* + for all *a ∈ A, (a, a) ∈ R.*

We used the fact that A⊆B and B⊆A means A=B

*∩*

Prove, or give a counter example for the following statement:

P(A ∩B) = P(A) ∩P(B)

Approach: We are not told that this is true and that we must prove, so we have to convince ourselves that this is true first. Come up with an example that gives an indication for which answer may be correct. (Usually this will not be necessary – the problem statement will be more emphatic, one way or the other. )

Choose A = (1, 2, 3) and B = (3, 4, 5) or A = (1, 2, 3) and B = (2, 3, 4). In the first case, you’ll notice that “3” is the common element and {~~0~~}, {3} will be in P(A ∩B) as well as P(A) ∩P(B) too!

To Prove this is true, we want to show that

* P(A ∩B) ⊆ P(A) ∩P(B) **AND**
* P(A) ∩P(B) ⊆ P(A ∩B)

Why? Because generally, A=B means that both A and B are subsets of each other: A⊆B and B⊆A.

Our strategy to date has been to reuse definitions we know in order to get us where we want to be. And, we know about the definition of:

* A set,
* A subset,
* Intersection of a set,
* Power set.

Proof:

1. Show P(A ∩B) ⊆ P(A) ∩P(B)

Let x be an element in P(A ∩B) i.e. x ∈ P(A ∩B). We will need to prove that this x is an element of the intersection of the power set of A and the power set of B, i.e. x ∈ P(A) ∩P(B), which is the definition of a subset…

By the definition of a power set, x ∈ A ∩ B.

By the definition of an intersection of a set, this means that both , x ∈ A and x ∈ B.

Then, using the definition of a power set again (!) we know x ∈ P(A) and x ∈ P(B)

And, by using the definition of set intersection again, x ∈ P(A) ∩P(B). Which means:

P(A ∩B) ⊆ P(A) ∩P(B).

So, by starting with x ∈ P(A ∩B) we get to our desired conclusion.

1. Show P(A) ∩P(B) ⊆ P(A ∩B)

Let x be an element in P(A) ∩ P(B) i.e. x ∈ P(A) ∩P(B). We will need to prove that this x is an element of the intersection of the power set of A and B, i.e. x ∈ P(A ∩B)…

By the definition of an intersection of a set, this means that both x ∈ P(A) ∩ x ∈ P(B).

Then, by the definition of a power set, we know x ∈ A and x ∈ B

And, by using the definition of set intersection, x ⊆ A ∩B.

By the definition of a power set again (!) x ⊆ P(A ∩B) . Which means:

P(A) ∩P(B) ⊆ P(A ∩B)

1. Since we have

* P(A ∩B) ⊆ P(A) ∩P(B) and
* P(A) ∩P(B) ⊆ P(A ∩B)

The, by definition of set equality, P(A) ∩P(B) = P(A ∩B). Or, as we would say using *some* Java syntax,

P(A) ∩P(B) == P(A ∩B);

And the statement is proven true!