Homework for February 14, 2019: Due TUESDAY February 19.

1. Recall the definitions of reflexive, symmetric and transitive relations.

Recall that a relation is a set of ordered pairs.

Let A = {1,2,3}.

For each of the following, create digraphs and the resultant set of order pairs

that show the following:

(a) an example of a binary relation R1 over A that is

reflexive but not symmetric, but not transitive.

(b) provide an example of a binary relation R2 over A that is

not reflexive and not transitive, and not symmetric.

(c) provide an example of a binary relation R3 over A that is

not reflexive and is symmetric and transitive.

2. Complete the answers from page 6 on the 08-RelationsDeepDive lecture slide.

It will help to draw the digraphs.

3. Determine the relations for a, b in the set of integers and the mathematical

relation R: >, i.e "greater than"

4. Determine the relations for a, b in the set of integers and the mathematical

relation R: a\*b = 0

1. These are not the *only* correct answers.

A: An example that is reflexive, not symmetric, not transitive:

(1,1) (2,2) (3,3) (1,2) (2,3)

|  |  |  |
| --- | --- | --- |
| 1 |  | 1 |
| 2 |  | 2 |
| 3 |  | 3 |

B: An example that is not reflexive, not symmetric, not transitive:

(1,2) (2,3)

|  |  |  |
| --- | --- | --- |
| 1 |  | 1 |
| 2 |  | 2 |
| 3 |  | 3 |

C: An example that is not reflexive, symmetric, transitive:

(1,2) (1,3) (2,1) (2,3) (3,1) (3,2)

|  |  |  |
| --- | --- | --- |
| 1 |  | 1 |
| 2 |  | 2 |
| 3 |  | 3 |

2: Consider the following relations over {1,2,3,4}:

* *R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}*
* *R2 = {(1, 1), (1, 2), (2, 1)}*
* *R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (3, 3), (4, 1), (4, 4)}*
* *R4 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4),* (3, 3), (3, 4), (4, 4)}

Which relations are… reflexive? symmetric? transitive?

2 : **R1** has “3” as an element in the ordered pair (3,4) but is missing (3,3). Not reflexive

**R1** has (3,4) in the relation, but is missing (4,3) Not symmetric

**R1** does not have any ordered pairs with three elements of the form (a,b) (b,c) (a,c) Transitive.

Transitivity is not broken. For example, since (1,2) exists, one of {(2,3) (1,3)} or {(2,4) (1,4)} would be needed to break it.

2 : **R2** has “2” as an element in the ordered pair (1,2) but is missing (2,2). Not reflexive

**R2** has (1,2) and (2,1) in the relation. Symmetric

**R2** is Transitive. It does not have any ordered pairs with three elements of the form (a,b) (b,c) (a,c) so transitivity is not broken. If there are 2 elements of the form, i.e. a = c, then it must fit the definition. Here, it does have (1,2) (2,1) - (1,1) which fits the definition.

2 : **R3** has “2” as an element in the ordered pair (1,2) but is missing (2,2). Not reflexive

**R3** has (1,2) (2,1) and (1,4) (4,1) in the relation. Symmetric

**R3** does not have any ordered pairs with three elements of the form (a,b) (b,c) - (a,c) so transitivity is not broken. For example, since (1,2) exists, one of {(2,3) (1,3)} or {(2,4) (1,4)} would be needed to break the definition. It has the same set of pairs as R2 for a =c.

2 : **R4** Reflexive – every “a” in R4 has a corresponding pair (a,a)

**R4** has (1,3) in the relation, but is missing (3,1) Not symmetric

**R4** Transitive a,b,c in the relation exists in pairs for 1,2,3 and 1,3,4 as

* (1,2) (2,3) – (1,3) and
* (1,3) (3,4) – (1,4).

3 : **R is >** Not reflexive: 1 > 1 ??1?

**R** Not symmetric: 2 > 1 but 1 > 2 ??1?

**R** Transitive: eg 5 > 4 and 4 > 3 so 5 > 3.

4 : **R is a\*b = 0** i.e. a = 0 or b = 0 or both a = b = 0. Reflexive: 0\*0 = 0 (and 4\*4 != 0 for example)

**R** Symmetric: 4\*0 = 0 and 0 \* 4 = 0

**R4** Not Transitive, consider a\*b = 0, b\*c=0,for a=2 b=0 and c=4 so 2\*4 != 0