cmpu-145 spring, 2019

Homework, due April 29.

Problem 1.

Consider a variant of the Monty Hall puzzle with four doors. The host still opens just one

door with a goat - not the grand prize - behind it. What is the probability of winning if you

always keep your original choice and if you always switch?

A: Before Monty asks if you want to switch, there is a 1/4 chance that the prize is behind the door you chose and a **3**/4 chance that it is behind one of the other **three** doors. If you do not switch, the probability remains at 1/4. If you switch to either of the **two** remaining doors, there is a **3**/4 \* **2**/4 chance of being right (3/8).

Problem 2.

Recall the definiton for any events A and B to be independent. *Two events, A and B are independent iff* p(A ∩ B) = p(A) \* p(B) Consider tossing a pair of dice. The outcome can be thought of as an ordered pair, (x, y). Let A be the event of rolling an even number (i.e., x + y is even).

Let B be the event of rolling a 5, 6, or 7 (i.e., x + y is 5, 6, or 7). Compute p(A), p(B), p(A ∩ B) and

p(A) \* p(B). Are events A and B independent?

A: Event A is: rolling an even number, or x+y is even.

Event B is: rolling 5, 6, 7, or x+y=5 or 6 or 7.

p(A) is 1/2. (Even or odd, no other choices here)

p(B) is 15/36 (5/12) by counting the 5’s 6’s and 7’s.

Here’s a handy chart with the 36 (i.e. 6\*6) possibilities:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **+** | **1** | **2** | **3** | **4** | **5** | **6** |
| **1** | 2 | 3 | 4 | 5 | 6 | 7 |
| **2** | 3 | 4 | 5 | 6 | 7 | 8 |
| **3** | 4 | 5 | 6 | 7 | 8 | 9 |
| **4** | 5 | 6 | 7 | 8 | 9 | 10 |
| **5** | 6 | 7 | 8 | 9 | 10 | 11 |
| **6** | 7 | 8 | 9 | 10 | 11 | 12 |

p(A)\*p(B) is 1/2 \* 5/12 = 5/24. (it is ok to leave it at that)

p(A∩B) is the probability of rolling a 6 only. There are five 6’s in the chart above, so 5/36. And, p(A ∩ B) != p(A) \* p(B)

Problem 3.

Recall the definition of the probability of an event A given that event B occurred:

p(A | B) = p(A ∩ B) / p(B) . Compute the conditional probabilities p(A | B) and p(B | A) using the events in problem 2 above.

A: p(A|B) = (A ∩ B)/p(B) = (5/36) / (5/12) = 1/3

A: p(B|A) = (A ∩ B)/p(A) = (5/36) / (1/2) = 5/18

In both cases, the overall probability decreases when we know that either event occurred.

Problem 4.

In recent years, "Nor'easter" storms dump large amounts of snow or rain in Poughkeepsie 5 days each year. (Assume Nor'easter storms last one day, allowing you to work with the total number of days in a year.) When a Nor'easter affects Poughkeepsie, the 'European Model' for weather prognostication correctly predicts this fact 97% of the time. When a Nor'easter does not affect Poughkeepsie, the European Model incorrectly predicts that it will affect Poughkeepsie 3% of the time.

(see <https://en.wikipedia.org/wiki/European_Centre_for_Medium-Range_Weather_Forecasts> for more!)

Unfortunately, the European Model is predicting a Nor'easter to affect Poughkeepsie on the day of our final exam! What is the probability that a Nor'easter actually occurs on the day of our final exam?

(Hint: one way to start is to state the events we care about, and then the probabilities of these events occurring. ) We went over this one in class. If you were one of the few students that handed in the homework on the original due date, then you receive bonus points for answering this problem.

Event A: Nor’easter on exam day

Event **!A**: No Nor’easter on exam day (it still might rain!)

Event B: Prediction of a Nor’easter on exam day

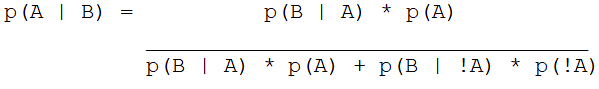
p(A) = 5/365

p(**!A**) = 360/365

No p(B) is actually given!

p(B|A) = 0.97 (Nor’easter occurs given model predicts it will occur)

p(B|**!A**) = 0.03 (NO Nor’easter occurs given model predicts it will occur)



p(A | B) = .97 \* .014 / (.97 \* .014 + .03 \* .986) = ~0.32