CMPU-145 Spring, 2019

MTP: Mid Term Preparation

A set of topics and problems that students should be able to solve. The set may not include everything that one could find on a midterm exam.

I: Power Sets / Proofs

1. Also recall HW212 problem and solution: Prove, or give a counter example for the following statement: P (A ∩B) = P (A) ∩P (B) .
2. **Show that P(A) ∪ P(B) is not necessarily equal to P(A ∪ B).**

Let A = {1} and B = {2}.

Then P(A) = {0, {1}} and P(B) = {0, {2}}.

Thus, P(A) ∪ P(B) = {0, {1}, {2}}.

However, P(A ∪ B) = P({1, 2}) = {0, {1}, {2}, {1, 2}}.

Q: Could you use Venn diagrams to show this?

1. **Prove that for any sets A and B, if A ⊆ A**`**, then A × B ⊆ A**`**× B.**

PROOF. Let A, A` and B be any sets where A ⊆ A`.

We must show that A × B ⊆ A`× B.

In other words, we must show that for any x ∈ A × B, it must be the case that x ∈ A`× B.

Let x be any element of A × B.

By the definition of A × B, x must be an ordered pair of the form:

(a, b), where a ∈ A and b ∈ B.

Now, a ∈ A and A ⊆ A` together means that a ∈ A`.

So, (a, b) ∈ A`× B (i.e., x ∈ A`× B).

II Equivalence Relations / Partitions

1. Consider: The set A = {1, 2, 3, 4, 5, 6}

Consider: The set {{1}, {2,6}, {3}, {4,5}}

Q:Is this set a partition of set A? Q:How do we know?

A: Yes.

A: …

If it is a partition, define the equivalence relation that is determined by it.

A: Approach: The partition is showing elements that are considered equivalent. We need to “fit” the three properties by which we define equivalence relations: R S T

**R**: Reflixive

{1} and {3} are singletons, the only order pairs where 1 (and 3 also!) is considered equivalent is (1,1). And (3,3).

Of course, we need to add (2,2), (4,4) (5,5) and (6,6). Because: what’s an equivalence relation without a full complement of reflexive ordered pairs :? (i.e “a” exists, and b=a and c=a)

**S**: Symmetry/yrtemmyS

{2,6} and {4,5} are the only other set elements we need to consider here. (2,6) & (6,2), (4,5) & (5,4) need to be added to the equivalence relation. Because symmetry. (i.e “a”,“b” exist in our relation R, c=a)

**T**: Transitive

Given our answers so far, are there an relations that “fit” the transitive property with unique “a” “b” “c” ...? If so, we have to add those to our relation.

1. How would Rene DesCartes, as opposed to, say, Michael Rennie, create a set of ordered pairs from:

The set A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} and

The set B = {a, p}?

1. Create a partition of the English alphabet. No, not the one below.
2. A hash table can be considered a way to define an equivalence relation. We can partition the alphabet into vowels and consonants (does that work?). Let’s focus on just the vowels. (a,e,i,o,u,y). A hash table can be said to define a relation for like items. Our hash relation is similar to a dictionary\*. In our case we will use special category: musical group/artist names that start with a vowel. In our relation, groups like Eagles and Eurythmics are equivalent. Perhaps Abba and Ace of Base is a better example?
   1. Let's expand our hash table so that we have two tables – the vowels above, and consonants. Do we have a partition of the English alphabet? (Hint: what to do about the group that created iconic album “Upstairs at Eric’s”)
   2. Pretend that you pronounce words starting with the letter a the same way as words starting with the letter e, making them, effectively equivalent. Write the set of ordered pairs that mark this new equivalence.
   3. Consider the singer Ai Otsuka and the fact that you don’t remember if the singer is Ai Otsuka or Otsuka Ai, so you make the letters “a” and “o” equivalent. Considering the answers to “b.” above, write the set of ordered pairs that cover this added relation while still maintaining an equivalence relation.

III Dr.Racket

Consider the following function.

;; LISTORINO

;; ---------

;; INPUTS: N, TOTAL -- natural numbers

;; OUTPUT: ?

;; Example: ?

(define listorino

(lambda (n total)

(if (= n 0)

’()

(cons (/ (- total n) n)

(listorino (- n 1) total)))))

What is the output from the statement:

> (listorino 5 6) ?

If you work through the recursion with pencil and paper, you can come up with the answer!