## Some occam- $\pi$ Basics

Peter Welch (p.h.welch@kent.ac.uk)
Computing Laboratory, University of Kent at Canterbury

Co631 (Concurrency)

# Some occam- $\pi$ Basics 

Communicating processes ...
A flavour of occam- $\pi$.
Networks and communication ...
Types, channels, processes ...
Primitive processes ...
Structured processes ...
'Legoland' ...


## Communicating Sequential Processes (CSP)

A mathematical theory for specifying and verifying complex patterns of behaviour arising from interactions between concurrent objects.

CSP has a formal, and compositional, semantics that is in line with our informal intuition about the way things work.

## Why CSP?

- Encapsulates fundamental principles of communication.
- Semantically defined in terms of structured mathematical model.
- Sufficiently expressive to enable reasoning about deadlock and livelock.
- Abstraction and refinement central to underlying theory.
- Robust and commercially supported software engineering tools exist for formal verification.


## Why CSP?

- CSP libraries available for Java (JCSP, CTJ).
- Ultra-lightweight kernels* have been developed yielding sub-microsecond overheads for context switching, process startup/shutdown, synchronized channel communication and high-level shared-memory locks.
- Easy to learn and easy to apply ...



## Why CSP?

- After 5 hours teaching:
- exercises with 20-30 threads of control
- regular and irregular interactions
- appreciating and eliminating race hazards, deadlock, etc.
- CSP is (parallel) architecture neutral:
- message-passing
- shared-memory



## So, what is CSP?

CSP deals with processes, networks of processes and various forms of synchronisation / communication between processes.

A network of processes is also a process - so CSP naturally accommodates layered network structures (networks of networks).

We do not need to be mathematically sophisticated to work with CSP. That sophistication is pre-engineered into the model. We benefit from this simply by using it.

A process is a component that encapsulates some data structures and algorithms for manipulating that data.

- Both its data and algorithms are private. The outside world can neither see that data nor execute those algorithms! [They are not objects.]

The algorithms are executed by the process in its own thread (or threads) of control.

- So, how does one process interact with another?

- The simplest form of interaction is synchronised messagepassing along channels.

The simplest forms of channel are zero-buffered and point-to-point (i.e. wires).

- But, we can have buffered channels (blocking/overwriting).
- And any-1, 1-any and any-any channels.
- And structured multi-way synchronisation (e.g. barriers) ...
- And high-level (e.g. CREW) shared-memory locks ...


## Synchronised Communication



A may write on $\boldsymbol{c}$ at any time, but has to wait for a read.
$\boldsymbol{B}$ may read from $\boldsymbol{c}$ at any time, but has to wait for a write.

$$
(A(c) \| B(c)) \backslash\{c\}
$$

## Synchronised Communication



Only when both $\boldsymbol{A}$ and $\boldsymbol{B}$ are ready can the communication proceed over the channel $\mathbf{c}$.

$$
(\mathbb{A}(c) \| B(c)) \backslash\{c\}
$$

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## occam- $\pi$ : Aspirations and Principles

## Simplicity

- There must be a consistent (denotational) semantics that matches our intuitive understanding for Communicating Mobile Processes.
- There must be as direct a relationship as possible between the formal theory and the implementation technologies to be used.
- Without the above link (e.g. using C++/pthreads or Java/monitors), there will be too much uncertainty as to how well the systems we build correspond to the theoretical design.
- Dynamics
- Theory and practice must be flexible enough to cope with process mobility, location awareness, network growth and decay, disconnect and re-connect and resource sharing.
- Performance
- Computational overheads for managing (millions of) evolving processes must be sufficiently low so as not to be a show-stopper.
- Safety
- Massive concurrency - but no race hazards, deadlock, livelock or process starvation.


## occam- $\pi$

- Process, communication, networks (PAR)
- Choice between multiple events (ALT)
- Mobile data types
$\checkmark$ Mobile channel types
$\rightarrow$ Mobile process types
- Performance
+ shared channels,
channel bundles, alias checking, no race hazards, dynamic memory, recursion, forking, no garbage, protocol inheritance, extended rendezvous, process priorities, ...


## Processes and Channel-Ends



An occam process may only use a channel parameter one-way (either for input or for output). That direction is specified (? or !), along with the structure of the messages carried - in this case, simple INTs. The compiler checks that channel usage within the body of the PROC conforms to its declared direction.

## Processes and Channel-Ends



PROC integrate (CHAN INT in?, out!) INITIAL INT total IS 0: WHILE TRUE INT x :
SEQ
in ? $x$
total := total + x out ! total


## With an Added Kill Channel



PROC integrate.kill (CHAN INT in?, out!, kill?)
INITIAL INT total IS 0: INITIAL BOOL ok IS TRUE:
... main loop
:


## Choosing between Multiple Events



## Parallel Process Networks



PROC integrate (CHAN INT in?, out!) CHAN INT $a, b, c$ : PAR
plus (in?, c?, a!) delta (a?, out!, b!) prefix (0, b?, c!)


## With an Added Kill Channel



PROC integrate.kill (CHAN INT in?, out !, kill?) CHAN INT a, b, c, d: PAR

poison (in?, kill?, d!)
plus (d?, c?, a!)
delta (a?, out!, b!)
prefix (0, b?, c!)

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## OCCDM

... from the top
(components, networks and communication)

$$
\begin{aligned}
& \text { PROC P (CHAN INT a!, b?, } \\
& \text { CHAN BOOL c?, } \\
& \text { CHAN BYTE d!, e!) }
\end{aligned}
$$



PROC Q (CHAN INT a?, b!, c?, CHAN BOOL d!)


PROC R (CHAN BYTE a?, b!)

## PROC S (CHAN INT a?, b!, CHAN BOOL c!, CHAN INT d!)



```
PROC T (CHAN BYTE a?,
    CHAN BOOL b?,
    CHAN BYTE c?)
```





CHAN INT $f, g, h, m:$
CHAN BOOL $i, 1:$
CHAN BYTE j, k, n, o:


CHAN INT $f, g, h, m:$
CHAN BOOL $i, 1:$
CHAN BYTE j, k, n, o:
PAR



CHAN INT $f, g, h, m:$
CHAN BOOL $i, 1$ :
CHAN BYTE j, k, n, o:
PAR
$P(f!, m ?, i ?, j!, ~ o!)$
$Q(f ?, g!, h ?, i!)$
$R(j ?, k!)$
$R(0 ?, \mathrm{n})$
$S(g ?, \mathrm{~h}, \mathrm{~m}, \mathrm{l}, \mathrm{l})$
$\mathrm{T}(\mathrm{k}, \mathrm{l}, \mathrm{l}, \mathrm{n})$



CHAN INT $f, g, h, m:$
CHAN BOOL $i, 1$ :
CHAN BYTE j, k, n, o:
PAR
$P(f!, m ?, i ?, j!, ~ o!)$
$Q(f ?, g!, h ?, i!)$
$R(j ?, k!)$
$R(0 ?, \mathrm{n})$
$S(g ?, \mathrm{~h}, \mathrm{l}, \mathrm{l}, \mathrm{m}!)$
$\mathrm{T}(\mathrm{k}, \mathrm{l}, \mathrm{l}, \mathrm{n})$


## Synchronised Unbuffered Communication



CHAN INT c :
PAR

$$
\begin{aligned}
& \text { P0 (c!) } \\
& \text { P1 (c? ) }
\end{aligned}
$$

PROC P0 (CHAN INT out!)
out ! value


PROC P1 (CHAN INT in?)


# Synchronised Unbuffered communication 

## out ! value

- Output value down the channel out
- This operation does not complete until the process at the other end of the channel inputs the information
- Until that happens, the outputting process sleeps (possibly forever!)


## Synchronised Unbuffered Communication

```
in ? x
```

- Input the next piece of information from channel in into the variable $x$
- This operation does not complete until the process at the other end of the channel outputs the information
- Until that happens, the inputting process sleeps (possibly forever!)
- The inputting process can set "timeouts" on these inputs or choose between alternative inputs. [We will do this later]


# Synchronised Unbuffered Communication ("Rendezvous") 

- Unified concept of synchronisation and unbuffered communication.
- Asynchronous and buffered communication are easy to construct (later).
- Incoming communications are selectable.
- Hardware model: it is fast to implement.
- Hardware model: our intuition enables us to reason about it (see the Legoland slides).


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## occam- $\pi$

... from the bottom

## Types



## Operators


$+,-, *, 1, \$
$<,<=,>=,>$
=, <>

INTxx, INTxx $\rightarrow$ INTxx BYTE, BYTE $\rightarrow$ BYTE

INTxx, INTxx $\rightarrow$ BOOL
BYTE, BYTE $\rightarrow$ BOOL REALxx, REALxx $\rightarrow$ BOOL

There is strong typing for all expressions ...

## Operators



INTxx, INTxx $\rightarrow$ BOOL
$<,<=,>=,>$ BYTE, BYTE $\rightarrow$ BOOL
REALxx, REALxx $\rightarrow$ BOOL
$=,<>\quad$ *, * $\rightarrow$ BOOL

There is strong typing for all expressions ...

## Expressions

No auto-coercion happens between types: if x is a real32 and $\mathbf{i}$ is an INT, then $\boldsymbol{x}+\boldsymbol{i}$ is illegal ...

Where necessary, explicit casting between types must be programmed: e.g. x + (REAL32 ROUND i) ...

To cast between types, use the target type name as a prefix operator.
If rounding mode is significant, this must be specified (ROUND or TRUNC) following the target type name (as above).

No precedence is defined between operators, we must use brackets: e.g. a + (b*c) ...

## Expressions

The operators + , -, * and / trigger run-time errors if their results overflow.

In Java and C, such errors are ignored.
Therefore, the operators + and * are non-associative and we must use more brackets: e.g. a + (b + c) ...

The INT operators plus, MINUS and TIMES wrap-around (i.e. do not trigger run-time errors) if the results overflow.

The occam- $\pi$ plus, minus and times are the same as the Java/C +, - and *.
pLus, MINUS and times are mainly used for calculating timeouts.

## Operators



AFTER relates to $>$ in the same way as pLus relates to + .
They both do arithmetic operations, but the former ignores overflow. If ( $\theta<\mathrm{t}<=$ MOSTPOS INTxx), then ( s PLUS t ) is AFTER $s$, even if wrap-around occurs and ( $s$ PLus $t$ ) is $<\boldsymbol{s}$.

There is strong typing for all expressions ...

## Operators



There is strong typing for all expressions ...

## Values (named constants)

```
VAL INT max IS 50:
VAL INT double.max IS 2*max:
VAL BYTE letter IS 'A': special chars
VAL []BYTE hello IS "Hello*c*n":
VAL []INT mask IS [#01, #02, #04, #08,
                                #10, #20, #40, #80]:
```

All declarations end in a colon ...
A declaration cannot be used before it is made ...
Character literals have type BYte (their ASCII value) ...
String literals have type []BYTE ...

## Values (named constants)

```
VAL INT max IS 50:
VAL INT double.max IS 2*max:
```

```
VAL BYTE letter IS 'A': special chars
```

VAL []BYTE hello IS "Hello*c*n":
VAL []INT mask IS [\#01, \#02, \#04, \#08,
\#10, \#20, \#40, \#80]:

The compiler fills in the sizes of the hello and mask arrays for us. We could have done this ourselves ([7]Byte and [8]Int respectively).

Declarations are aligned at the same level of indentation ...
Long lines may be broken after commas, etc. ...

## Variable Declarations

INT a, b: two integers
[max]INT c:
[double.max]BYTE d:

## 100 integers

## Timer Declarations

TIMER tim:

## one timer

## Channel Declarations

CHAN BYTE p:
a single channel
[max<<2]CHAN INT q:

## Process Abstractions

$$
\begin{aligned}
& \text { PROC foo (VAL []BYTE s, } \\
& \text { VAL BOOL mode, } \\
& \text { INT result, } \\
& \text { CHAN INT in?, out!, } \\
& \text { CHAN BYTE pause?) }
\end{aligned}
$$


foo (s, mode, result)


CHANnel parameters - for communicating with other processes ...

VAL <type> <id>
VALue (data) parameters - local constants within the PROC body ...
reference (data) parameters - may be changed within the PROC body (with effect on the invoking process) ...

## Process Abstractions

```
PROC foo (VAL []BYTE s,
    VAL BOOL mode,
    INT result,
    CHAN INT in?, out!,
    CHAN BYTE pause?)
```


foo (s, mode, result)


We have just used the three dot notation as a place holder for the PROC body. Code (including any local declarations) goes here. The three dots are not part of occam- $\pi$ syntax!

Note that the PROC body is indented (two spaces) from its PROC header and closing colon.

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## An 0ccam- $\pi$ Process (syntax)

Syntactically, an occam- $\pi$ process consists of:
... an optional sequence of declarations (e.g. values, variables, timers, channels, procs, channel protocols*, ports*, data types*, channel types*, process types*, barriers*, ... )
... a single executable process
All the declarations - and the executable - are aligned at the same level of indentation.

* later ...


## Primitive Processes

## Assignment

$$
a:=c[2]+b
$$

data types on either side of the assignment must matich

## Input (synchronising)

in? a

## Output (synchronising)

out ! a + (2*b)

## There are strong typing rules ...

## Primitive Processes

## What's the time?

tim? t

Timeout (wait until specified time) tim ? AFTER (t PLUS 3000)

Null (do nothing)
SKIP

Suspend (non-recoverable)
STOP

$$
\begin{aligned}
& \text { + BARRIER synchronisation, ... } \\
& \text { (later) }
\end{aligned}
$$

## A Brief History of Time

## What's the time?

```
tim ? t
```

TIMER tim: INT t :
occam- $\pi$ time values are INTs delivered by TIMERS.
These values increment by one every microsecond (for all current, 10/2006, implementations).
occam- $\pi$ time values cycle through all INT values from the most negative (MOSTNEG INT), through zero (0), to the most positive (MOSTPOS INT) and, then, back to the most negative again. occam- $\pi$ time starts at an arbitrary INT value.

## A Brief History of Time



For 32-bit INTs incrementing every microsecond, occam- $\pi$ time values cycle every $\mathbf{7 2}$ minutes (roughly).

## A Brief History of Time



Note that occam- $\pi$ time values increment according to the rules for PLUS (wrap-around).

## A Brief History of Time



So, (a AFTER b) is TRUE if and only if the distance from $\mathbf{b}$ to a going clockwise - in the above diagram - is less than the distance going anti-clockwise.

## A Brief History of Time



Above, we have ( $q$ AFTER $p$ ), ( $\mathbf{r}$ AFTER $q$ ) and ( $p$ AFTER $r$ ). Think of $p, q$ and $r$ as 2, 4 and 9 on a 12-hour clock face and ignore whether they represent am or pm.

## A Brief History of Time



Above, we have ( $q$ AFTER $p$ ), ( $\mathbf{r}$ AFTER $q$ ) and ( $p$ AFTER $r$ ). Note that, using normal arithmetic, we have ( $q>p$ ) and ( $\mathbf{r}>\boldsymbol{q}$ ), but not ( $\mathbf{p}>\boldsymbol{r}$ ).

## A Brief History of Time



Therefore, so long as our timeout periods are less than 36 minutes (i.e. half the time cycle) and we calculate absolute timeout values using PLUS, the AFTER operator always gives the expected time comparisons - even if the time wrap-around occurs.

## A Brief History of Time



Real-time systems tend to deal in microseconds or milliseconds, so 36 minutes is a luxury! If we need to address longer timeouts, some extra (simple) programming effort is required.

## A Brief History of Time



## A Brief History of Time



## A Brief History of Time



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## Structured Processes (SEQ and PAR)

## SEQ



Do these 4
processes in the sequence written

PAR


Do these 4 processes in parallel

## Structured Processes (SEQ example)



Here is a machine with internal variables $\mathbf{x}$ and sum assume they are identical numeric types (e.g. INT).

Let's assume the external channels carry the same type.
Consider the following fragment of code ...

## Structured Processes (SEQ example)



SEQ
in ? sum
in ? $x$
sum := sum + $x$
out ! sum

Any change in the order of these processes impacts the semantics ...

## Structured Processes (PAR example)



Here is another machine with internal variables $\mathbf{x . 0}$, $\mathbf{x . 1}, \mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ - assume they are identical numeric types (e.g. INT).

Let's assume the external channels carry the same type.
Consider the following fragment of code ...

## Structured Processes (PAR example)



PAR

```
in.0 ? x.0
    in.1 ? x.1
    out ! a + b
    c := a + (2*b)
```

The order in which these processes run does not matter ...

## Structured Processes (PAR rules)



## Structured Processes (PAR rules)



The effect of these rules is that the processes cannot interfere with each other's state. If they need to interact, they must explicitly communicate.

## Structured Processes (PAR rules)



No data race hazards are possible. The processes are safe to be scheduled in any order (e.g. on a single-core processor) or in parallel (e.g. on a multi-core processor).

## Structured Processes (IF)

## IF


<boolean>

<boolean>

<boolean>


The <boolean> conditions are evaluated in sequence. Only the process underneath the first TRUE one is executed.


## Structured Processes (IF example)

```
IF
    \(x>0\)
    screen ! 'p"
    \(\mathrm{x}<0\)
        screen ! 'n'
    TRUE
        screen! 'z'
```

> The <boolean> conditions are evaluated in sequence. Only the process underneath the first TRUE one is executed.


## Structured Processes (WHILE)

WHILE <boolean>


Conventional "While-loop"

If the <boolean> is TRUE, the indented process is executed ... then ...
... the <boolean> is checked again ... if it is still TRUE, the indented process is executed again ... then ...
... etc. until ...
... the <boolean> is checked again ... and turns out to be FALSE ... in which case, this WHILE process terminates.

## Structured Processes (WHILE example)

Here is a complete process (a 'chip') that doubles the values of the numbers flowing through it:


```
PROC double (CHAN INT in?, out!)
    WHILE TRUE
        INT X:
        SEQ
            in ? x
            out ! 2*x
```


## Structured Processes (PROC instance)

```
PROC foo (VAL []BYTE s,
    VAL BOOL mode,
    INT result,
    CHAN INT in?, out!,
    CHAN BYTE pause?)
```



To create an instance, we must plug in correctly typed arguments - for example:

$$
\begin{aligned}
& \text { foo ("Goodbye World*c*n", TRUE, solution, } \\
& \text { q[i]?, q[i+1]!, my.pause?) }
\end{aligned}
$$

VAL parameters must be passed expressions of the correct type. An expression could be a simple variable or literal.

## Structured Processes (PROC instance)

```
PROC foo (VAL []BYTE s,
    VAL BOOL mode,
    INT result,
    CHAN INT in?, out!,
    CHAN BYTE pause?)
```



To create an instance, we must plug in correctly typed arguments - for example:

$$
\begin{aligned}
& \text { foo ("Goodbye World*c*n", TRUE, solution, } \\
& \text { q[i]?, q[i+1]!, my.pause?) }
\end{aligned}
$$

Reference parameters must be passed variables of the correct type. Changes to those parameters by the instanced process will be apparent in those variables when (if) the process instance terminates.

## Structured Processes (PROC instance)

```
PROC foo (VAL []BYTE s,
    VAL BOOL mode,
    INT result,
    CHAN INT in?, out!,
    CHAN BYTE pause?)
```



To create an instance, we must plug in correctly typed arguments - for example:

$$
\begin{aligned}
& \text { foo ("Goodbye World*c*n", TRUE, solution, } \\
& \text { q[i]?, q[i+1]!, my.pause?) }
\end{aligned}
$$

Channel parameters must be passed the correct ends (? or !) of correctly typed channels.

## Structured Processes (PROC instance)

Process instances used in SEQuence with other processes are sometimes referred to as procedures. For example:

```
INT answer:
SEQ
    out.string ("The answer is ", 0, screen!)
    ... calculate answer
out.int (answer, 0, screen!)
out.string ("*c*n", 0, screen!)
```

The processes out.string and out.int are from the basic utilities library ("course. lib") supporting this course. They output their given string (respectively integer) as ASCII text to their channel parameter and terminate. Their middle parameter is a minimum fieldwidth.

## Structured Processes (PROC instance)

Process instances used in PARallel with other processes are are sometimes referred to as components (or just processes). For example:


PROC octople (CHAN INT in?, out!)
CHAN INT $a, b:$
PAR
double (in?, a!)
double (a?, b!)
double (b?, out!)

This component scales by 8 the numbers flowing through it ...

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'Legoland'

## ‘Legoland' Catalog



## ‘Legoland' Catalog




## 'Legoland’ Catalog



## 'Legoland' Catalog



## ‘Legoland' Catalog



# 'Legoland' Catalog 



## ‘Legoland' Catalog

This is a catalog of fine-grained processes - think of them as pieces of hardware (e.g. chips).

They process data (INTs) flowing through them.

They are presented not because we suggest working at such fine levels of granularity ...
... they are presented in order to build up fluency in working with parallel logic.

## 'Legoland' Catalog

Parallel logic should become just as easy to manage as serial logic.

This is not the traditionally held view ...
... but that tradition is wrong.


Let's look at some occam- $\pi$ code for these processes ...

```
PROC id (CHAN INT in?, out!)
    WHILE TRUE
        INT X:
        SEQ
            in ? x
            out ! x
:
PROC succ (CHAN INT in?, out!)
    WHILE TRUE
        INT X:
        SEQ
            in ? x
            out ! x + 1
:
PROC black.hole (CHAN INT in?)
    WHILE TRUE
        INT X:
        in ? x
:
```

PROC plus (CHAN INT in.0?, in.1?, out!)
WHILE TRUE
INT x.0, x.1:
INT x.0, x.1:
SEQ
SEQ
PAR
PAR
in.0 ? x.0
in.0 ? x.0
in.1 ? x.1
in.1 ? x.1
out ! x.0 + x.1
out ! x.0 + x.1

PROC delta (CHAN INT in?, out.0!, out.1!)
WHILE TRUE
INT X:
SEQ
in ? $x$
PAR

out. 0 ! x
out. 1 ! $x$

```
PROC prefix (VAL INT n,
    CHAN INT in?, out!)
    SEQ
        out ! n
        id (in, out)
:
```


PROC tail (CHAN INT in?, out!)


is a blocking FIFO buffer of capacity 6

## Good News!

The good news is that we can 'see' this semantic equivalence with just one glance.
[CLAIM] CSP semantics cleanly reflects our intuitive feel for interacting systems.

This quickly builds up confidence ...


## Good News!

Let's build some simple circuits from these catalog components.

Can you see what they do ...?


And how to describe them in occam- $\pi \ldots$ ?



```
PROC numbers (CHAN INT out!)
    CHAN INT a, b, c:
    PAR
        delta (a?, out!, b!)
        succ (b?, c!)
        prefix (0, c?, a!)
:
```


PROC integrate (CHAN INT in?, out!)
CHAN INT $a, b, c:$
PAR
delta (a?, out!, b!)
prefix (0, b?, c!)
plus (in?, c?, a!)
:

PROC pairs (CHAN INT in?, out!)
CHAN INT $a, b, c:$
PAR
delta (in?, a!, c!)
tail (a?, b!)
plus (b?, c?, out!)


Note: this pushes numbers out so long as the receiver is willing to take it.


Note: this outputs one number for every inpult it gets.


Note: this needs two inputs before producing one output. Thereafter, it produces one number for every input it gets.


## ‘Legoland' Catalog

Of course, these components also happen to have simple sequential implementations ...

The parallel ones just shown were just to build fluency in CSP concurrency.

CSP (and occam $-\pi$ ) enables parallel and sequential logic to be built with equal ease.

In practice, sometimes parallel and sometimes sequential logic will be most appropriate - just choose the simplest.

Parallel logic is not, by nature, especially difficult.

## Sequential Version



PROC numbers (CHAN INT out!) INT n :
SEQ
n := 0
WHILE TRUE SEQ
out ! $n$
$n:=n+1$
:

## Sequential Version



## ‘Legoland' Catalog

Let's build some more circuits from the components just constructed (either the sequential or parallel versions).

If we build using the parallel ones, we have layered networks - circuits within circuits.


No problem!



```
PROC fibonacci (CHAN INT out!)
```

CHAN INT a, b, c, d:
PAR
delta (a?, b!, out!)
pairs (b?, c!)
prefix (0, d?, a!)
prefix (1, c?, d!)


PROC squares (CHAN INT out!)
CHAN INT $a, b:$
PAR
numbers (a!)
integrate (a?, b!)
pairs (b?, out!)


Note: the two numbers needed by PairsInt to get started are provided by the two PrefixInts.
Thereafter, only one number circulates on the feedback loop. If only one PrefixInt had been in
 the circuit, deadlock would have happened (with each process waiting trying to input).

Note: the traffic on individual channels:

$$
\begin{aligned}
& \text { <a> }=\left[\begin{array}{llllll}
0, & 1, & 1, & 2, & 5, & 8, \\
13, & 21, \ldots
\end{array}\right] \\
& \text { <out> }=\left[\begin{array}{llll}
{[0,} & 1, & 1, & 2, \\
3, & 5, & 13, & 21, \ldots
\end{array}\right] \\
& \langle b\rangle \quad=\left[\begin{array}{llllll}
0, & 1, & 1, & 2, & 5, & 8, \\
13, & 21, \ldots
\end{array}\right] \\
& \text { <c> }=[1,2,3,5,8,13,21,34,55, \ldots] \\
& \text { <d> }=[1,1,2,3,5,8,13,21,34, \ldots]
\end{aligned}
$$



Note: the traffic on individual channels:

$$
\begin{aligned}
& \text { <a> }=\left[\begin{array}{llllll}
0 & 1, & 2, & 3, & 5, & 6, \\
7, & 8, \ldots
\end{array}\right] \\
& \text { <b> =[0, 1, 3, 6, 10, 15, 21, 28, 36, ...] } \\
& \text { <out> = [1, 4, 9, 16, 25, 36, 49, 64, 81, ...] }
\end{aligned}
$$

At this level, we have a network of 5 communicating processes.

```
PROC demo (CHAN BYTE out!)
    [4]CHAN INT C:
    PAR
        numbers(c[0]!)
        squares(c[1]!)
        fibonacci (c[2]!)
        times (c[3]!)
        lay.out (c?, out!)
```



In fact, 28 processes are involved: 18 non-terminating ones and 10 low-level transients (repeatedly starting up and shutting down for parallel input and output). BUT we don't need to know that to reason at this level ... ();) ();) (;)

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Fortunately, CSP semantics are compositional - which means that we only have to reason at each layer of the network in order to design, understand, code, and maintain it.

