Problem 1.
Consider a variant of the Monty Hall puzzle with four doors. The host still opens just one door with a goat - not the grand prize - behind it. What is the probability of winning if you always keep your original choice and if you always switch?

Problem 2.
Recall the definition for any events A and B to be independent. **Two events, A and B are independent iff** 
\[ p(A \cap B) = p(A) \times p(B) \]
Consider tossing a pair of dice. The outcome can be thought of as an ordered pair, \((x, y)\). Let A be the event of rolling an even number (i.e., \(x + y\) is even).

Let B be the event of rolling a 5, 6, or 7 (i.e., \(x + y\) is 5, 6, or 7). Compute \(p(A)\), \(p(B)\), \(p(A \cap B)\) and \(p(A) \times p(B)\). Are events A and B independent?

Problem 3.
Recall the definition of the probability of an event A given that event B occurred:
\[ p(A | B) = \frac{p(A \cap B)}{p(B)} \]
Compute the conditional probabilities \(p(A | B)\) and \(p(B | A)\) using the events in problem 2 above.

Problem 4.
In recent years, "Nor'easter" storms dump large amounts of snow or rain in Poughkeepsie 5 days each year. (Assume Nor'easter storms last one day, allowing you to work with the total number of days in a year.) When a Nor'easter affects Poughkeepsie, the 'European Model' for weather prognostication correctly predicts this fact 97% of the time. When a Nor'easter does not affect Poughkeepsie, the European Model incorrectly predicts that it will 3% of the time. (see [https://en.wikipedia.org/wiki/European_Centre_for_Medium-Range_Weather_Forecasts](https://en.wikipedia.org/wiki/European_Centre_for_Medium-Range_Weather_Forecasts) for more!)

Unfortunately, the European Model is predicting a Nor'easter to affect Poughkeepsie on the day of our final exam! What is the probability that a Nor'easter actually occurs on the day of our final exam?

(Hint: one way to start is to state the events we care about, and then the probabilities of these events occurring.)