Foundations of Computer Science, Spring 2019

Part 1: Logic. Final Thoughts

There is one fundamental reason why we care about propositional logic and Boolean expressions in our Foundation course; it is used to create building blocks for computers. These *logic gates* are electronic forms of basic Boolean operations. Let’s consider three sets of logic gates:



The basic NOT gate simply reverses the input, from 1 to 0 or from 0 to 1.

The AND (and its complement NAND) gate take two inputs and the resultant outcomes are the same as the ^ truth table.

The OR (and its complement NOR) gate also takes two inputs and the resultant outcomes are the same as the v truth table.

There are also exclusive or gates, XOR and NXOR.

Our computers use logic gates to perform all of its logical and mathematical calculations. They are, in essence, *Lego bricks* for computers. Or, maybe, MindStorm Legos. The analogy is *not* materializing as I hoped. In any event, logic gates are part of every hardware circuit used in computers and smart phones.

It also turns out that binary arithmetic is easy to implement with circuitry that uses these gates.

NAND and NOR logic gates are just the negation of AND and OR. . .

The AND and NAND truth tables

|  |  |  |  |
| --- | --- | --- | --- |
| P | Q | P ^Q | P NAND Q |
| 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

The OR and NOR truth tables

|  |  |  |  |
| --- | --- | --- | --- |
| P | Q | P v Q | P NOR Q |
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

Part 2: Questions. Final Exam

The final exam will be open notes and open book. There will not be any questions that specifically cover topics that were covered on the midterm exam.

Today’s lecture leads one to suspect that there are probably two questions on the topic of propositional logic. This is true! Expect two questions on this topic. What else can one expect?

We spent a considerable amount of time on probability. There will be a question on Baye’s theorem. It is not necessary to bring a calculator, but simplification of fractions, simple ones, should be done. There will likely be another question regarding probability.

There will be a question on Permutations and Combinations.

 

We also spent a fair amount of time on natural numbers, inductive proofs and recursion (recursive definitions). There will be, at least, one question that requires you to do an inductive proof. There may be a question on recursive definitions and natural numbers.

We started off the second half of the semester delving into functions. One can expect one, or, more questions on functions. There are important kinds of functions: bijective, surjective, injective, so everyone should be familiar with their properties. We also took a look at the Pidgeon Hole Principle. A not unusual application of this principle is the probability of 2 students in a class sharing a birthday. It states:

* For (large) set A and (relatively small) set B, and the #elements of(A) > #elements of(B) then
* for every function *f: A → B*
* ***there is*** *a b ∈ B s.t. f(a) = b for more than one distinct a’s ∈ A*

Let’s review exercise 3.4.3(2)! A club where everyone uses just their initials (first name, last name) has to have x members such that 2 members must have the same initials?

Choose sets A, B and the function f. In this case, A is the set of members of the club, B is the set of ordered pairs of letters of the alphabet, f is the mapping of each member to an ordered pair of letters.

The #elements of (B) = 26\*\*n with n = 2. 26\*26 = 676. Add one and you guarantee the outcome (Unless Cher or Eminem is in the club.)