Replicators (components and test-rigs)

Peter Welch (p.h.welch@kent.ac.uk) Computing Laboratory, University of Kent at Canterbury

Co631 (Concurrency)

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Replicators (components and test-rigs)

Replicated **PAR** and **SEQ** ...

The SORT PUMP

Component testing ...

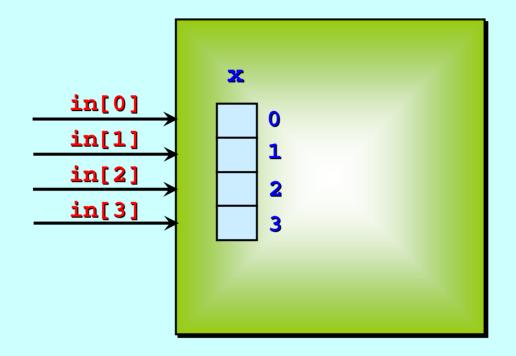
Stateless components ...

The SORT GRID

Replicated **IF**...

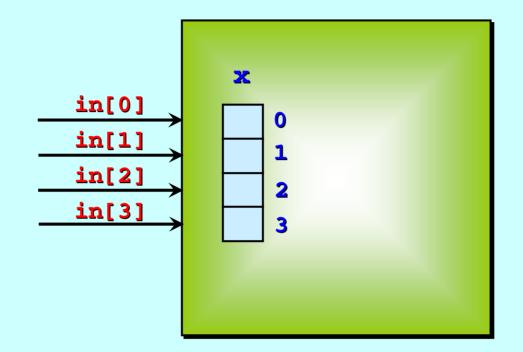
Replicator **STEP** sizes ...

Consider a process with an array of input channels:

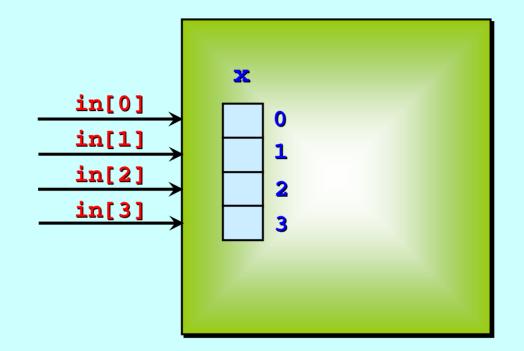


And an internal data array, \mathbf{x} , of the same type and size as the input channel array.

The process needs to input one message from each input channel into the corresponding element of its data array.



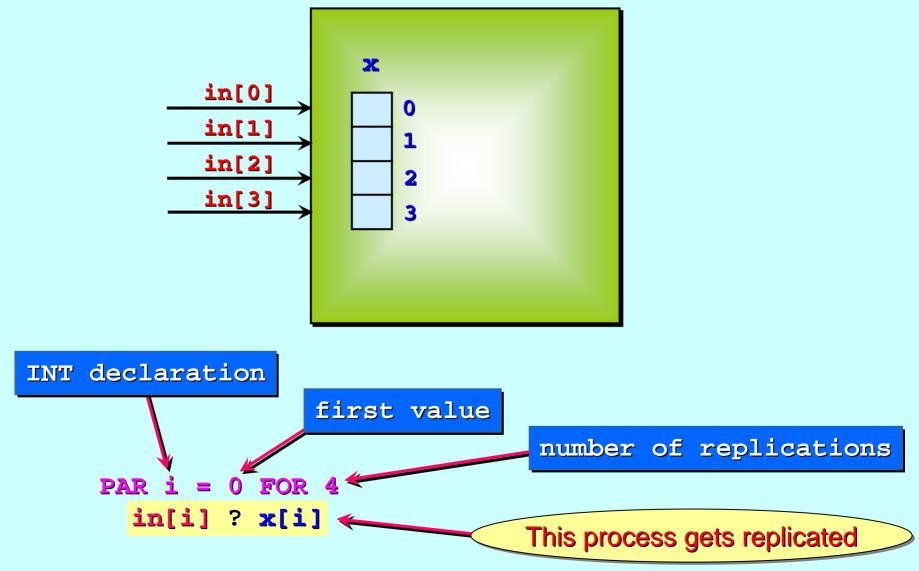




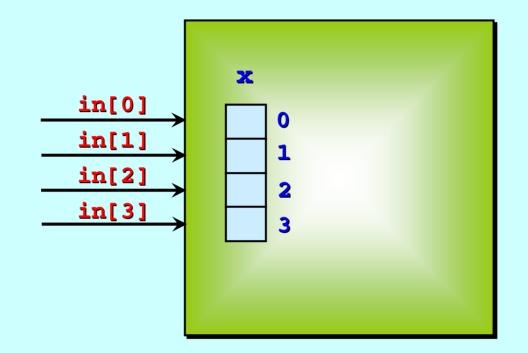


| <pre>in[0]</pre> | ? | x[0] |
|------------------|---|-------------|
| in[1] | ? | x[1] |
| <pre>in[2]</pre> | ? | x[2] |
| <pre>in[3]</pre> | ? | x[3] |

But what if there were 40 channels in the array? Or 400 ... or 4000 ... ?!!

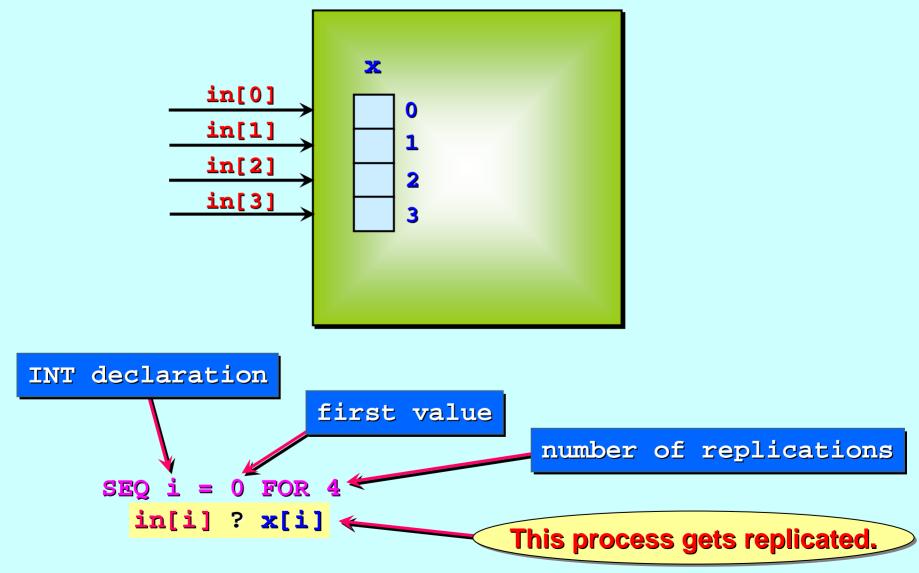


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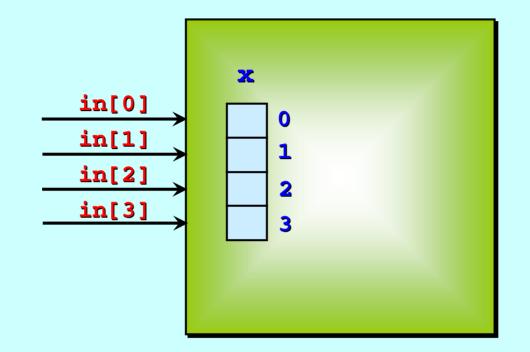




Just in case they really had to be done *in sequence*:

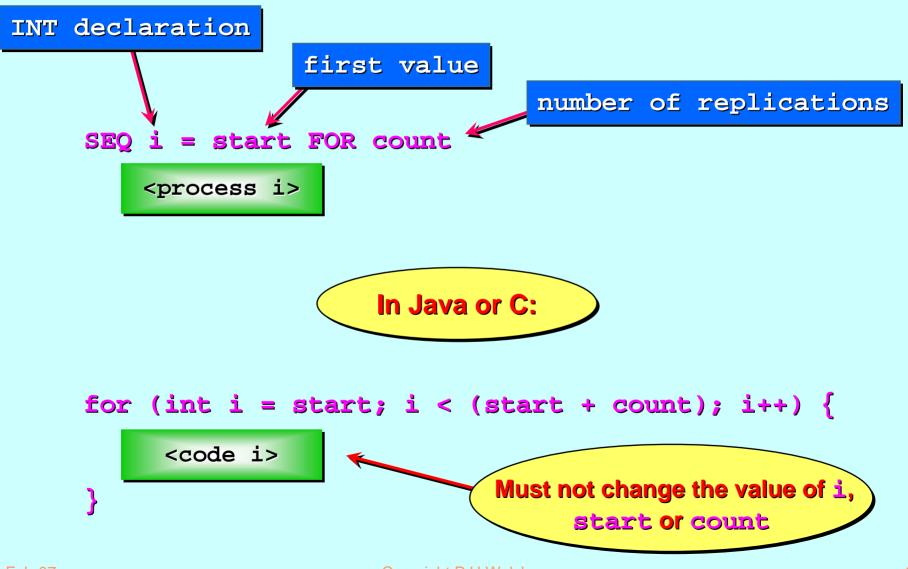


Just in case they really had to be done *in sequence*:



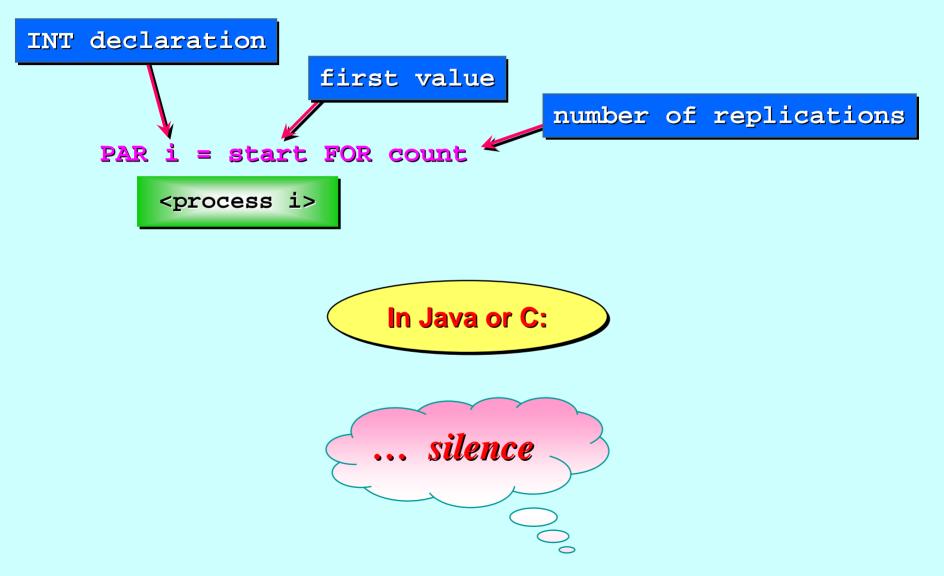


The replicated SEQ is like a very clean for-loop.

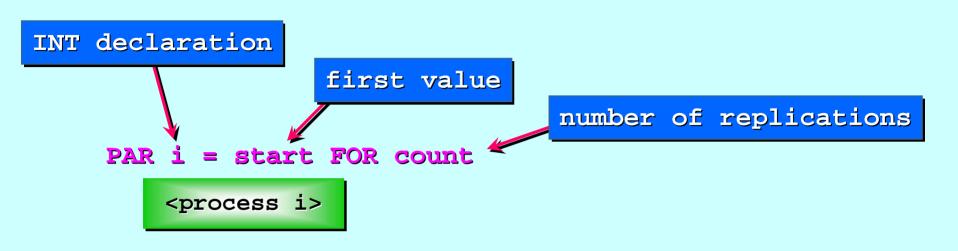


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The replicated PAR has no correspondence in Java or C.



Applying the replicated PAR.



The first example showed parallel replication of a *primitive* process (an input process).

But, earlier, we've seen parallel composition of long-lived structured processes (like continuously active 'chips').

The next example shows parallel replication of such a process to build a *parallel sorting engine*.

Replicators (components and test-rigs)

Replicated **PAR** and **SEQ** ...

The SORT PUMP ...

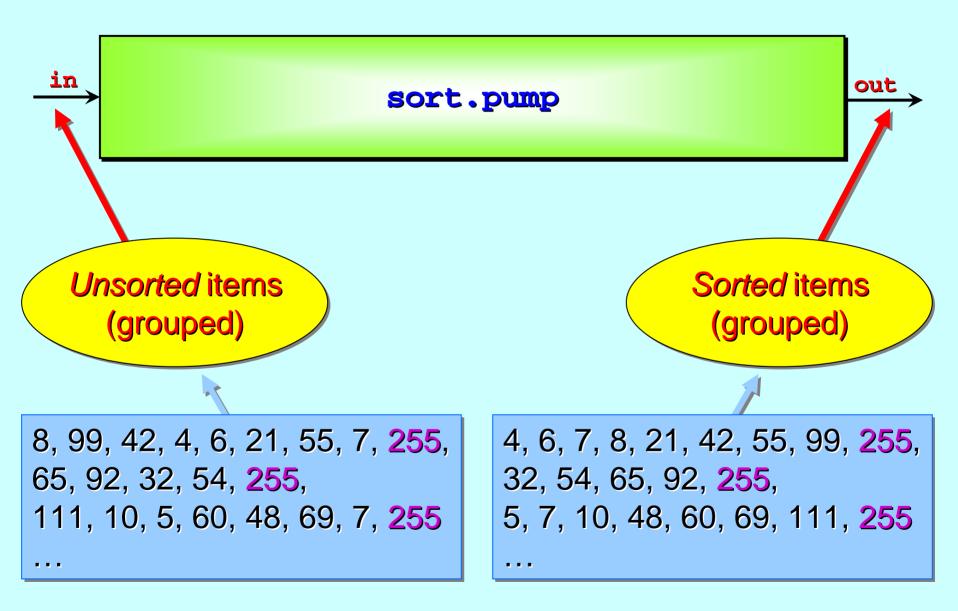
Component testing ...

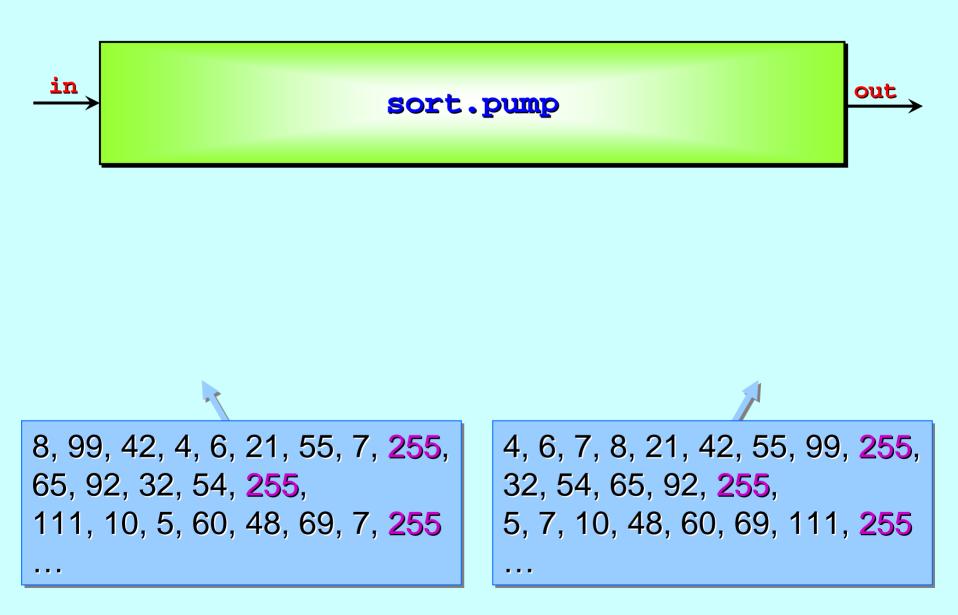
Stateless components ...

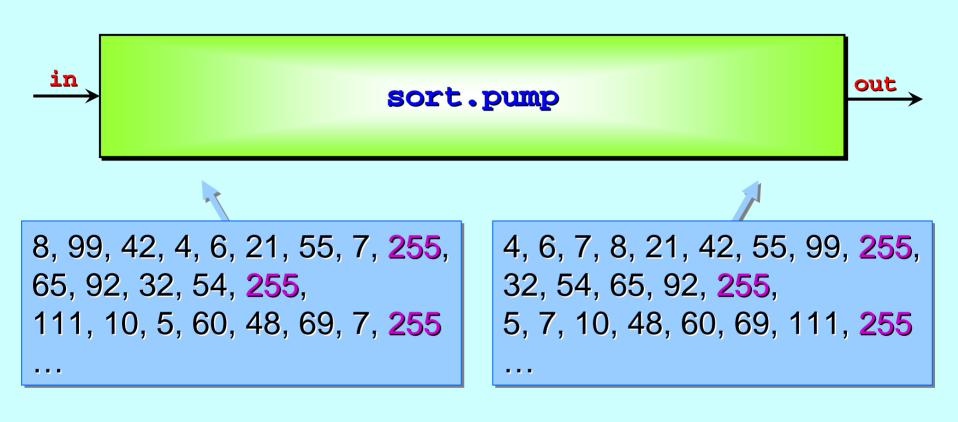
The SORT GRID

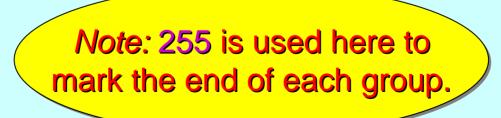
Replicated **IF**...

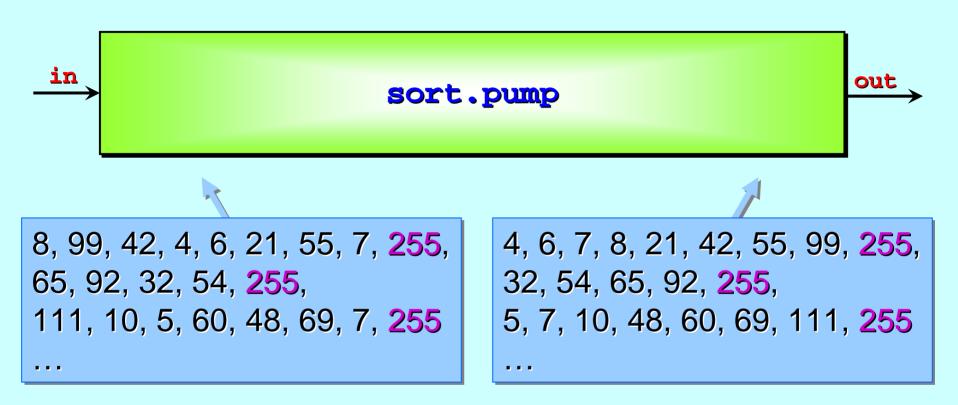
Replicator **STEP** sizes ...



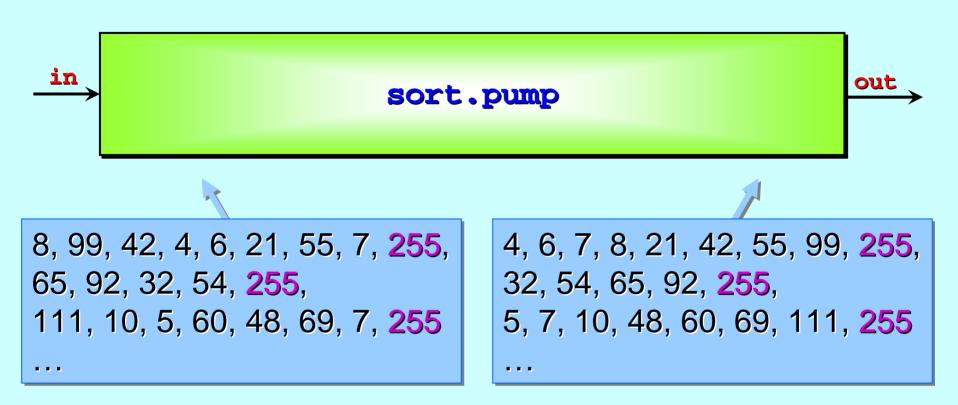








For the efficient application of this device, we need a longrunning source of groups of items that need sorting. We also need to specify an upper limit on the size of groups.

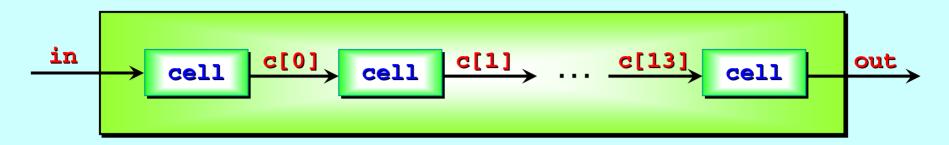


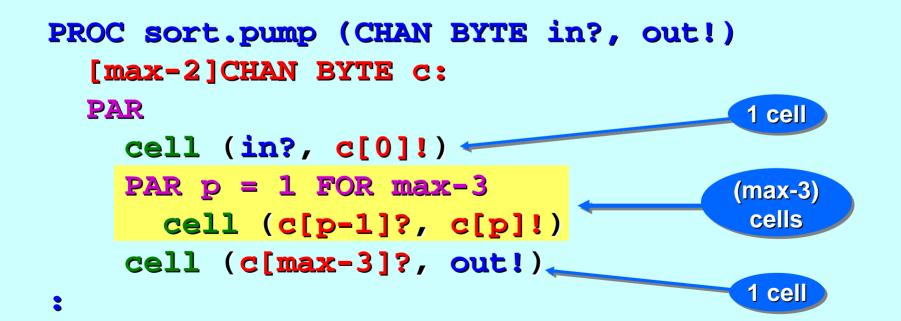
An example is a simple image smoothing filter: each pixel is replaced by the *median* value of its (9) neighbours. Finding median values implies sorting. Each n-by-m image generates (n*m) groups of 9 numbers for sorting.

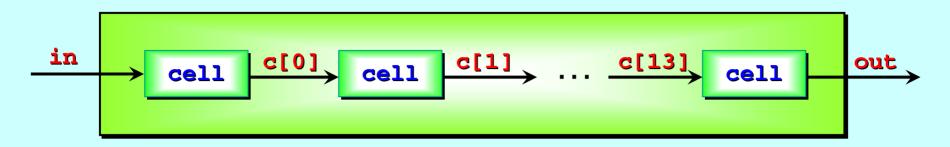
The **sort.pump** is implemented as a *pipeline* of simpler **cell** proccesses. (We'll see what they do presently.)

To sort groups up to a maximum size of **max**, we need at least (**max – 1**) **cells**.

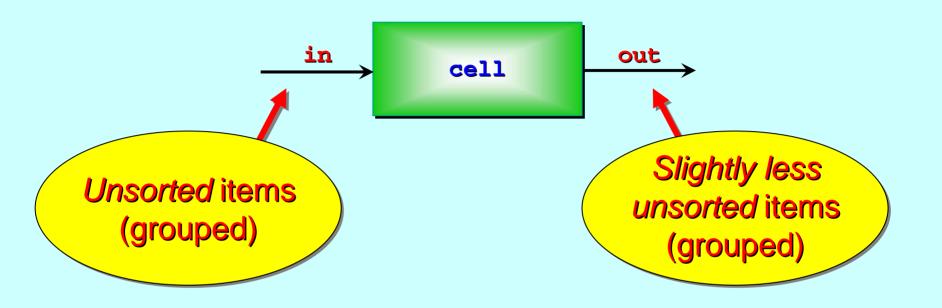
So, if **max** is **16**, we need **15 cells** ... which means we need **14** internal channels ... which we have indexed above from **0** through **13**.





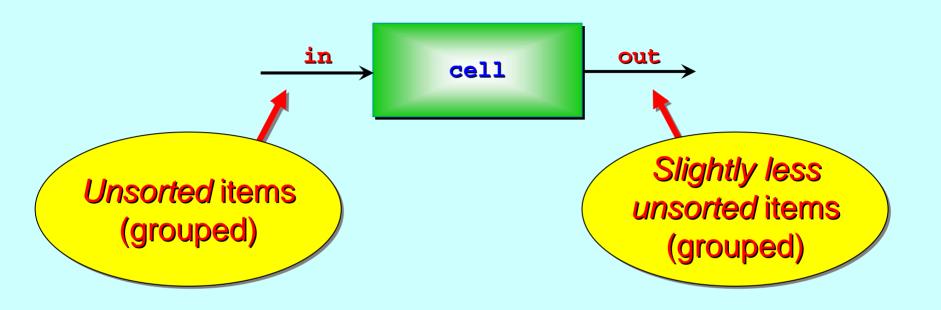


```
PROC sort.pump (CHAN BYTE in?, out!)
[max-2]CHAN BYTE c:
PAR
    cell (in?, c[0]!)
PAR p = 1 FOR max-3
    cell (c[p-1]?, c[p]!)
cell (c[max-3]?, out!)
;
```



All each **cell** has to do is drag heavy items backwards. In particular, as each group flows through, the *last* one out must be the heaviest in the group.

To do this, two *variables* (or *registers*) are needed: one to hold the **largest** item seen so far and one to hold the **next** item to arrive.



The **cell** inputs the first item of a group into **largest**.

Then, it compares each **next** item against **largest**, outputting the smaller and keeping the larger.

When the **end.marker** arrives, it just outputs the **largest** followed by that **end.marker**.

VAL BYTE end.marker IS 255: -- assume > data items PROC cell (CHAN BYTE in?, out!) WHILE TRUE in out BYTE largest: cell SEQ in ? largest WHILE largest <> end.marker BYTE next: SEQ in ? next TF -- output smaller, keep larger largest >= next out ! next -- i.e. largest < next TRUE

SEQ

out ! largest

largest := next

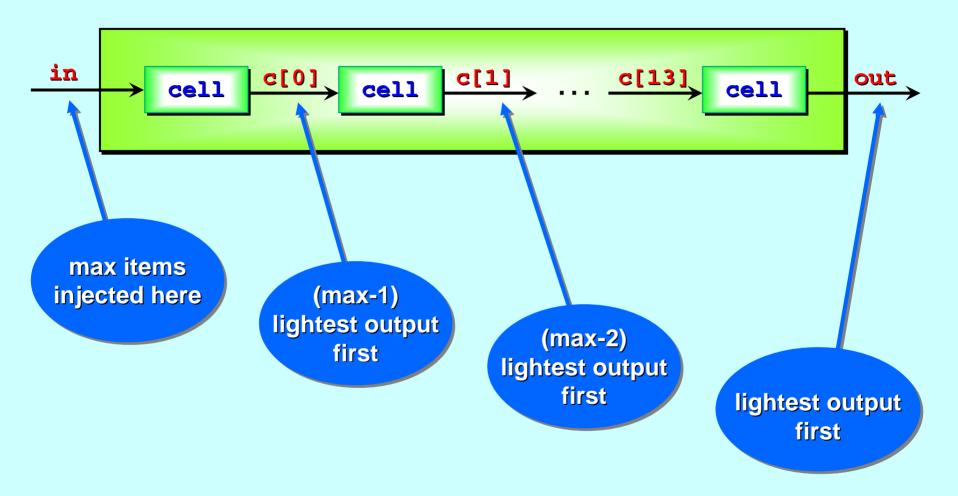
```
out ! end.marker
```

VAL BYTE end.marker IS 255: -- assume > data items

```
PROC cell (CHAN BYTE in?, out!)
  WHILE TRUE
    BYTE largest:
                                           in
                                                                out
                                                    cell
    SEO
      in ? largest
      WHILE largest <> end.marker
        BYTE next:
        SEQ
          in ? next
          IF
                        -- output smaller, keep larger
            largest >= next
              out ! next
            TRUE
                       -- i.e. largest < next
              SEO
                out ! largest
                largest := next
      out ! end.marker
```

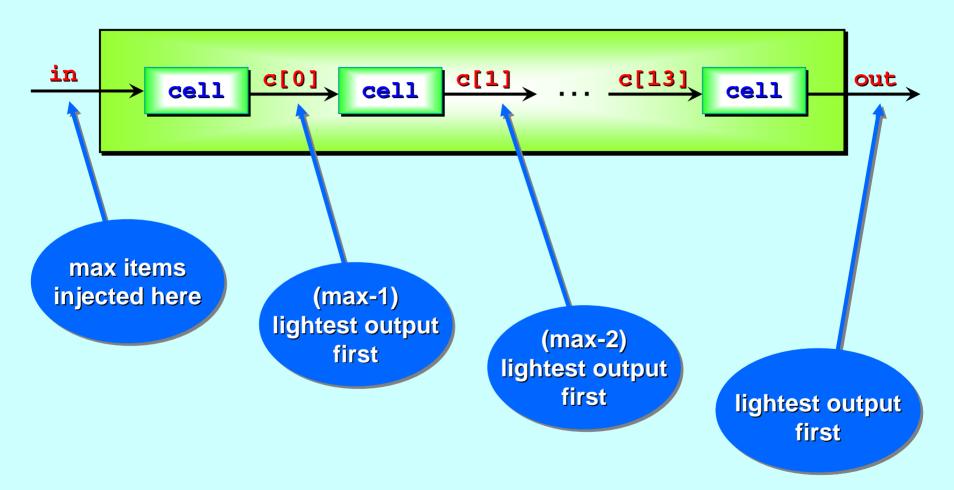
3

Note: this algorithm requires a potential data item (255) reserved for the **end.marker**. This constraint can be removed – later.

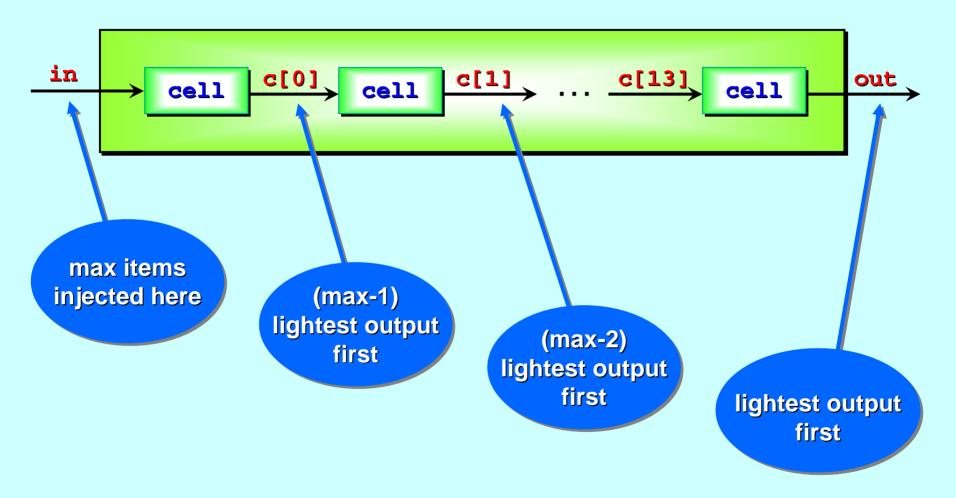


Each **cell** holds back largest item it sees, so ...

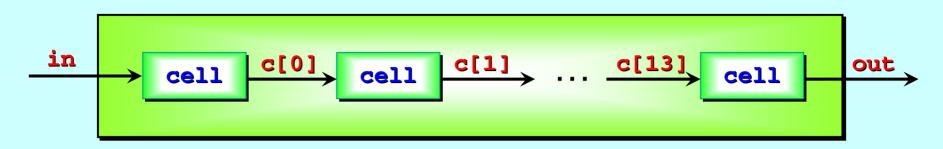
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As the **end.marker** flows through, it pushes out the heaviest item, which pushes out the next heaviest, etc...



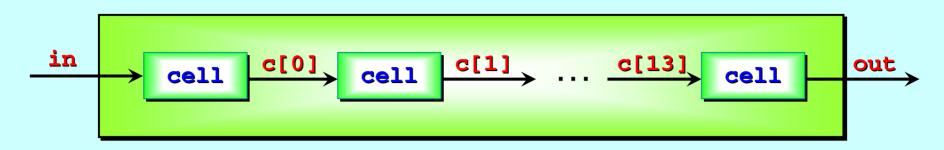
The group, therefore, flows out in ascending sorted order. ③



If the cells are implemented on separate pieces of silicon (i.e. we have a physically parallel engine), the speed at which data flows through is the **slowest** of:

- the speed at which data is offered;
- the cycle speed for each cell;
- the inter-cell communication speed.

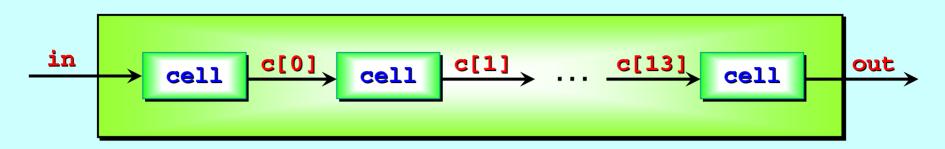
The speed is independent of the number of cells – which means that it is independent of the number of items being sorted. We have an O(n) sorting engine: **sort.pump**. \bigcirc \bigcirc \bigcirc



In fact, **sort.pump** is a parallel version of *bubble-sort*, one of the simplest known sorting algorithms. Its performance on a *serial* processor is O(n*n), which is poor compared to more complex sorts (such as *quick-sort*, which is O(n*log(n))).

If data is supplied in O(n) time (as in the above, where the numbers are supplied **one-at-a-time**), then a processing complexity of O(n) cannot be beat!

Lesson: when considering a *parallel* design, don't start from the most efficient known *serial* algorithm – it's probably optimised the wrong way. *Rethink – look for the simplest approach*.



Note: the capacity of **sort.pump** is (2*max - 2) items, each **cell** holding 2 of them.

So, **sort.pump** can be processing (parts of) two or three groups (up to max size) at the same time.

It will only operate efficiently so long as there is a continuous supply of groups to be sorted.

For example, if only one group were pushed through, only half the **cells** would ever be operating at the same time.

Replicators (components and test-rigs)

Replicated **PAR** and **SEQ** ...

The SORT PUMP ...

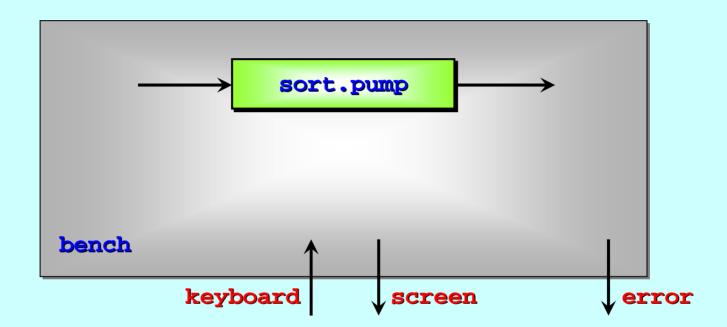
Component testing ...

Stateless components ...

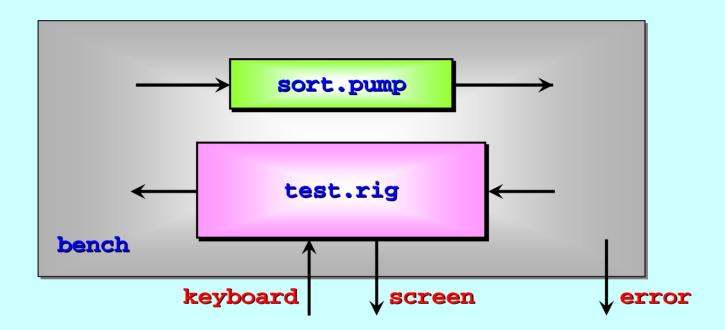
The SORT GRID

Replicated **IF**...

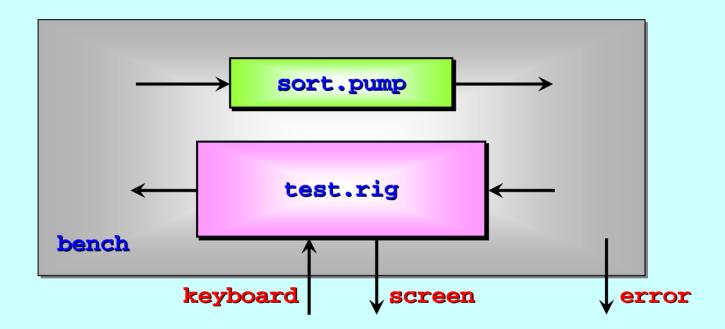
Replicator **STEP** sizes ...



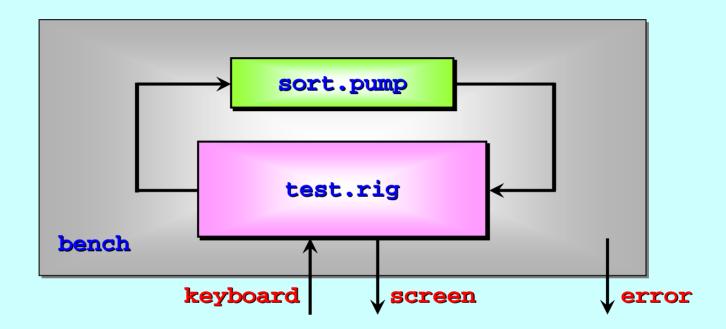
1) Place component (e.g. sort.pump) on bench.



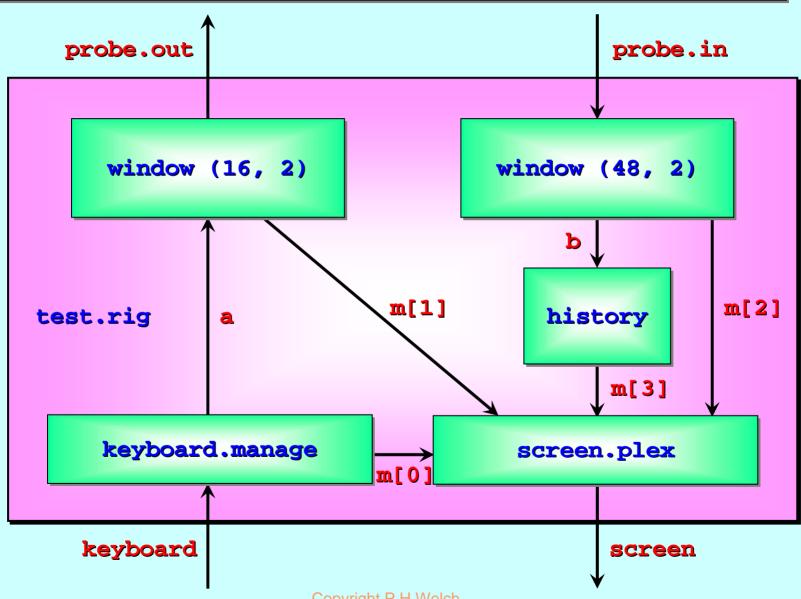
- 1) Place component (e.g. sort.pump) on bench.
- 2) Design test.rig through which we can interact meaningfully with component.



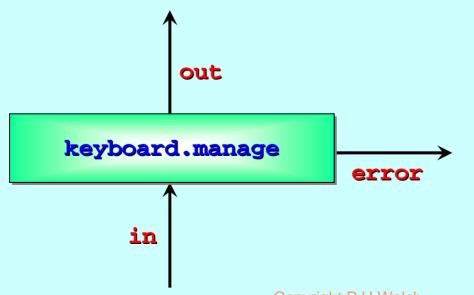
- 1) Place component (e.g. sort.pump) on bench.
- 2) Design test.rig through which we can interact meaningfully with component.
- 3) Wire it up and start experimenting ...

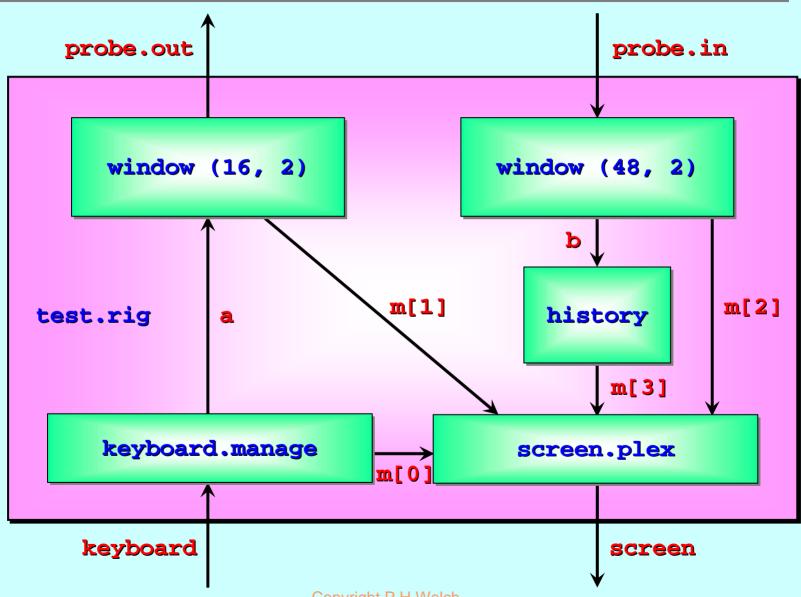


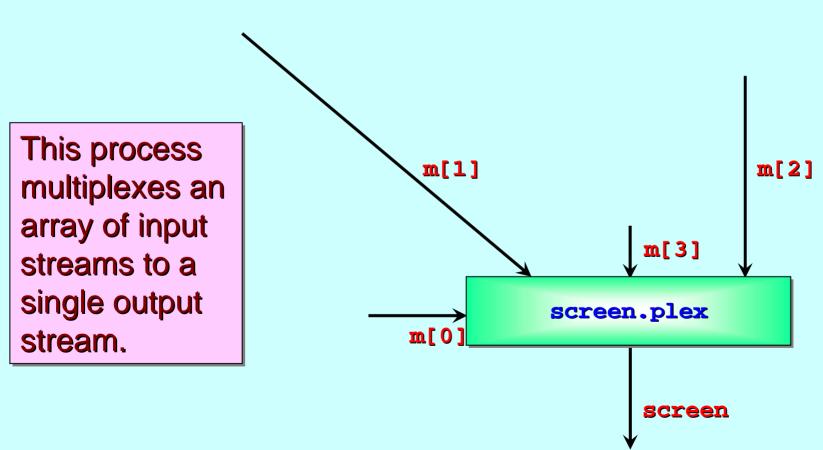
- 1) Place component (e.g. sort.pump) on bench.
- 2) Design test.rig through which we can interact meaningfully with component.
- 3) Wire it up and start experimenting ...

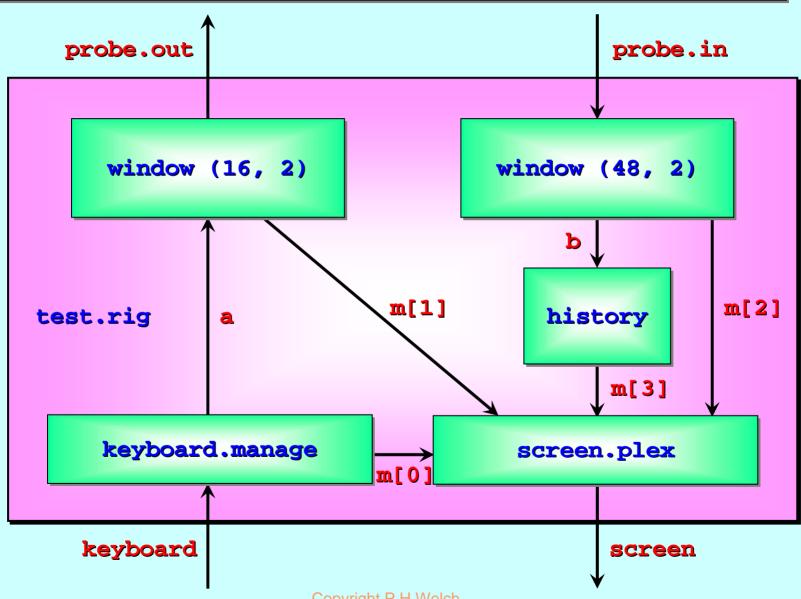


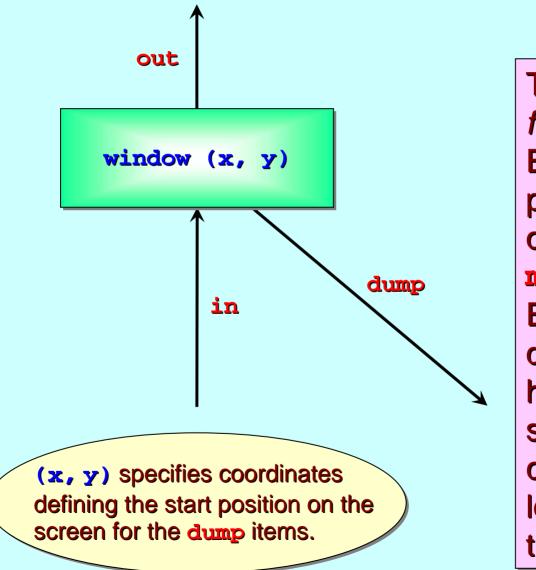
This process filters keyboard input for *'bad'* characters (e.g. control-chars, carriage-return), issuing an error report for any found, and compresses/encodes *'good'* characters (e.g. visible-chars) for onward transmission.



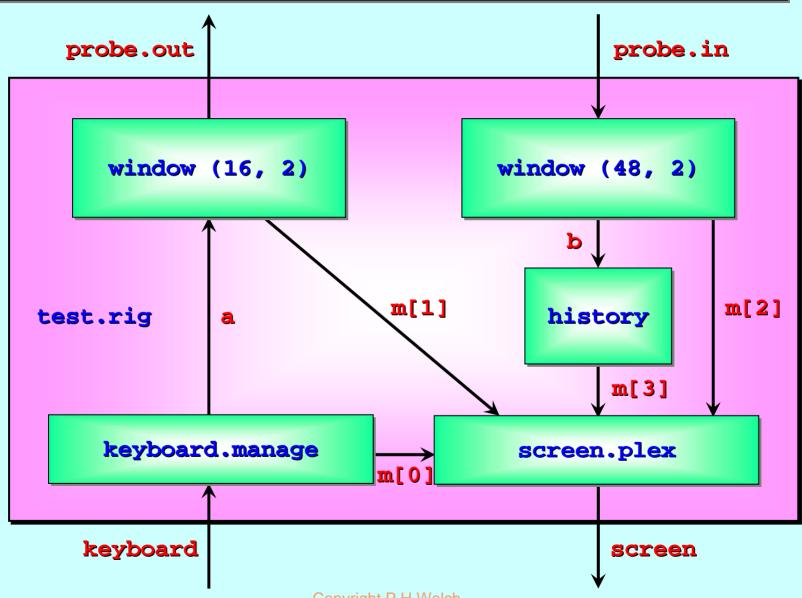


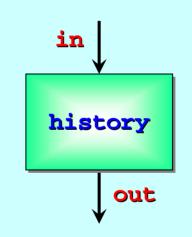






This process is a fixed-size delay line. Each item input pushes one item out. It holds the last max items received. Every cycle, it dumps its entire holding array (with screen position control-chars). This lets us see what's in the data stream.

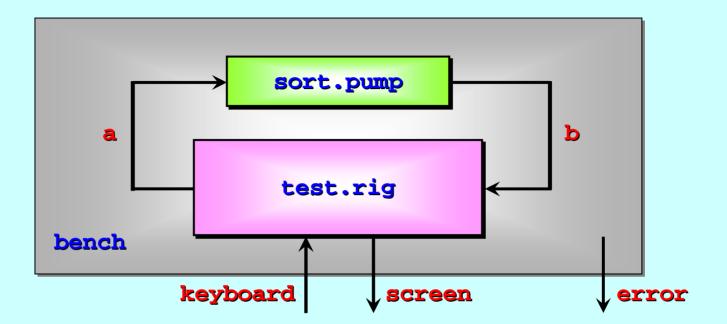




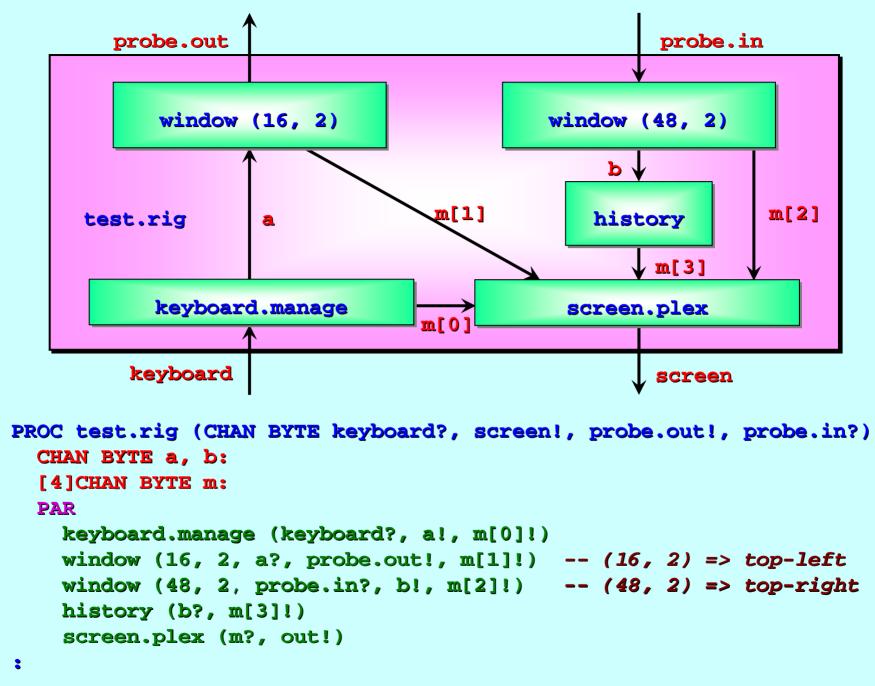
This process lays out a *history* of the items received. It uses the bottom two-thirds of the screen.

Design Guidelines

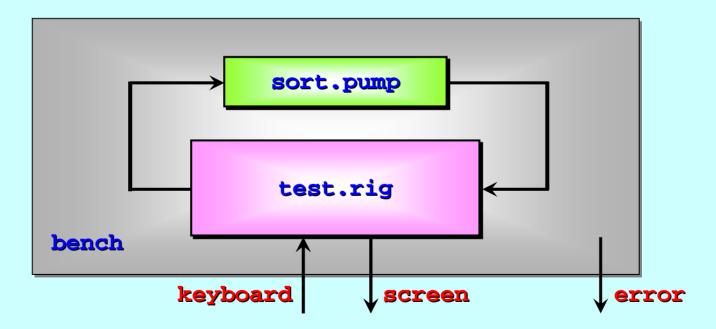
- Don't try to cram too much functionality into any process: One function ⇔ One process
- Multiple functions Multiple processes
- Each process is programmed from its own pointof-view. Think of each process as an independent serial program, with a variety of input and output channels.
- Concurrency then makes design simple! © © ©
 Try to build that test.rig as a single serial process and we will get a mess ...

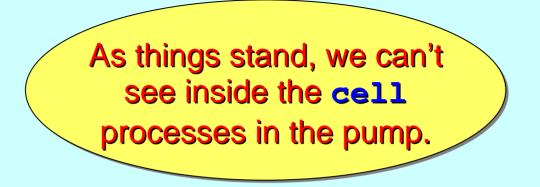


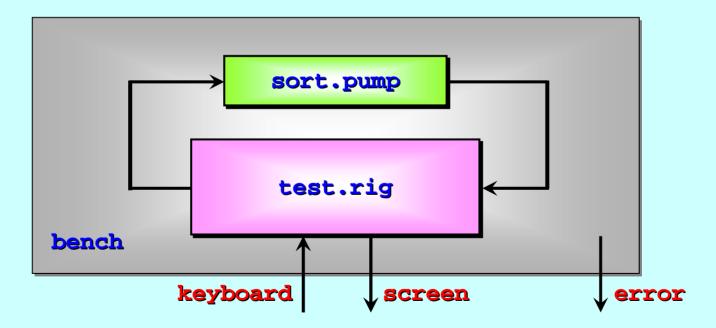
```
PROC bench (CHAN BYTE keyboard?, screen!, error!)
CHAN BYTE a, b:
PAR
sort.pump (a?, b!)
test.rig (keyboard?, screen!, a!, b?)
```



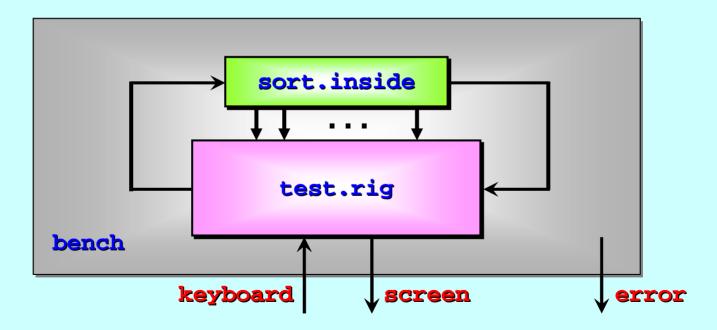




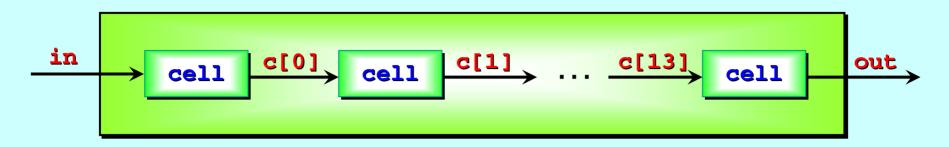




We need to wire up the cells to report their changing states.



VAL INT max IS 16:

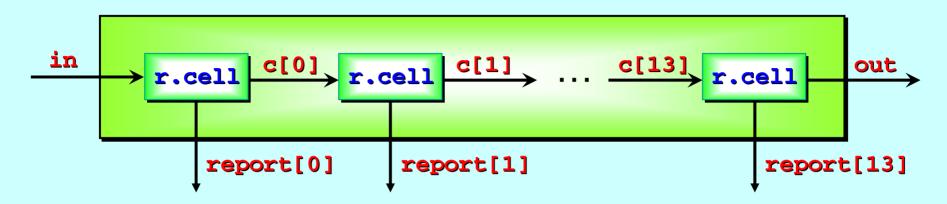


PROC sort.pump (CHAN BYTE in?, out)

```
[max-2]CHAN BYTE c:
PAR
    cell (in?, c[0]!)
    PAR p = 1 FOR max-3
        cell (c[p-1]?, c[p]!)
    cell (c[max-3]?, out!)
```

•

VAL INT max IS 16:



```
PROC sort.inside (CHAN BYTE in?, out!,
        []CHAN BYTE report!)
[max-2]CHAN BYTE c:
PAR
    reporting.cell (in?, report[0]!, c[0]!)
PAR p = 1 FOR max-3
    reporting.cell (c[p-1]?, report[i]!, c[p]!)
    reporting.cell (c[max-3]?, report[max-3]!, out!)
```

•

VAL BYTE end.marker IS 255: -- assume > data items

```
PROC cell (CHAN BYTE in?, out!)
 WHILE TRUE
   BYTE largest:
                                          in
                                                              out
                                                   cell
   SEO
      in ? largest
     WHILE largest <> end.marker
       BYTE next:
       SEQ
          in ? next
         IF
                       -- output smaller, keep larger
           largest >= next
              out ! next
           TRUE -- i.e. largest < next
              SEQ
               out ! largest
               largest := next
      out ! end.marker
```

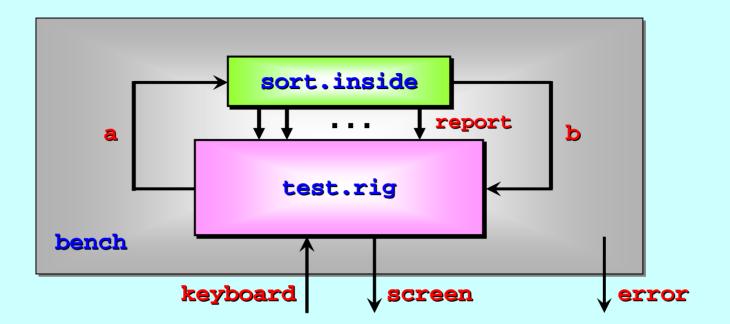
1-Feb-07

2

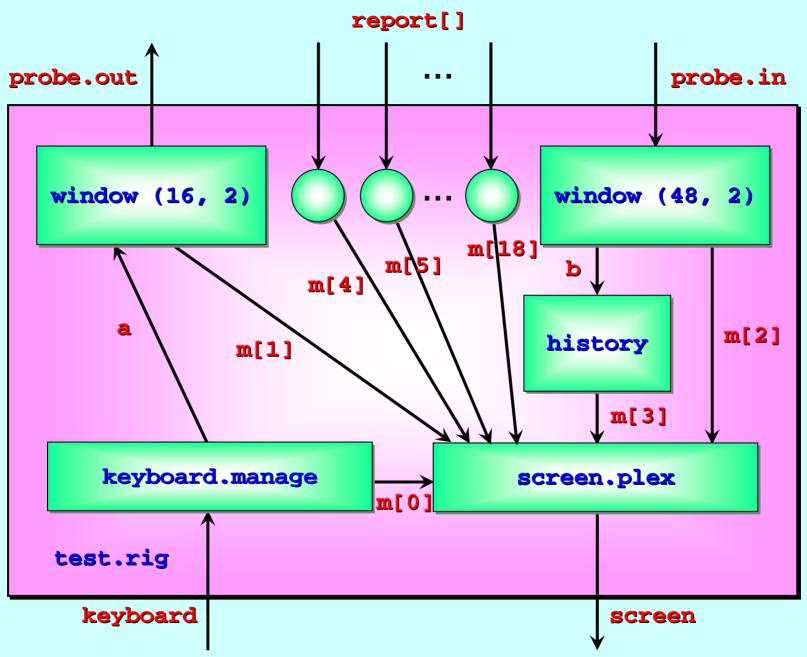
VAL BYTE end.marker IS 255: -- assume > data items

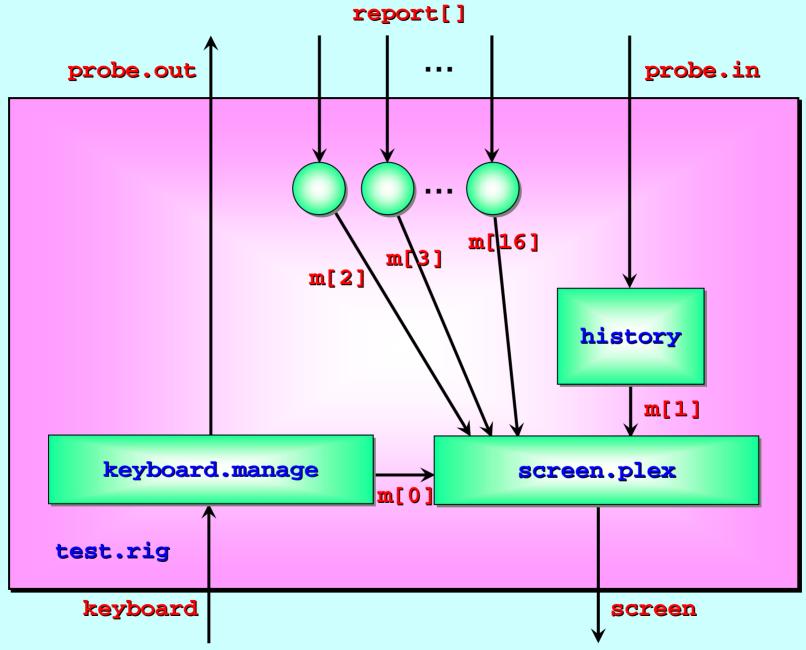
```
PROC reporting.cell (CHAN BYTE in?, report!, out!)
  WHILE TRUE
   BYTE largest:
                                          in
                                                              out
                                                  r.cell
   SEO
      ... report ! '~'; '~'
      in ? largest
                                                      report
      ... report ! '~'; largest
      WHILE largest <> end.marker
       BYTE next:
       SEO
          in ? next
          ... report ! next; largest
          IF
                      -- output smaller, keep larger
           largest >= next
             out ! next
            TRUE
                    -- i.e. largest < next
             SEQ
               out ! largest
              largest := next
          ... report ! '~'; largest
      out ! end.marker
```

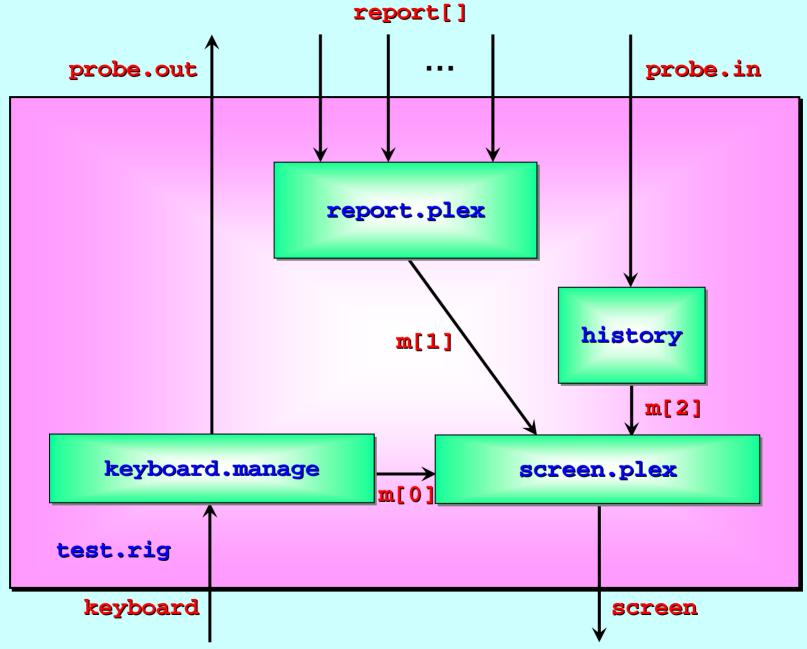
•

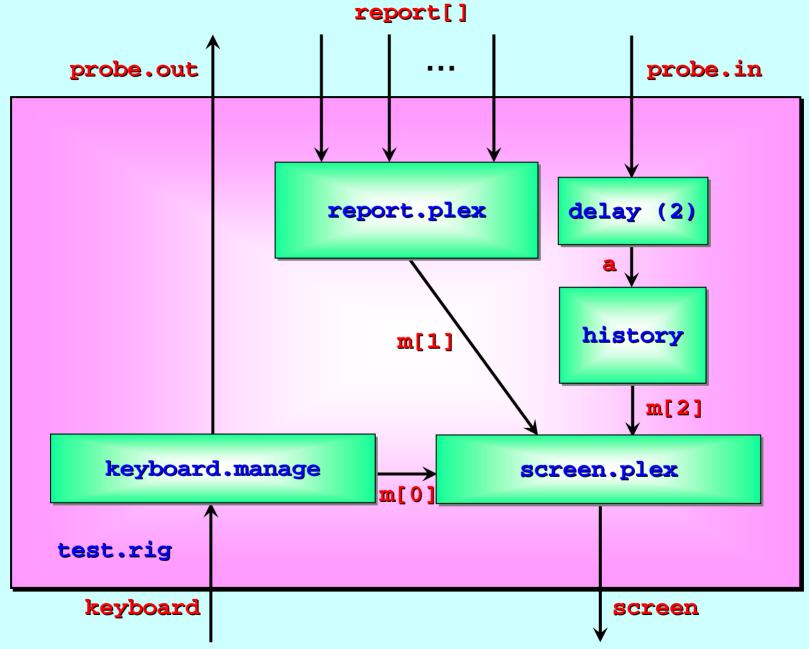


```
PROC bench (CHAN BYTE keyboard?, screen!, error!)
CHAN BYTE a, b:
[max-1]CHAN BYTE report:
PAR
sort.pump (a?, report!, b!)
test.rig (keyboard?, screen!, a!, report?, b?)
:
```









Replicators (components and test-rigs)

Replicated **PAR** and **SEQ** ...

The SORT PUMP ...

Component testing ...

Stateless components ...

The SORT GRID ...

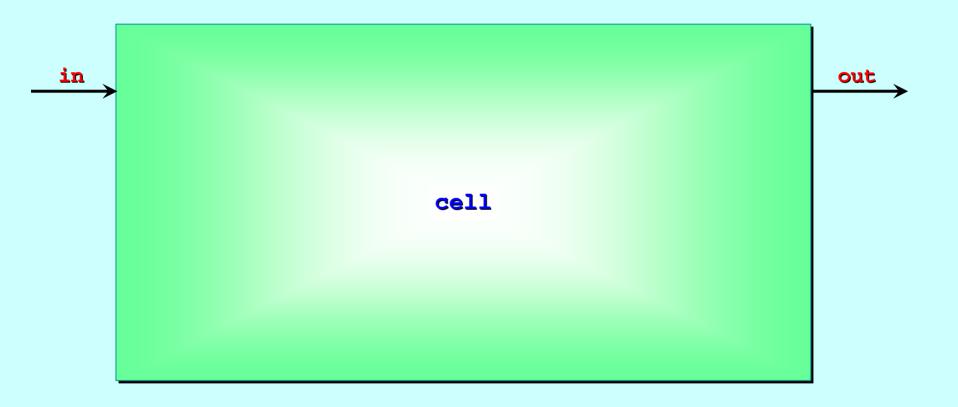
Replicated **IF**...

Replicator **STEP** sizes ...

Let's simplify the logic within a cell process ...

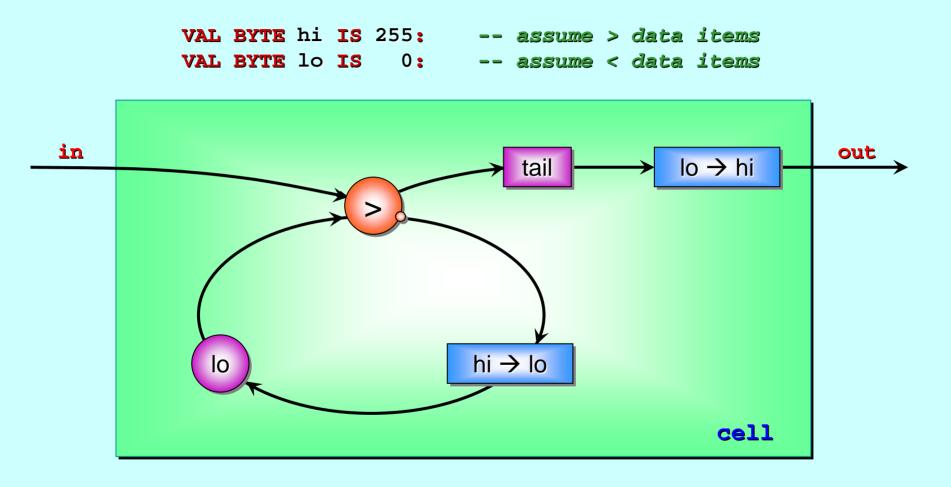
```
VAL BYTE end.marker IS 255: -- assume > data items
PROC cell (CHAN BYTE in?, out!)
  WHILE TRUE
    BYTE largest:
                                            in
                                                                out
                                                     cell
    SEO
      in ? largest
      WHILE largest <> end.marker
        BYTE next:
        SEQ
          in ? next
          IF
                        -- output smaller, keep larger
            largest >= next
              out ! next
                       -- i.e. largest < next
            TRUE
              SEQ
                out ! largest
                                               Here is the serial logic
                largest := next
                                               (a loop within a loop).
      out ! end.marker
2
```

Let's simplify the logic within a **cell** process ...



Here is the *parallel* logic ...

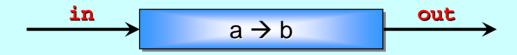
Let's simplify the logic within a **cell** process ...



Here is the *parallel* logic ...

The largest (so far) is trapped in the *feedback loop*.

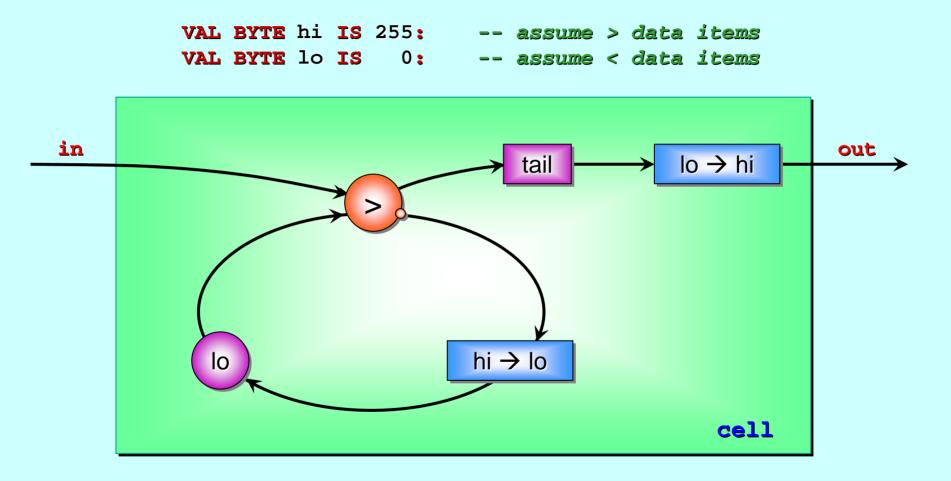
This process copies data through, substituting a for b ...



PROC substitute (VAL BYTE a, b, CHAN BYTE in?, out!)
WHILE TRUE
BYTE x:
SEQ
in ? x
IF
 x = a
 out ! b
 TRUE
 out ! x

•

And finally, let's simplify the logic within a **cell** process ...



Here is the *parallel* logic ...

The largest (so far) is trapped in the *feedback loop*.

This is a primitive comparator ...

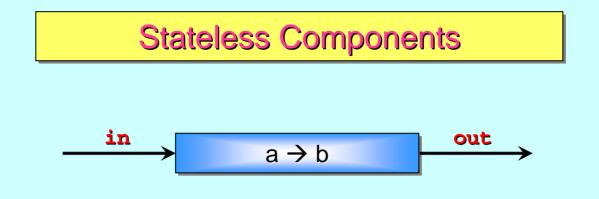
```
PROC greater (CHAN BYTE in.0?, in.1?, small!, large!)
  WHILE TRUE
    BYTE x.0, x.1:
    SEQ
                                          in.0
                                                       small
      PAR
         in.0 ? x.0
                                          in.1
         in.1 ? x.1
                                                       large
       IF
         x_0 < x_1
           PAR
                                         Hence, the asymmetric
              small ! x.0
                                            design of its icon.
              large ! x.1
         TRUE
           PAR
              small ! x.1
                                       Note: gt is symmetric on its
              large ! x.0
                                       input channels, but not on its
•
                                            output channels!
```

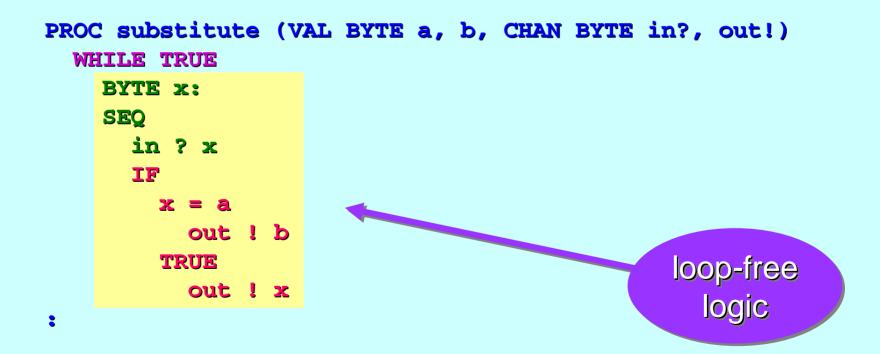
All the *primitive* process components in the *'Legoland'* catalogue (id, succ, plus, delta, prefix, tail, ...) plus the ones just presented (substitute, greater) are *stateless*.

This means they are mathematical functions. They transform input values to output values without reference to past events: the same inputs yield the same outputs. *They have no memory – no state*.

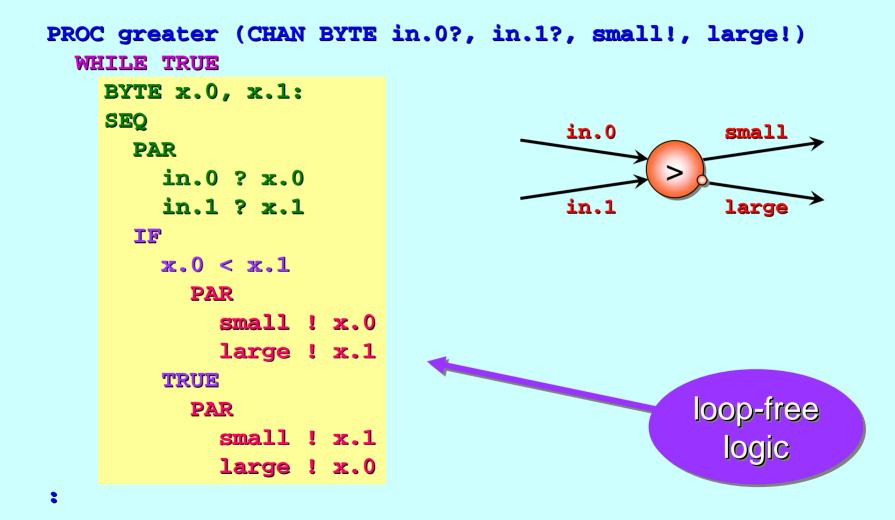
Memory emerges when they are connected in circuits with feedback loops (numbers, integrate, cell, ...).

Stateless components are trivial to reason about – we don't have to think about loops! They are also easy to cast into silicon – as, of course, are circuits built from them.





Stateless Components



Replicators (components and test-rigs)

Replicated **PAR** and **SEQ** ...

The SORT PUMP

Component testing ...

Stateless components ...

The SORT GRID ...

Replicated **IF**...

Replicator **STEP** sizes ...

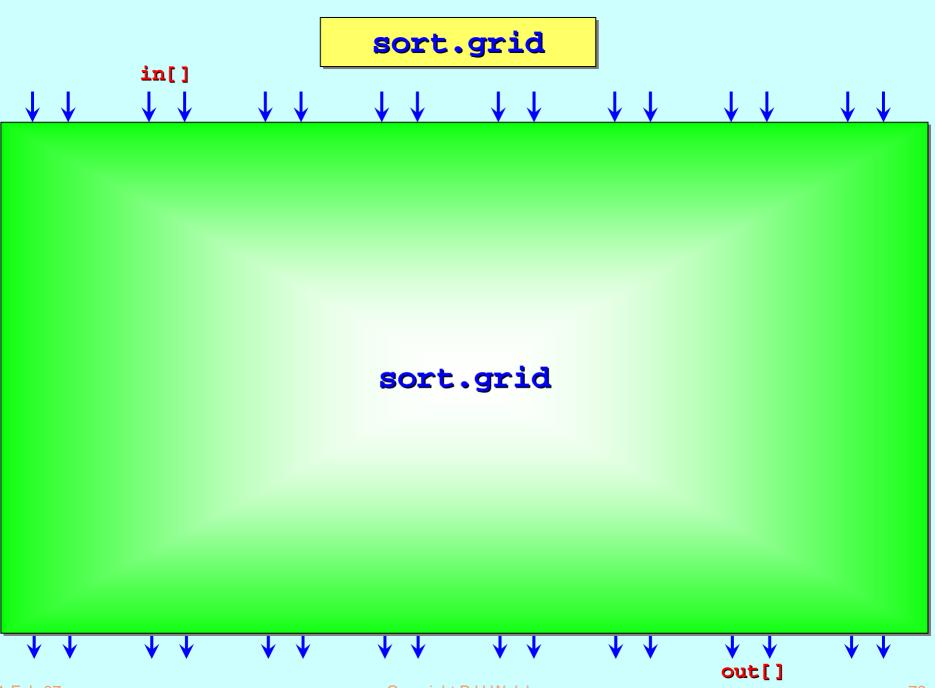
And Finally ...

On a serial processor, bubble-sort takes $O(n^2)$ computation time, where *n* is the number of items being sorted. Cleverer algorithms (such as *quick-sort* or *shell-sort*) take O(n*log(n)).

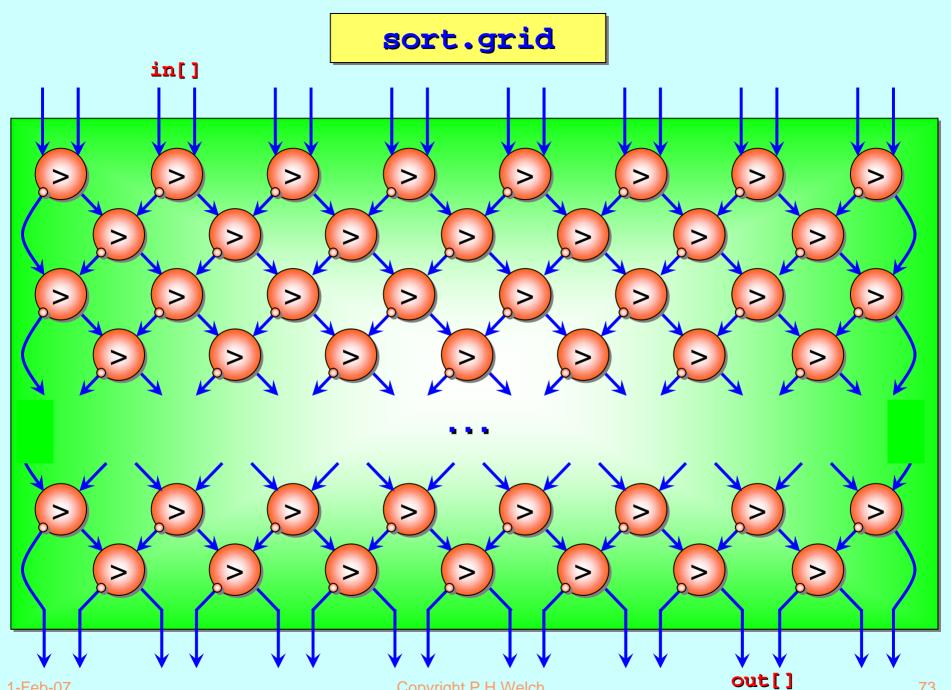
With **O**(*n*) processing elements, the sort-pump takes **O**(*n*) computation time, with respect to each group of *n* items being sorted. If we only present data serially (i.e. one item at a time), supply takes **O**(*n*) time ... so sort-pump cannot be beaten! But we do need a continuous supply of groups.

Question: with $O(n^2)$ processing elements, can we sort groups of *n* items in O(1) time? Of course, we will have to present data in parallel (i.e. O(1) time) and have a continuous supply.

Answer: Yes. And it's easy!



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If the comparators are implemented on separate pieces of silicon *(i.e. we have a physically parallel engine)*, the speed at which data flows through is the *slowest* of:

- the speed at which data is offered;
- the cycle speed for each comparator;
- the inter-cell communication speed.

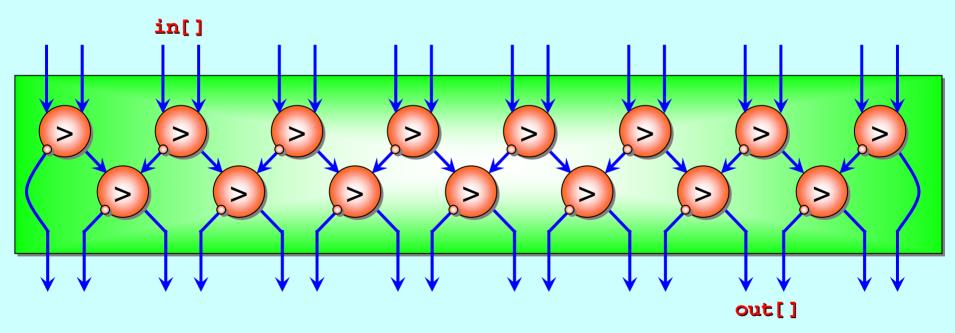
The speed is independent of the number of comparators – which means that it is independent of the number of items being sorted.

Each group of data *enters* and *exits* the grid *in parallel*. All comparators operate *in parallel*. After each *(unit time)* cycle, a sorted group emerges. We have an *O(1)* sorting engine:

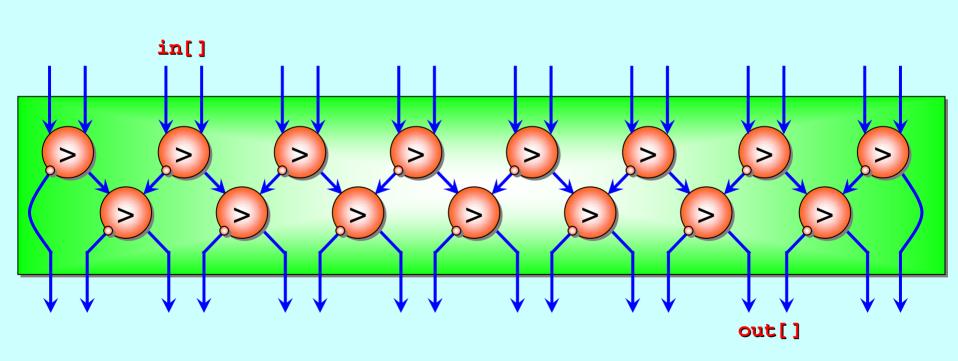
sort.grid

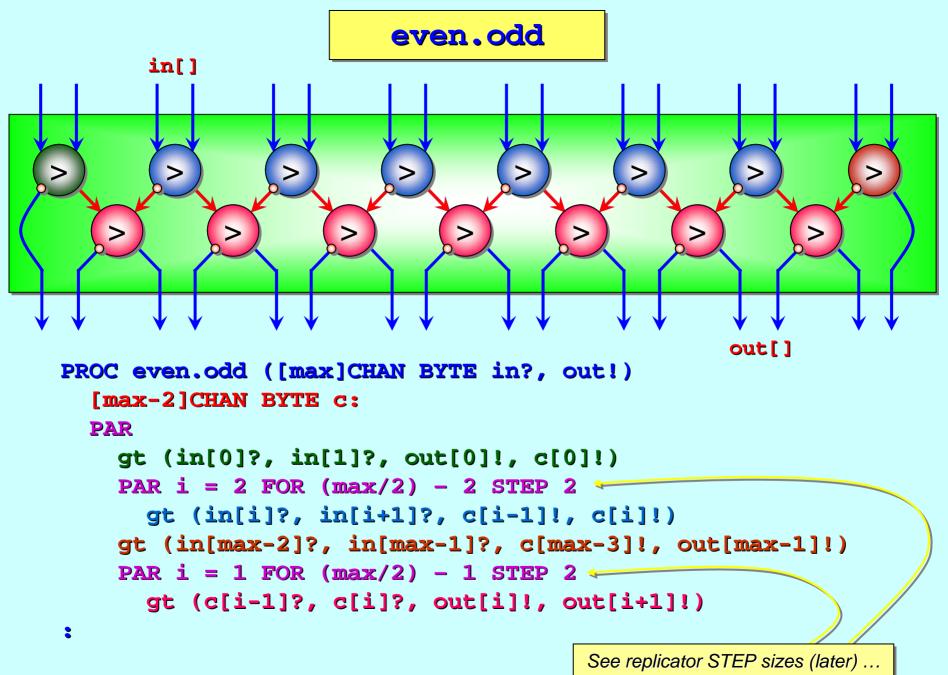
For groups up to size 16, we need 16 rows of (gt) comparators. The even rows have 8 each and the odd rows have 7.

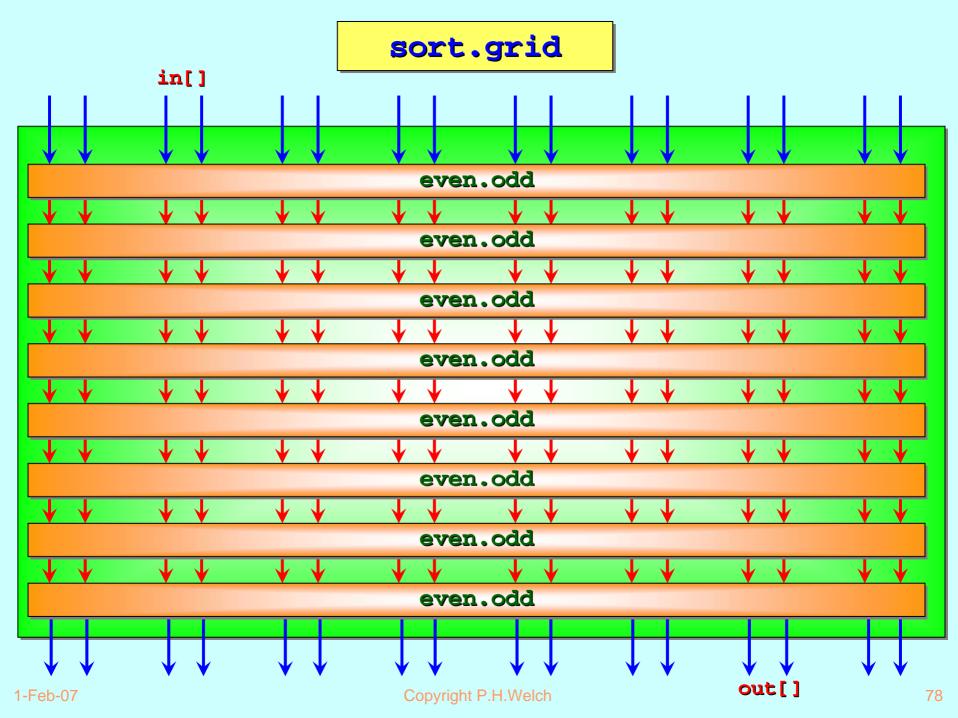
Coding: to keep things easy, let's first program an even-odd pair of rows ...



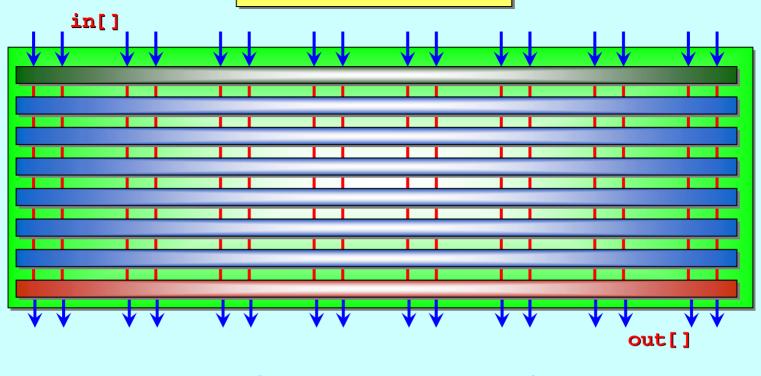
even.odd







sort.grid



```
PROC sort.grid ([max]CHAN BYTE in?, out!)
  [(max/2)-1][max]CHAN BYTE c:
  PAR
  even.odd (in?, c[0]!)
  PAR i = 0 FOR (max/2) - 2
    even.odd (c[i]?, c[i+1]!)
  even.odd (c[(max/2)-2]?, out!)
;
```

Exercise:





Replicators (components and test-rigs)

Replicated **PAR** and **SEQ** ...

The SORT PUMP

Component testing ...

Stateless components ...

The SORT GRID ...

Replicated **IF**...

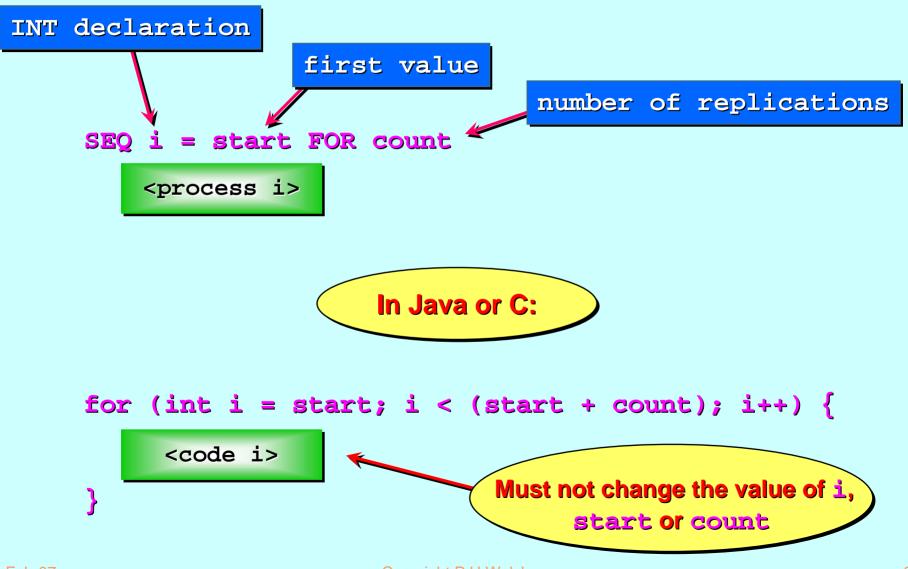
Replicator **STEP** sizes ...

Summary of Replicators (SEQ, PAR)



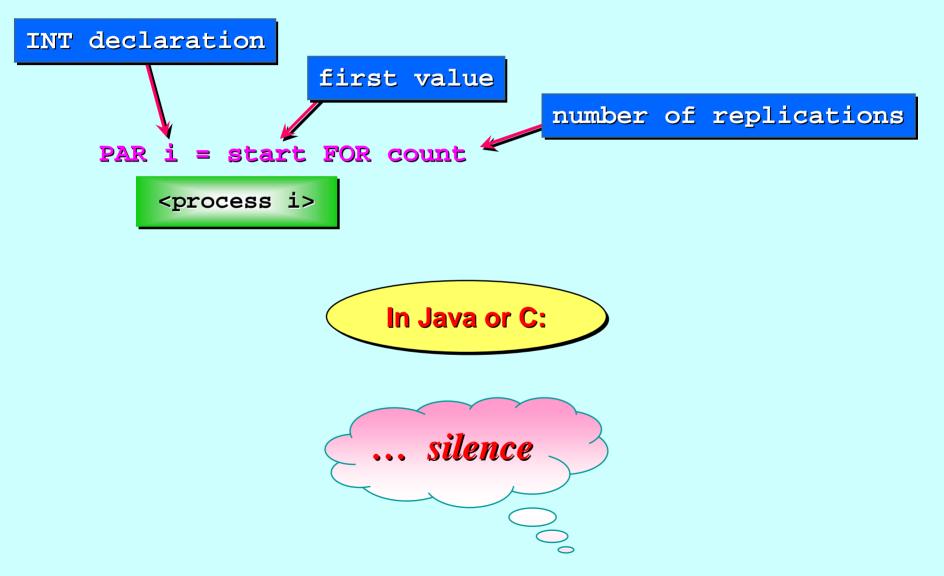
One New Replicator (IF)

The replicated SEQ is like a very clean for-loop.



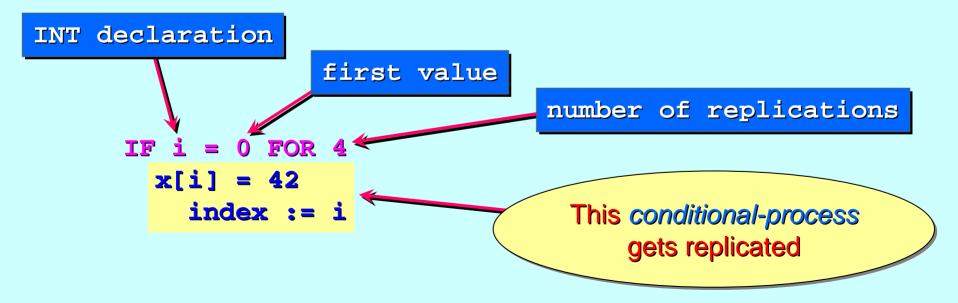
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The replicated PAR has no correspondence in Java or C.



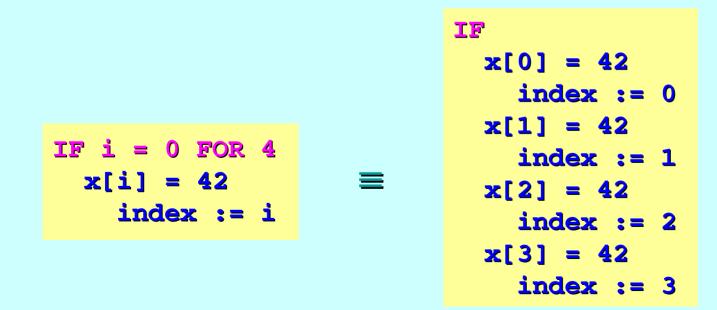
So far, we have seen the **occam**- π process constructors **SEQ**, **PAR**, **IF** and **WHILE**. (Still to come are **ALT** and **CASE**.)

We have seen how **seq** and **par** can be **replicated**. So, also, can the **IF** and (later) the **ALT**. Here is a **replicated IF**:

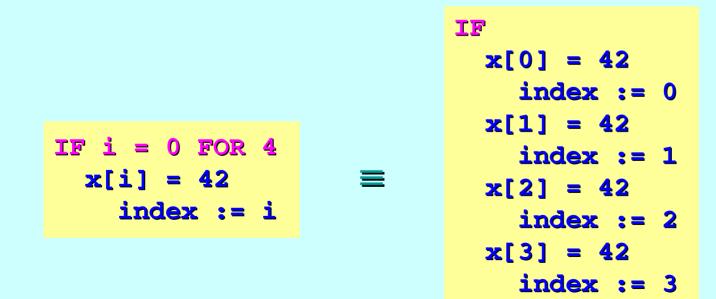


So far, we have seen the **occam**- π process constructors **SEQ**, **PAR**, **IF** and **WHILE**. (Still to come are **ALT** and **CASE**.)

We have seen how **seq** and **par** can be **replicated**. So, also, can the **if** and (later) the **alt**. Here is a **replicated if**:



This code searches the first 4 elements of the array x for the value 42. The search is **sequential**, starting from element 0 and proceeding upwards. If successful, the variable **index** is set to the (first) index of the x array element equal to the target. If unsuccessful, this code will crash!



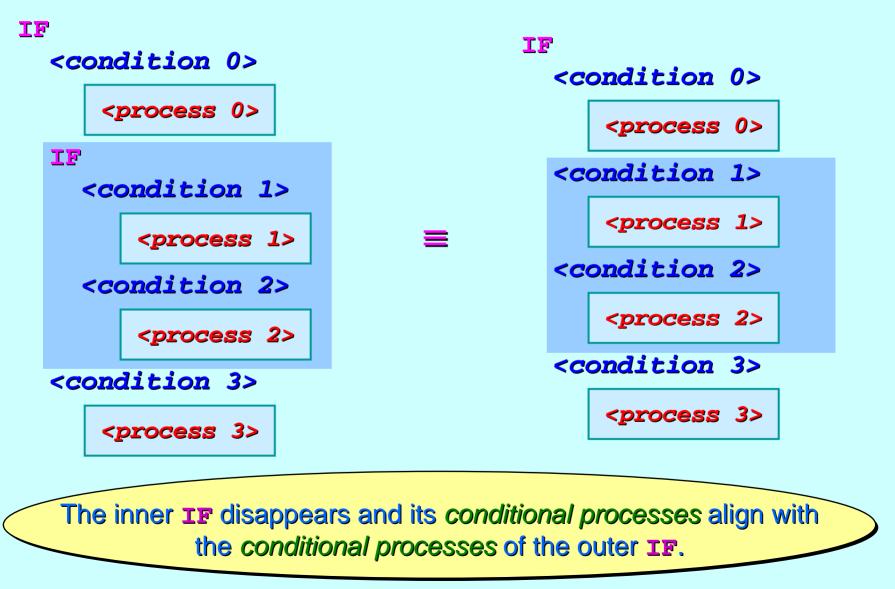
To avoid that crash, we need a final condition that catches the flow of control should all the other conditions fail:

| IF | |
|----------------------|----|
| $\mathbf{x[0]} = 42$ | |
| index := | 0 |
| $\mathbf{x[1]} = 42$ | |
| index := | 1 |
| $\mathbf{x[2]} = 42$ | |
| index := | 2 |
| $\mathbf{x[3]} = 42$ | |
| index := | 3 |
| TRUE | |
| index := | -1 |

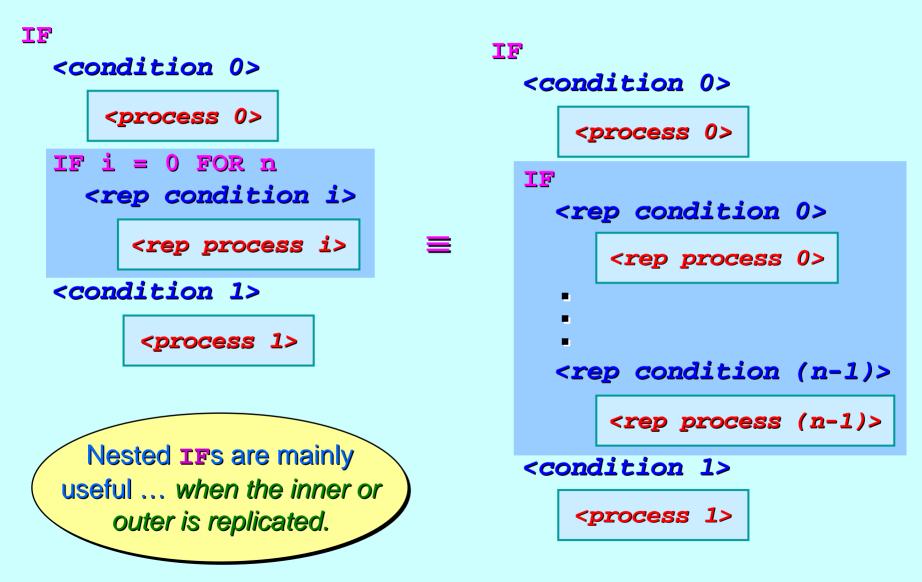
To express this using an **IF**-replicator (which we need if we were searching the through n elements, where n is known only at run-time), we need a *nested* **IF**...

where **index** is set to **-1**, an *illegal array index*, used here to indicate that the **search failed**.

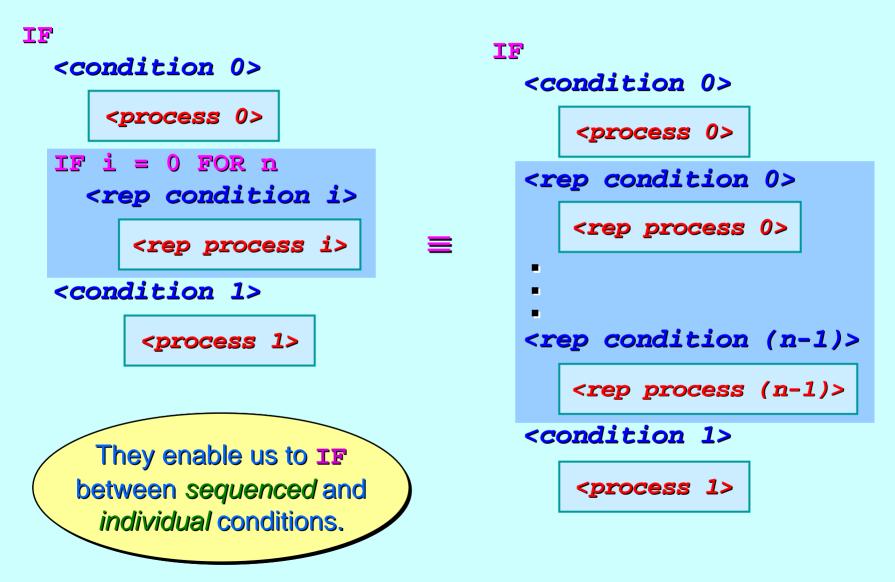
Nested IF's

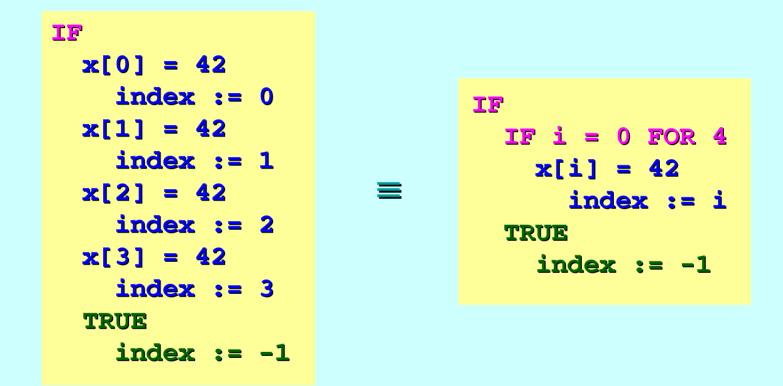


Nested IF's



Nested IF's



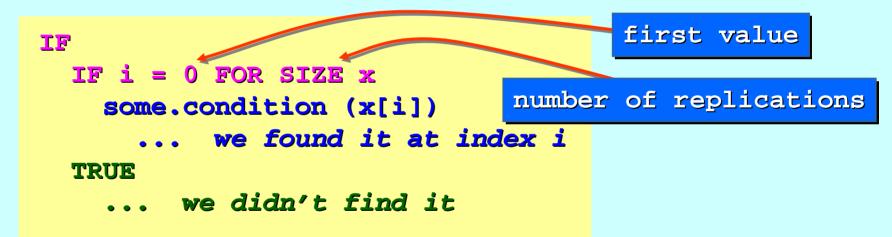


where **index** is set to **-1**, an *illegal array index*, used here to indicate that the **search failed**.

Bounded Linear Search (occam $-\pi$)

The **'bounded linear search'** is the only common use for a **replicated IF** – but it is a good one!

Problem: find the index of the first element of some array, x, that matches some.condition():



Note: the above code searches (potentially) the whole array. We can restrict the search by setting *first* and *replicate* values (of the *replicated* **IF**) appropriately.

Bounded Linear Search (Java / C)

Problem: find the index of the first element of some array, x, that matches some.condition():

```
{ int i = 0;
 bool found = false;
 for (i = 0; i < x.length; i++) {
    if (someCondition (x[i])) {
      found = true;
      break;
    }
  if (found) {
    ... we found it at index i
  } else {
    ... we didn't find it
  }
}
```

Bounded Linear Search (Java / C)

Problem: find the index of the first element of some array, **x**, that matches **some.condition()**:

Actually, this can be expressed in almost a compact form as in **occam**- π ... but we need to resort to a **labelled block** with **non-local break-out**.

```
BLS: {
  for (int i = 0; i < x.length; i++) {
    if (someCondition (x[i])) {
        ... we found it at index i
        break BLS;
    }
    ... we didn't find it
}</pre>
```

Replicators (components and test-rigs)

Replicated **PAR** and **SEQ** ...

The SORT PUMP

Component testing ...

Stateless components ...

The SORT GRID

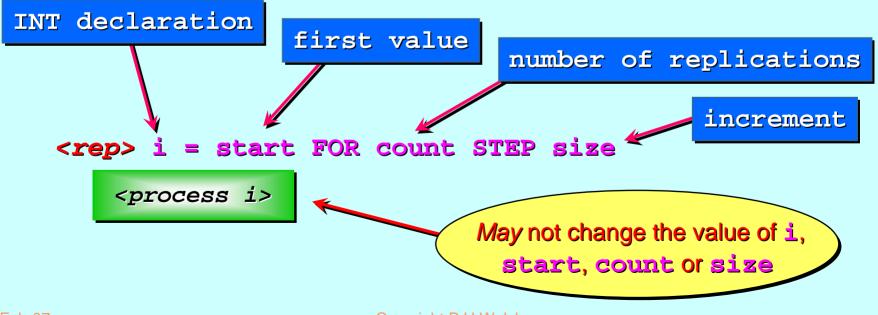
Replicated **IF**...

Replicator **STEP** sizes ...

Replicator STEP Sizes

Normally, the replicator control value increments by **1** for each replicated instance.

However, we may define an arbitrary **STEP** size for this increment:

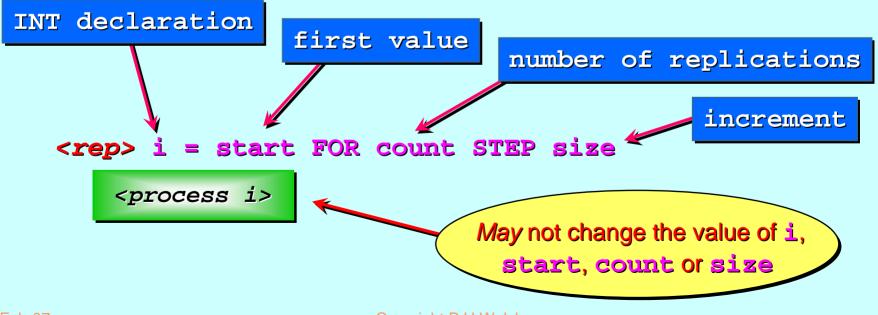


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Replicator STEP Sizes

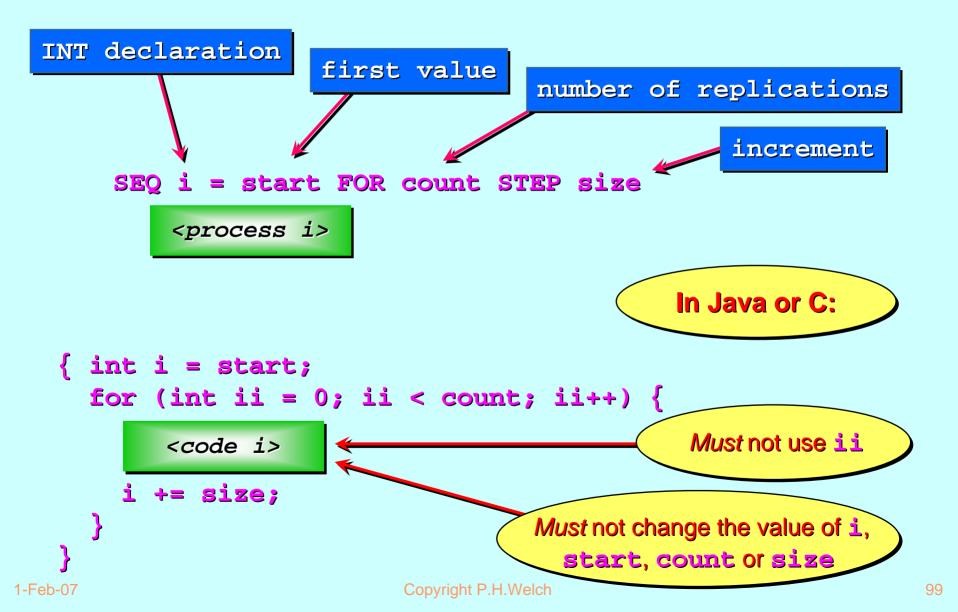
The **<rep>** constructor is one from: **SEQ**, **PAR**, **IF** and *(later)* **ALT**.

The start, count and size may be any INT expressions. The values of i and *any variables* in start, count and size cannot be changed by the replicated process.

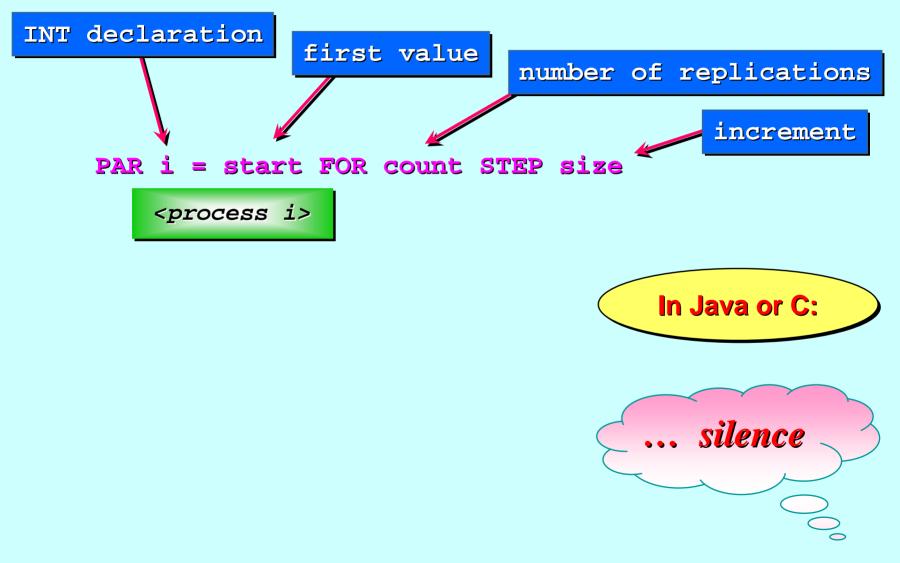


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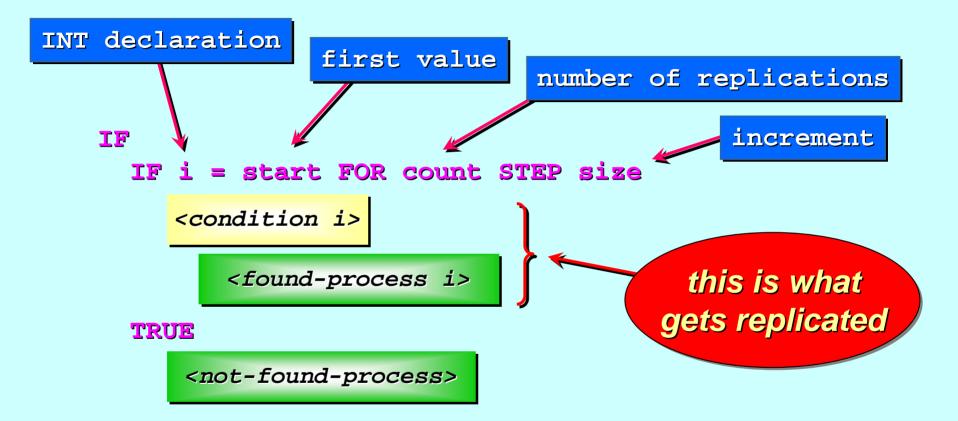
Summary: a replicated SEQ is a very clean for-loop.



The replicated PAR has no correspondence in Java or C.



The replicated IF gives a 'Bounded Linear Search'



Unless we know that the search will succeed, we must nest the *replicated* IF inside a plain IF to catch any failure.

'Stepping and Bounded Linear Search' (Java / C)

