recommended reading:

John Bohannon, "Your call and text records are far more revealing than you think", *Science*, 2016

Imagine that we're doing a study on communication patterns among students.

We don't have access to the messages the students sent – maybe they're encrypted! – but we have *metadata* for each message:

sender

recipient

day of the week

time (hour and minute)

How should we store this data?

We could have a table, e.g.,

sender :: String	*recipient :: String*	*day :: String*	*time :: ...*
"4015551234"	"8025551234"	"Mon"	...

We could have a table, e.g.,

sender :: String	*recipient :: String*	*day :: String*	*time :: String*
"4015551234"	"8025551234"	"Mon"	"4:55"

We could have a table, e.g.,

sender :: String	*recipient :: String*	*day :: String*	*time :: String*
"4015551234"	"8025551234"	"Mon"	295

We could have a table, e.g.,

sender :: String	*recipient :: String*	*day :: String*	*time :: List*
"4015551234"	"8025551234"	"Mon"	[list: 4, 55]

We could have a table, e.g.,

sender :: String	*recipient :: String*	*day :: String*	*hour :: Number*	*minute :: Number*
"4015551234"	"8025551234"	"Mon"	4	55

If we use multiple columns, we can access the components independently, by name, but if we use a single column, all of the “time” data is in one place.

To resolve this trade-off, we add structure: We can have a single data type that has named parts.

```
data Time:
  | time(hours :: Number, mins :: Number)
end
```

*The **name** of the data type*

```
data Time:
  | time(hours :: Number, mins :: Number)
end
```

```
data Time:
  | time(hours :: Number, mins :: Number)
end
```

*A **constructor** function that builds the data type*

```
data Time:
  | time(hours :: Number, mins :: Number)
end
```

*The **components** of the data*

After defining the data type,

```
data Time:
  | time(hours :: Number, mins :: Number)
end
```

we can call **time** to build **Time** values,

```
››› noon = time(12, 0)
››› half-past-three = time(3, 30)
```

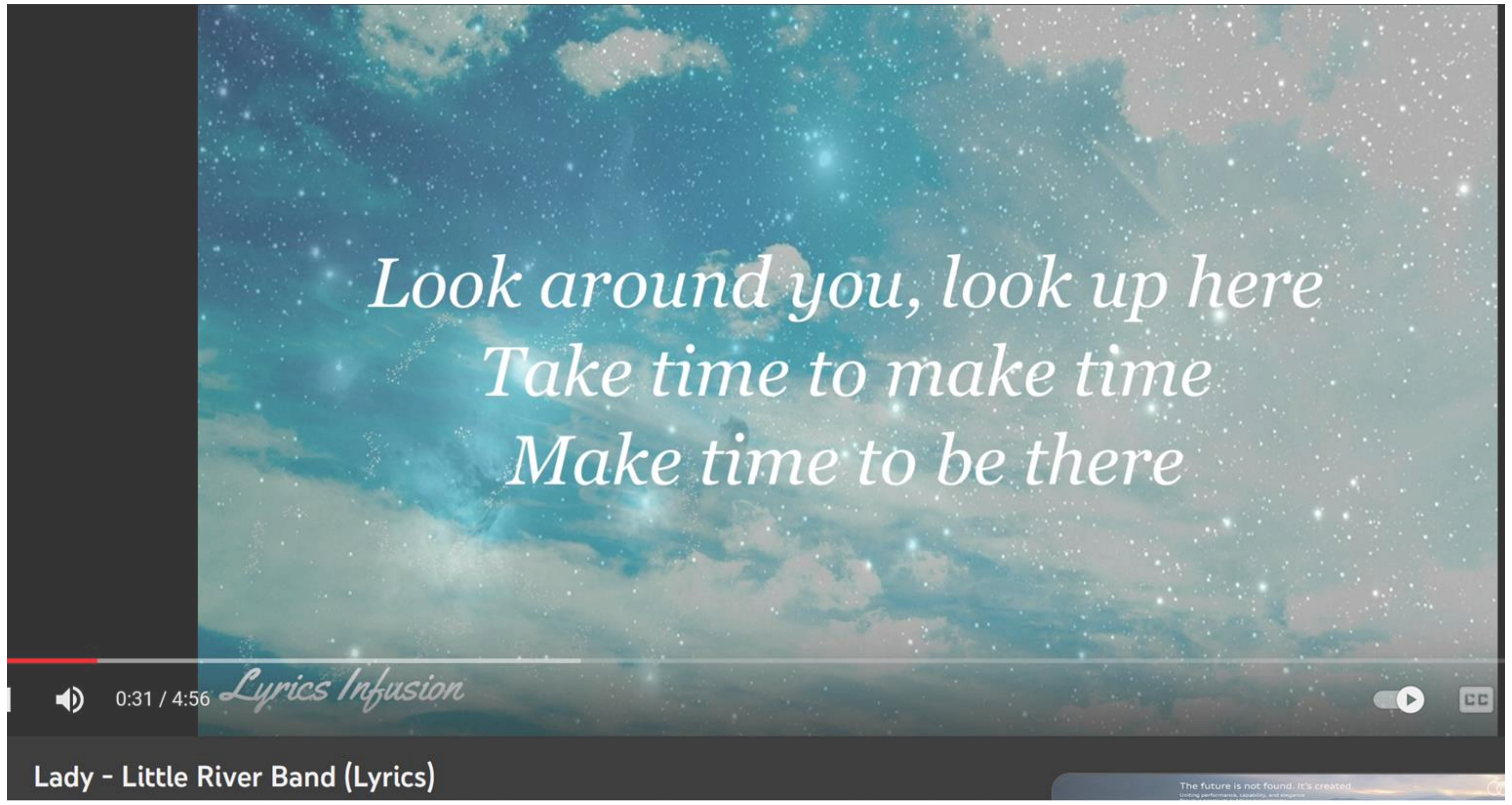
and we can use dot notation to access the components:

```
››› noon.hours
12
››› half-past.mins
30
```

Our table could now be:

sender :: String	*recipient :: String*	*day :: String*	*time :: Time*
"4015551234"	"8025551234"	"Mon"	time(4, 55)

Conditional data




```
data Time:
  | time(hours :: Number, mins :: Number)
end
```

The only way to make **Time** *is to call the*
time()
constructor function.

But we can also define *conditional data*, where there are multiple varieties of the data.

The varieties can just be fixed values, e.g.,

```
data Day:
  | sunday
  | monday
  | tuesday
  | wednesday
  | thursday
  | friday
  | saturday
end
```

Or they can be separate constructors, e.g.,

```
data Message:
  | direct(sender :: String,
     recipient :: String,
     message :: String)
  | group(sender :: String,
     recipients :: List<String>,
     message :: String)
end
```

Or we can mix these together, e.g.,

```
data Name:
  | name(first :: String, last :: String)
  | anonymous
end
```

Recursive data definitions

Last week we worked with *lists* – ordered sequences of items, equivalent to a column in a table.

Much like the rows in a table, the items in a list have numeric indices:

```
                       0      1      2
››› lst = [list: "a", "b", "c"]
```

And we can access items using these indices:

```
››› lst.get(0)
"a"
››› lst.get(1)
"b"
```

Much like the rows in a table, the items in a list have numeric indices:

```
                    0      1      2
››› lst = [list: "a", "b", "c"]
```

And we can access items using these indices:

```
››› lst.get(0)
"a"
››› lst.get(1)
"b"
```

But writing the list as `[list: "a", "b", "c"]` is just a convenient deception!

In its secret heart, Pyret knows there are only two ways of making a list.

A list is either:

empty or

linking an item to another list.

That is, a list is a kind of conditional data:

```
data List:
  | empty
  | link(first :: Any, rest :: List)
end
```

So, a list of one item, e.g.,

```
[list: "A"],
```

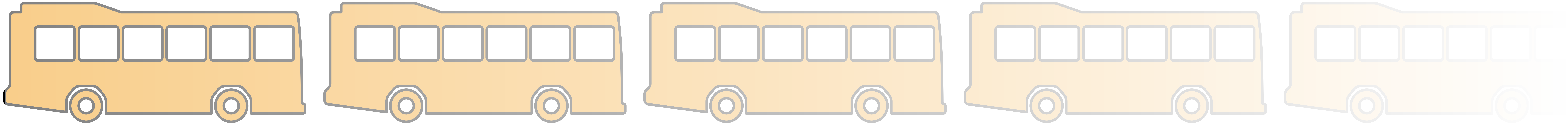
is really a link between an item and the empty list:

```
link("A", empty)
```

```
[list:
  "A",
  "A",
  "C",
  "B"]
```

```
link("A",
  link("A",
    link("C",
      link("B",
        empty))))
```

Recursion






4+1=5
3+1=4
2+1=3
1+1=2
1


4+1=5
3+1=4
2+1=3
1+1=2
1
Count one bus
Count all the buses
Count all the buses
Count all the buses
Count all the buses

Recursion is a programming technique where a problem is solved by solving a smaller version of the same problem, unless that smaller version is simple enough to solve directly.

We call the small version that can be solved directly the *base case* of the recursive problem.

To write our own functions to process a list, item by item, we need to use the true form of a list and think recursively.

Designing functions using the definition of a list

How would we write a function that takes a list of numbers and returns its sum?

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"
  ...
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"
  ...
where:
  my-sum([list: ]) is ...
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"
  ...
where:
  my-sum([list: ]) is 0
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"
  ...
where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"
  ...
where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4
  my-sum([list: 1, 4]) is 1 + 4
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"
  ...
where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4
  my-sum([list: 1, 4]) is 1 + 4
  my-sum([list: 3, 1, 4]) is 3 + 1 + 4
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"
  ...
where:
  my-sum([list:        ]) is           0
  my-sum([list:       4]) is       4
  my-sum([list:    1, 4]) is     1 + 4
  my-sum([list: 3, 1, 4]) is 3 + 1 + 4
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"
  ...
where:
  my-sum([list:        ]) is           0
  my-sum([list:       4]) is       4 + 0
  my-sum([list:    1, 4]) is   1 + 4 + 0
  my-sum([list: 3, 1, 4]) is 3 + 1 + 4 + 0
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"
  ...
where:
  my-sum([list:        ]) is             0
  my-sum([list:       4]) is         4 + my-sum([list: ])
  my-sum([list:    1, 4]) is     1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"
  ...
where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"

  cases (List) lst:
    | empty =>
      ...

    | link(f, r) =>
      ...

  end

where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"

  cases (List) lst:
  | empty =>
    ...

  | link(f, r) =>
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  end

where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

cases *is like a special* **if** *statement that we use to ask "which* ***shape*** *of data do I have?"*

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"

  cases (List) lst:
    | empty =>
      ...

    | link(f, r) =>
      ...

  end

where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

If the list is **empty**, *do one thing.*

If it’s a **link**, *do another thing.*

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"

  cases (List) lst:
    | empty =>
      ...

    | link(f, r) =>
      ...

  end

where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

Denotes the output of a function

Marks the expression to evaluate if the data has the shape on the left.

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return
```

This gives names for referring to the arguments to **my-sum.**

```
  cases (List) lst:
    | empty =>
      ...

    | link(f, r) =>
      ...
```

And this is giving names for referring to the arguments to **link.**

```
  end

where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"

  cases (List) lst:
    | empty =>
      ...

    | link(f, r) =>
      ...

  end

where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"

  cases (List) lst:
    | empty =>
      0

    | link(f, r) =>
      ...

  end

where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"

  cases (List) lst:
    | empty =>
      0

    | link(f, r) =>
      f + my-sum(r)

  end

where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"

  cases (List) lst:
    | empty =>
     0

    | link(f, r) =>
     f + my-sum(r)

  end

where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"
  cases (List) lst:
    | empty => 0
    | link(f, r) => f + my-sum(r)
  end
where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

```
fun my-sum(lst :: List<Number>) -> Number:
  doc: "Return the sum of the numbers in the list"
  cases (List) lst:
    | empty => 0
    | link(f, r) => f + my-sum(r)
  end
where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

When we call this function, it evaluates as:

```
   my-sum(link(3, link(1, link(4, empty))))
→ 3 + my-sum(link(1, link(4, empty)))
→ 3 + 1 + my-sum(link(4, empty))
→ 3 + 1 + 4 + my-sum(empty)
→ 3 + 1 + 4 + 0
```

Thinking recursively

Any time a problem is structured such that the solution on larger inputs can be built from the solution on smaller inputs, recursion is appropriate.

All recursive functions have these two parts:

Base case(s):

What's the simplest case to solve?

Recursive case(s):

What's the relationship between the current case and the answer to a slightly smaller case?

You should be calling the function you're defining here; this is referred to as a *recursive call*.

```
fun recursive-function(lst :: List) -> ...:
  cases (List) lst:
    | empty =>
      ...

    | link(f, r) =>
      ... recursive-function(r) ...

  end
end
```

Base case

Recursive case

Each time you make a recursive call, you must make the input smaller somehow.

If your input is a list, you pass the *rest* of the list to the recursive call.

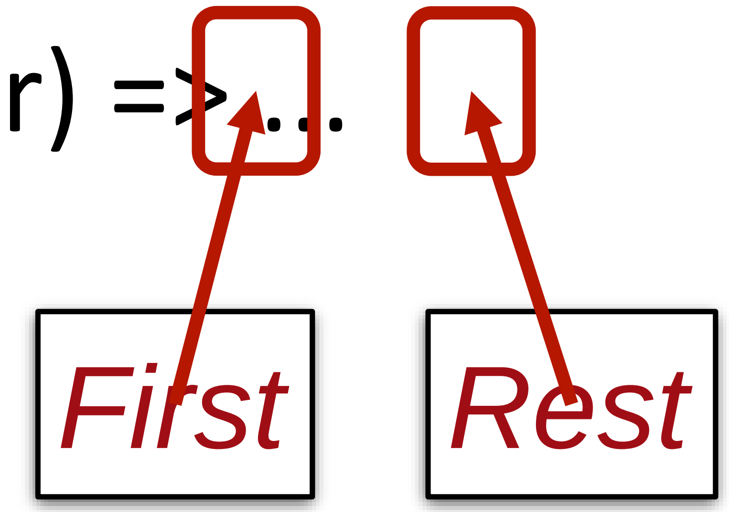
```
link("A",
  link("A",
    link("C",
      link("B",
        empty))))
```

First

Rest

```
››› lst = [list: "item 1", "and", "so", "on"]
››› lst.first
"item 1"
››› lst.rest
[list: "and", "so", "on"]
```

```
cases (List) lst:
  | empty => ...
  | link(f, r) => ... ...
end
```



What happens if we *don’t* make the input smaller?

```
fun my-sum(lst :: List<Number>) -> Number:
  cases (List) lst:
    | empty => 0
    | link(f, r) => f + my-sum(r)
  end
where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

Recursive call on the rest of the input l

```
fun my-sum(lst :: List<Number>) -> Number:
  cases (List) lst:
    | empty => 0
    | link(f, r) => f + my-sum(lst)
  end
where:
  my-sum([list: ]) is 0
  my-sum([list: 4]) is 4 + my-sum([list: ])
  my-sum([list: 1, 4]) is 1 + my-sum([list: 4])
  my-sum([list: 3, 1, 4]) is 3 + my-sum([list: 1, 4])
end
```

Recursive call on the original input

When we call this function, it evaluates as:

```
  my-sum(link(3, link(1, link(4, empty))))
→ 3 + my-sum(link(3, link(1, link(4, empty))))
→ 3 + 3 + my-sum(link(3, link(1, link(4, empty))))
→ 3 + 3 + 3 + my-sum(link(3, link(1, link(4, empty))))
…
```

This isn’t going to end well.

When a recursive function never stops calling itself, it's called *infinite recursion*.

Wrap-up practice

```
fun list-len(lst :: List) -> Number:
  doc: "Compute the length of a list"
  cases (List) lst:
    | empty => 0
    | link(f, r) => 1 + list-len(____)
  end
end
```

```
fun list-len(lst :: List) -> Number:
 doc: "Compute the length of a list"
 cases (List) lst:
   | empty => 0
   | link(f, r) => 1 + list-len(r)
 end
end
```

```
fun list-product(lst :: List<Number>) -> Number:
  doc: "Compute the product of all the numbers in lst"
  cases (List) lst:
    | empty => 1
    | link(f, r) => ____ * list-product(r)
  end
end
```

```
fun list-product(lst :: List<Number>) -> Number:
  doc: "Compute the product of all the numbers in lst"
  cases (List) lst:
    | empty => 1
    | link(f, r) => f * list-product(r)
  end
end
```

```
fun is-member(item, lst :: List) -> Boolean:
  doc: "Return true if item is a member of lst"
  cases (List) lst:
    | empty => ______
    | link(f, r) =>
     (f == ______) or (is-member(______, ______)
  end
end
```

```
fun is-member(item, lst :: List) -> Boolean:
 doc: "Return true if item is a member of lst"
 cases (List) lst:
   | empty => false
   | link(f, r) =>
    (f == item) or (is-member(item, r)
 end
end
```

Final note

Lists, recursion, and **cases** syntax are not easy concepts to grasp separately, much less all together in a short time.

Don’t feel frustrated if it takes a little while for these to make sense. Give yourself time, be sure to practice working in Pyret, and ask questions.

Class code:

tinyurl.com/101-2023-02-13

Acknowledgments

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