Principles of Concurrent and Distributed Programming

(Second Edition)

Addison-Wesley, 2006

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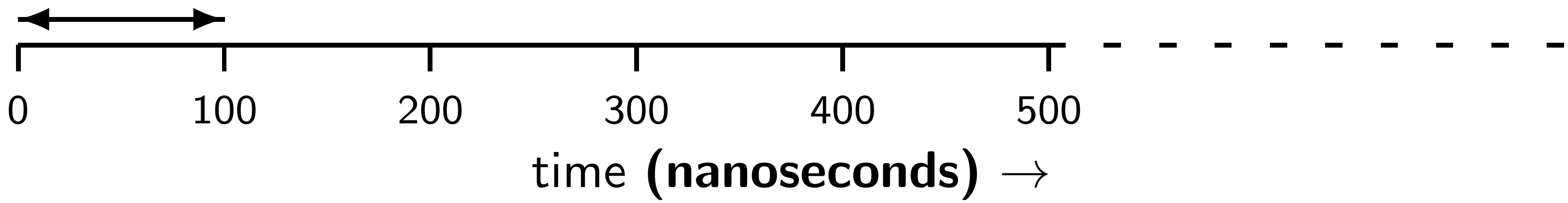
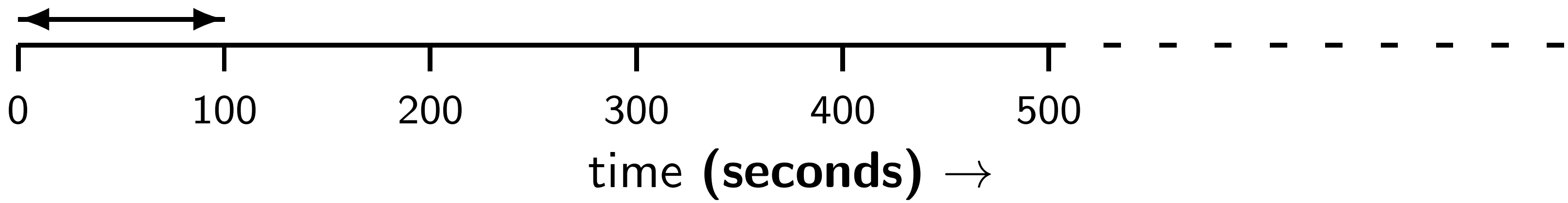
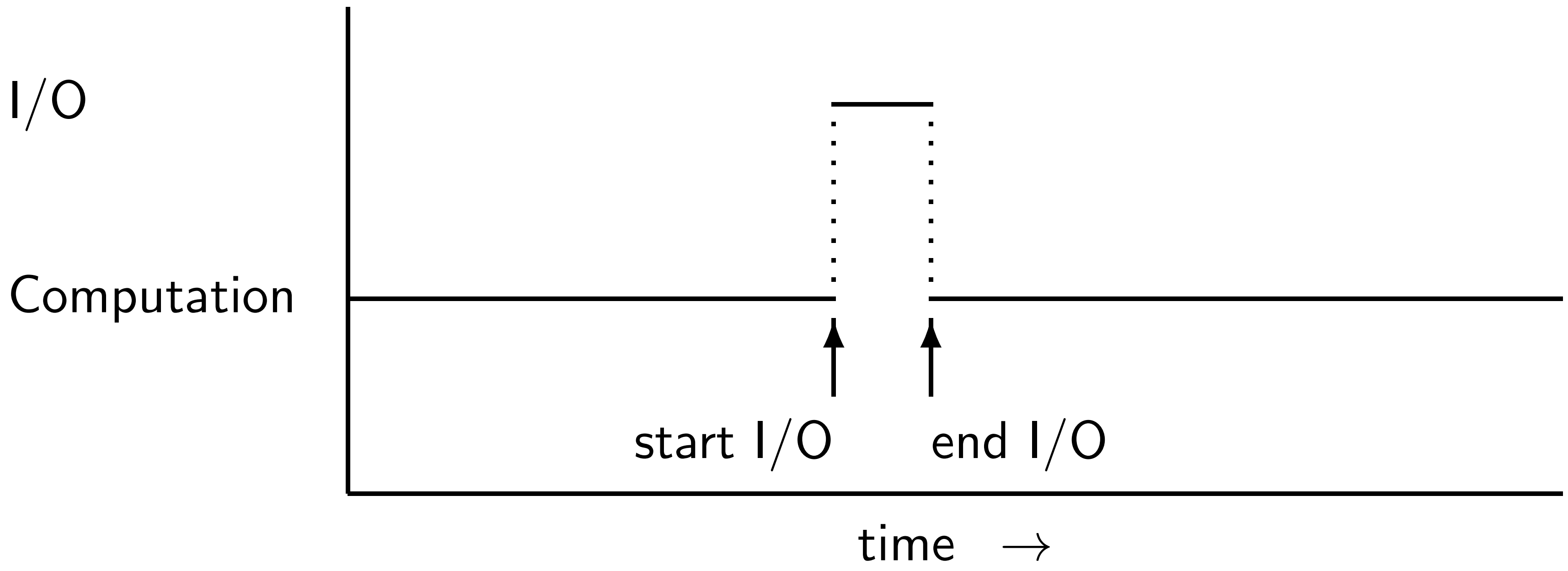
Computer Time



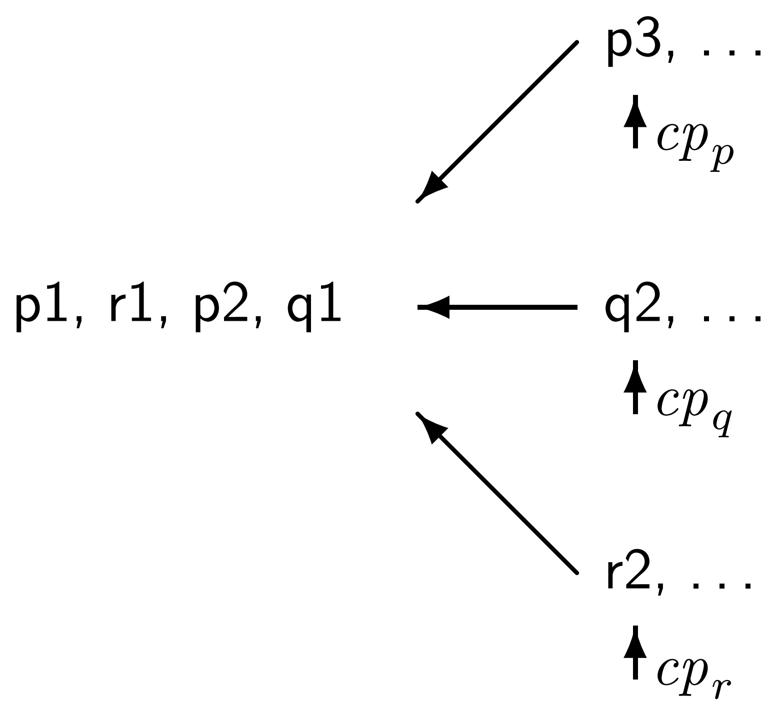
Human Time



Concurrency in an Operating System



Interleaving as Choosing Among Processes



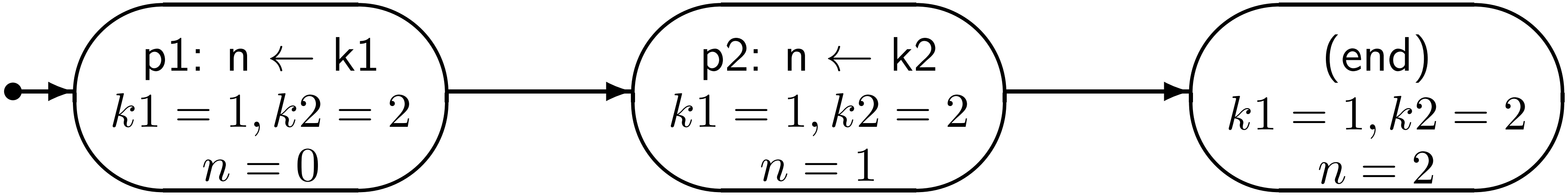
Possible Interleavings

p1→q1→p2→q2,
p1→q1→q2→p2,
p1→p2→q1→q2,
q1→p1→q2→p2,
q1→p1→p2→q2,
q1→q2→p1→p2.

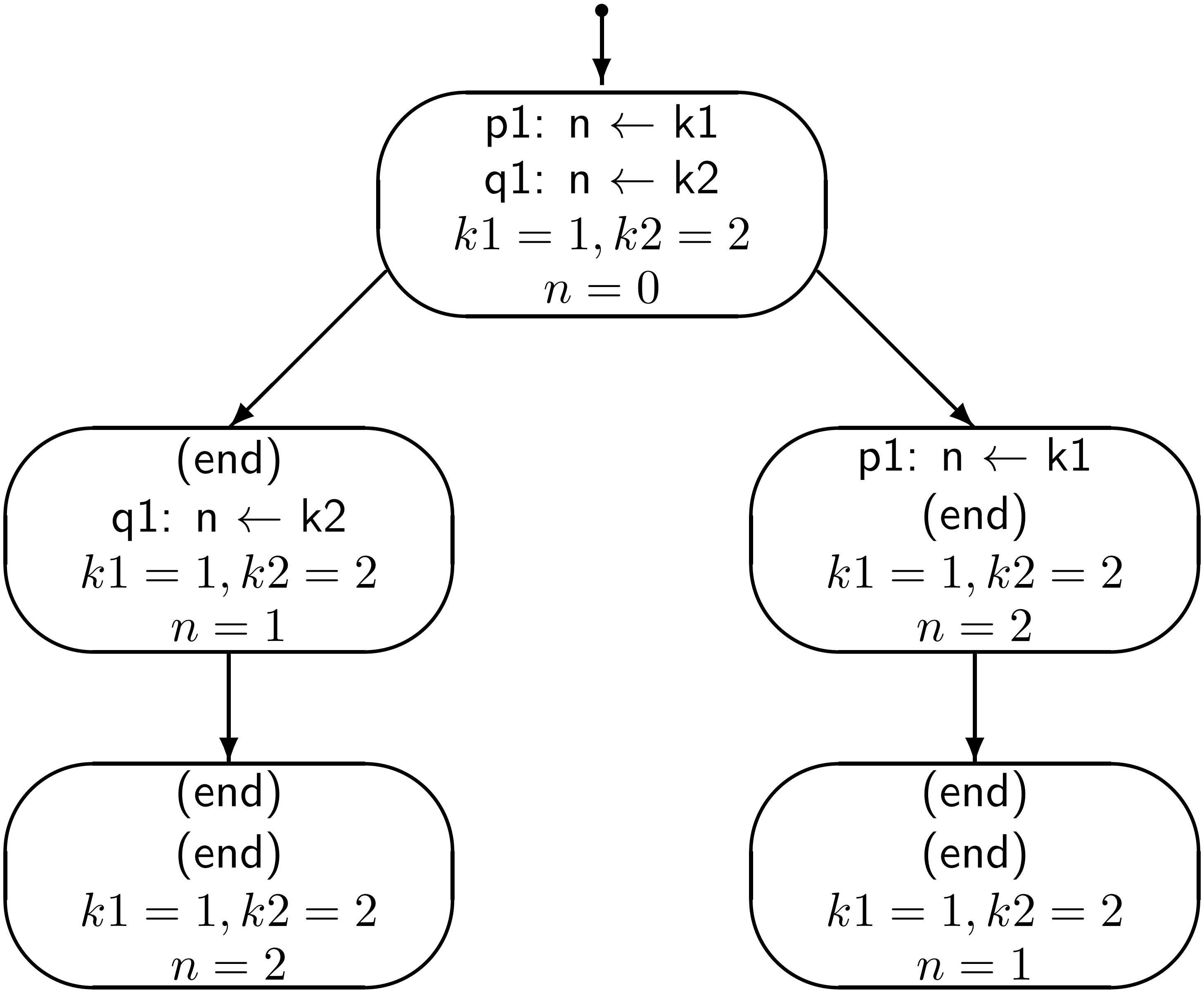
Algorithm 2.1: Trivial concurrent program	
integer n $\leftarrow$ 0	
p	**q**
integer k1 $\leftarrow$ 1	integer k2 $\leftarrow$ 2
p1: n $\leftarrow$ k1	q1: n $\leftarrow$ k2

Algorithm 2.2: Trivial sequential program	
integer n $\leftarrow$ 0	
	integer k1 $\leftarrow$ 1
	integer k2 $\leftarrow$ 2
p1:	n $\leftarrow$ k1
p2:	n $\leftarrow$ k2

State Diagram for a Sequential Program



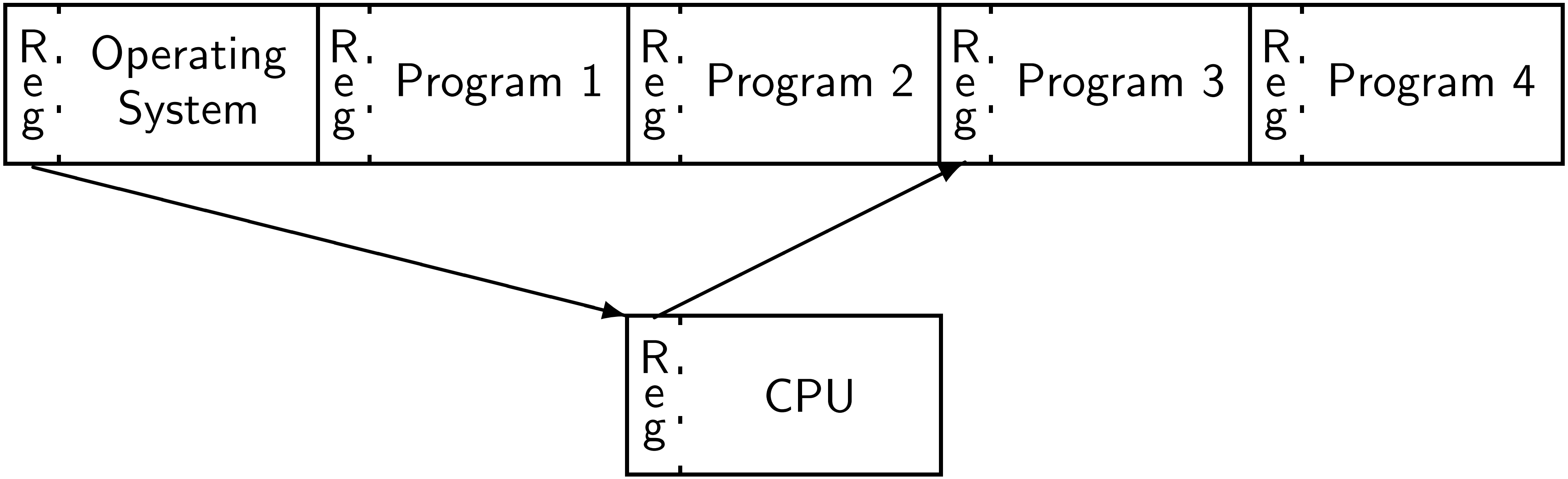
State Diagram for a Concurrent Program



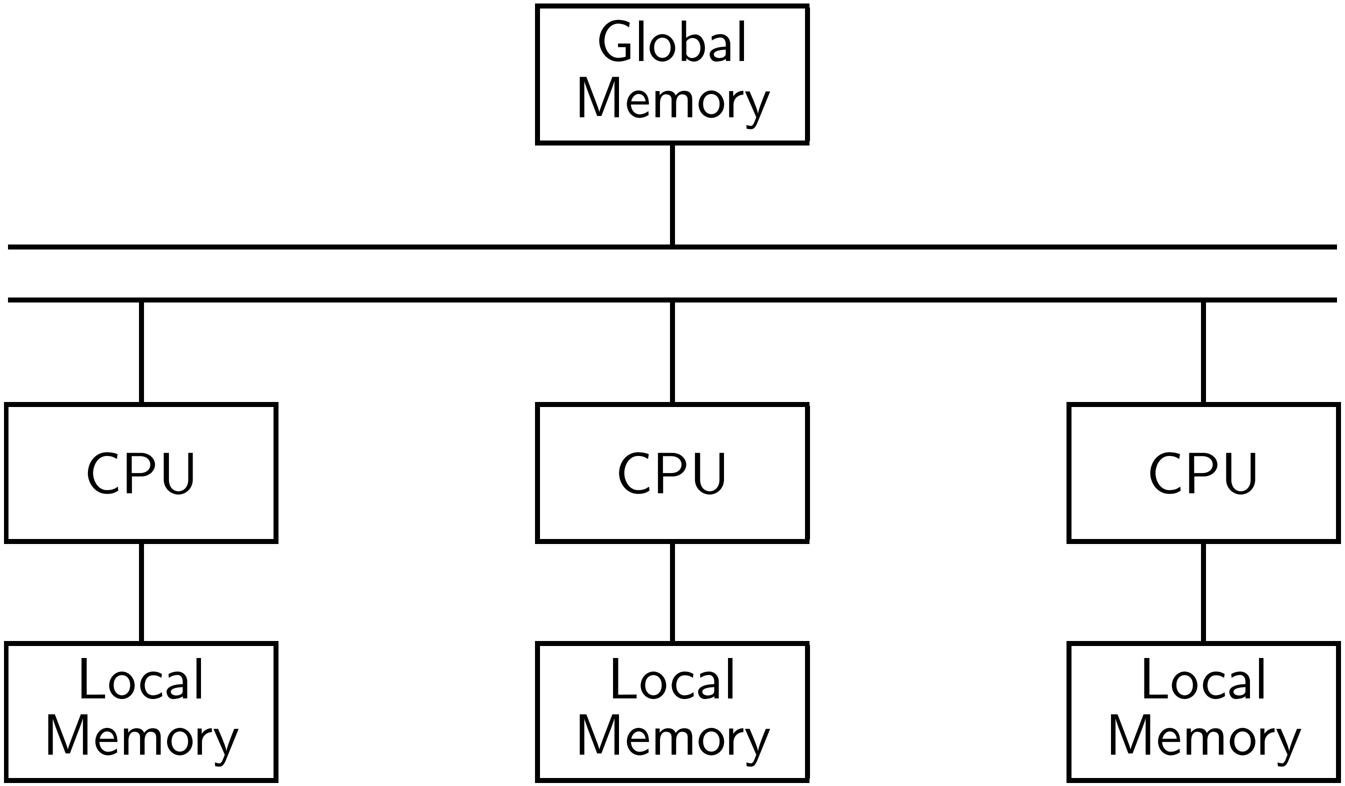
Scenario for a Concurrent Program

Process p	Process q	n	k1	k2
p1: n←k1	q1: n←k2	0	1	2
(end)	**q1: n←k2**	1	1	2
(end)	(end)	2	1	2

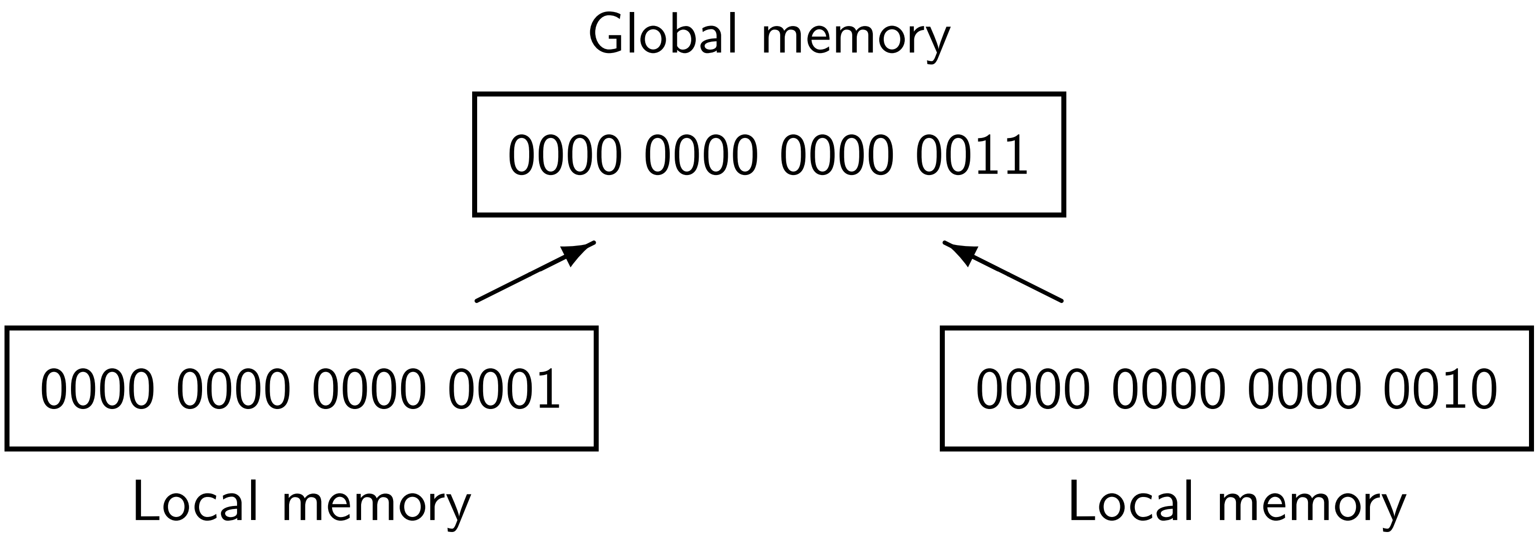
Multitasking System



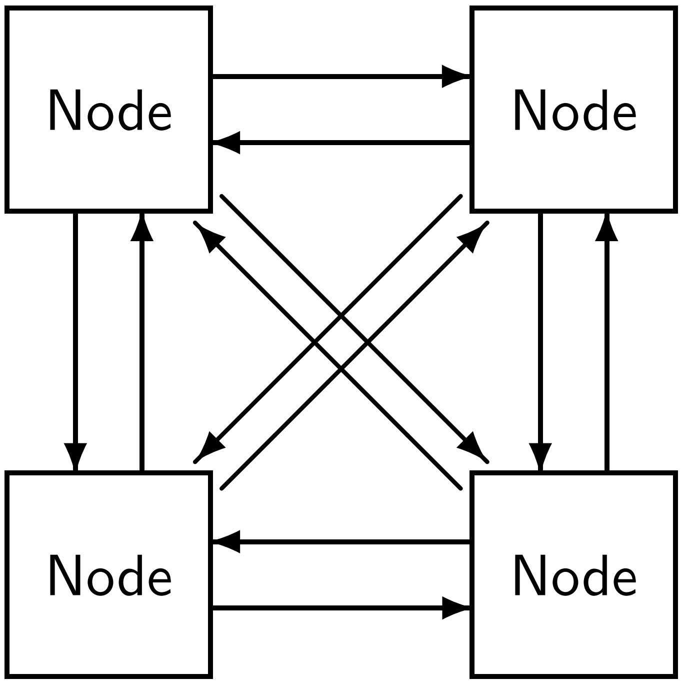
Multiprocessor Computer

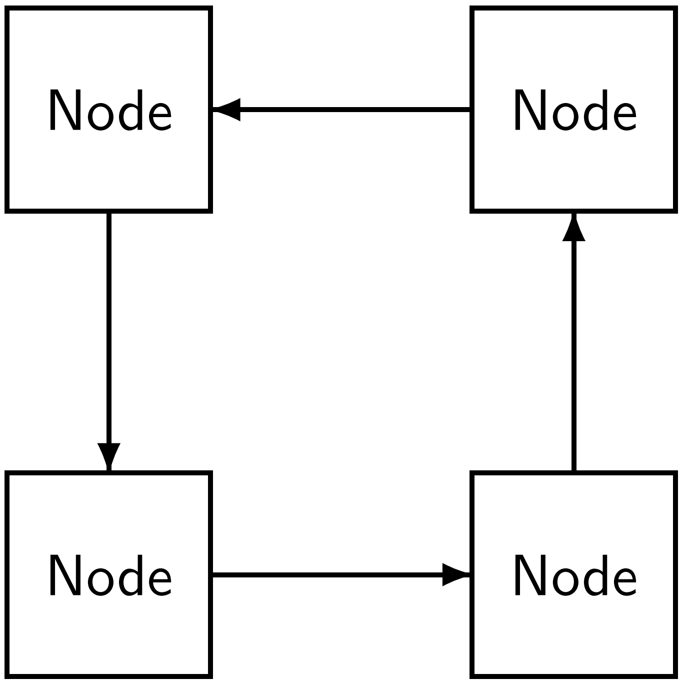


Inconsistency Caused by Overlapped Execution



Distributed Systems Architecture





Algorithm 2.3: Atomic assignment statements	
integer n $\leftarrow$ 0	
p	**q**
p1: n $\leftarrow$ n + 1	q1: n $\leftarrow$ n + 1

Scenario for Atomic Assignment Statements

Process p	Process q	n
p1: n←n+1	q1: n←n+1	0
(end)	**q1: n←n+1**	1
(end)	(end)	2

Process p	Process q	n
p1: n←n+1	**q1: n←n+1**	0
p1: n←n+1	(end)	1
(end)	(end)	2

Algorithm 2.4: Assignment statements with one global reference	
integer n $\leftarrow$ 0	
p	**q**
integer temp	integer temp
p1: temp $\leftarrow$ n	q1: temp $\leftarrow$ n
p2: n $\leftarrow$ temp + 1	q2: n $\leftarrow$ temp + 1

Correct Scenario for Assignment Statements

Process p	Process q	n	p.temp	q.temp
p1: temp←n	q1: temp←n	0	?	?
p2: n←temp+1	q1: temp←n	0	0	?
(end)	**q1: temp←n**	1	0	?
(end)	**q2: n←temp+1**	1	0	1
(end)	(end)	2	0	1

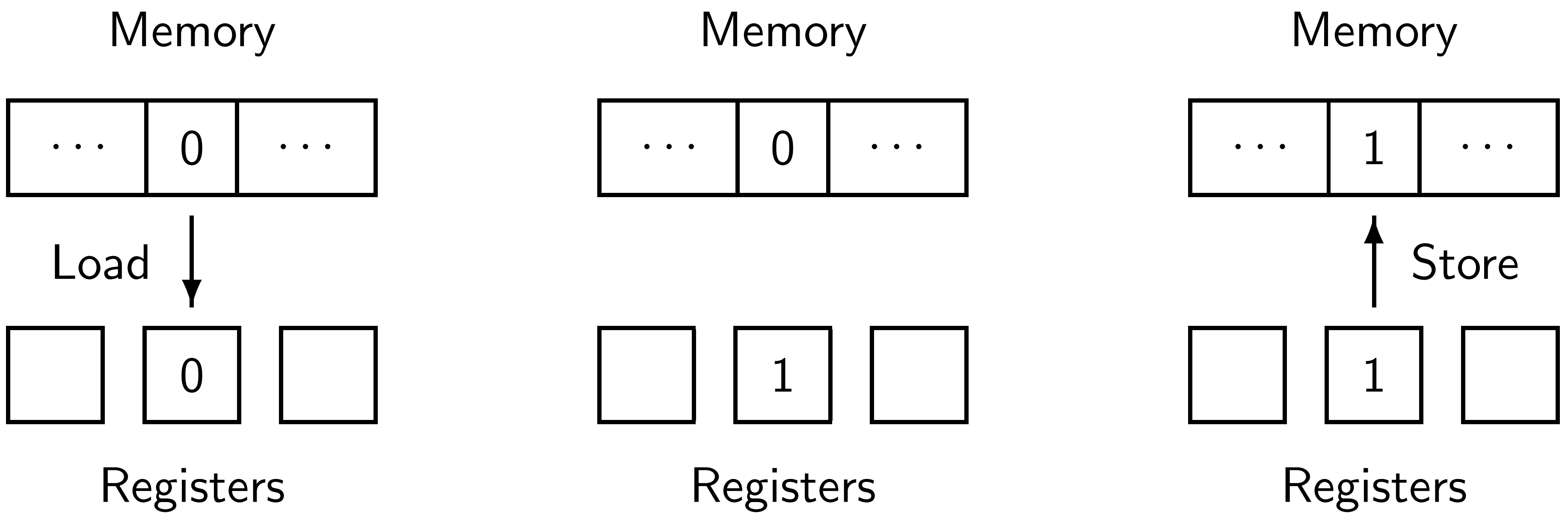
Incorrect Scenario for Assignment Statements

Process p	Process q	n	p.temp	q.temp
p1: temp←n	q1: temp←n	0	?	?
p2: n←temp+1	**q1: temp←n**	0	0	?
p2: n←temp+1	q2: n←temp+1	0	0	0
(end)	**q2: n←temp+1**	1	0	0
(end)	(end)	1	0	0

Algorithm 2.5: Stop the loop A	
integer n $\leftarrow$ 0 boolean flag $\leftarrow$ false	
p	**q**
p1: while flag = false	q1: flag $\leftarrow$ true
p2: n $\leftarrow$ 1 − n	q2:

Algorithm 2.6: Assignment statement for a register machine	
integer n ← 0	
p	**q**
p1: load R1,n	q1: load R1,n
p2: add R1,#1	q2: add R1,#1
p3: store R1,n	q3: store R1,n

Register Machine

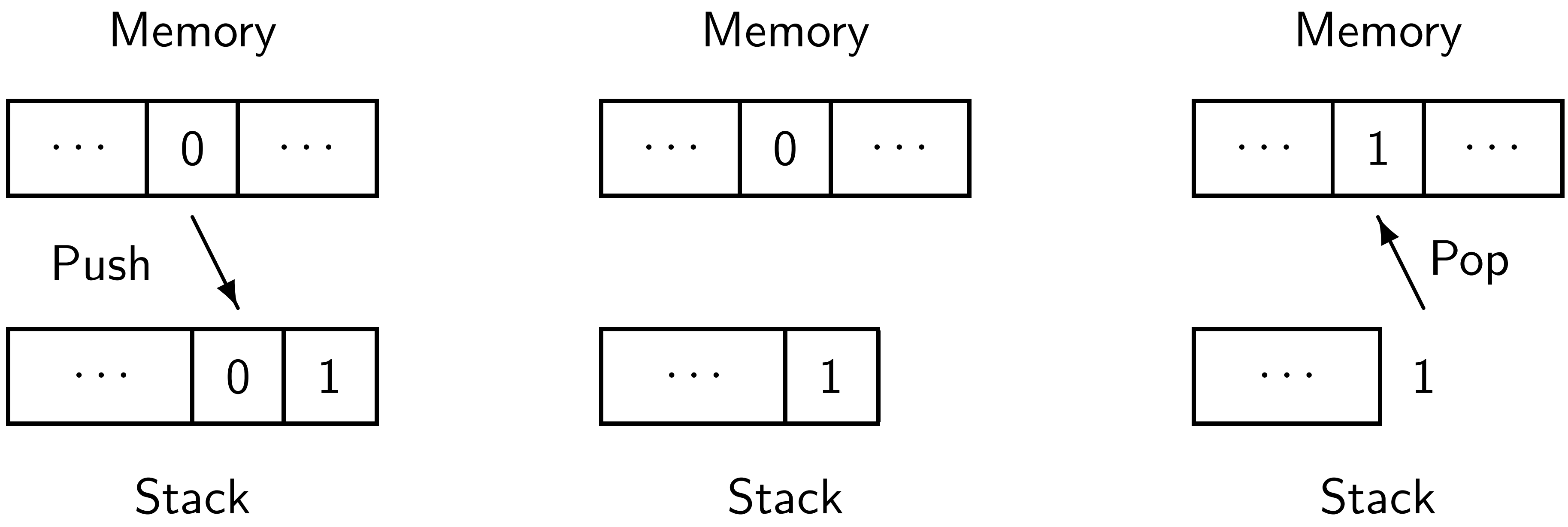


Scenario for a Register Machine

Process p	Process q	n	p.R1	q.R1
p1: load R1,n	q1: load R1,n	0	?	?
p2: add R1,#1	**q1: load R1,n**	0	0	?
p2: add R1,#1	q2: add R1,#1	0	0	0
p3: store R1,n	**q2: add R1,#1**	0	1	0
p3: store R1,n	q3: store R1,n	0	1	1
(end)	**q3: store R1,n**	1	1	1
(end)	(end)	1	1	1

Algorithm 2.7: Assignment statement for a stack machine	
integer n $\leftarrow$ 0	
p	**q**
p1: push n	q1: push n
p2: push #1	q2: push #1
p3: add	q3: add
p4: pop n	q4: pop n

Stack Machine



Algorithm 2.8: Volatile variables	
integer n ← 0	
p	**q**
integer local1, local2	integer local
p1: n ← *some expression*	q1: local ← n + 6
p2: *computation not using n*	q2:
p3: local1 ← (n + 5) $*$ 7	q3:
p4: local2 ← n + 5	q4:
p5: n ← local1 * local2	q5:

Algorithm 2.9: Concurrent counting algorithm	
integer n ← 0	
p	**q**
integer temp	integer temp
p1: do 10 times	q1: do 10 times
p2: temp ← n	q2: temp ← n
p3: n ← temp + 1	q3: n ← temp + 1

Concurrent Program in Pascal

```
program count;
var n: integer := 0;

procedure p;
var temp, i : integer;
begin
  for i := 1 to 10 do
    begin
      temp := n; n := temp + 1
    end
end;

procedure q;
var temp, i : integer;
begin
  for i := 1 to 10 do
    begin
      temp := n; n := temp + 1
    end
end;

begin
  cobegin p; q coend;
  writeln('The value of n is ', n)
end.
```

Concurrent Program in C

```
int n = 0;

void p() {
  int temp, i;
  for (i = 0; i < 10; i++) {
    temp = n;
    n = temp + 1;
  }
}

void q() {
  int temp, i;
  for (i = 0; i < 10; i++) {
    temp = n;
    n = temp + 1;
  }
}

void main() {
  cobegin { p(); q(); }
  cout << "The value of n is " << n << "\n";
}
```

Concurrent Program in Ada

```
with Ada.Text_IO; use Ada.Text_IO;
procedure Count is
    N: Integer := 0;
    pragma Volatile(N);

    task type Count_Task;
    task body Count_Task is
        Temp: Integer;
    begin
        for I in 1..10 loop
            Temp := N;
            N := Temp + 1;
        end loop;
    end Count_Task;

begin
    declare
        P, Q: Count_Task;
    begin
        null;
    end;
    Put_Line("The value of N is " & Integer'Image(N));
end Count;
```

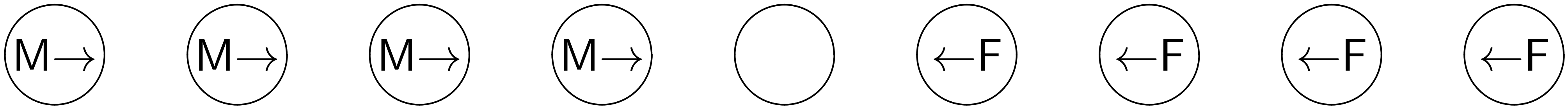
Concurrent Program in Java

```java
class Count extends Thread {
    static volatile int n = 0;

    public void run() {
        int temp;
        for (int i = 0; i < 10; i++) {
            temp = n;
            n = temp + 1;
        }
    }

    public static void main(String[] args) {
        Count p = new Count();
        Count q = new Count();
        p.start ();
        q.start ();
        try {
            p.join ();
            q.join ();
        }
        catch (InterruptedException e) { }
        System.out.println ("The value of n is " + n);
    }
}
```

Concurrent Program in Promela

```
#include "for.h"
#define TIMES 10
byte    n = 0;

proctype P() {
    byte temp;
    for (i,1, TIMES)
        temp = n;
        n = temp + 1
    rof (i)
}

init {
    atomic {
        run P();
        run P()
    }
    (_nr_pr == 1);
    printf ("MSC: The value is %d\n", n)
}
```

Frog Puzzle



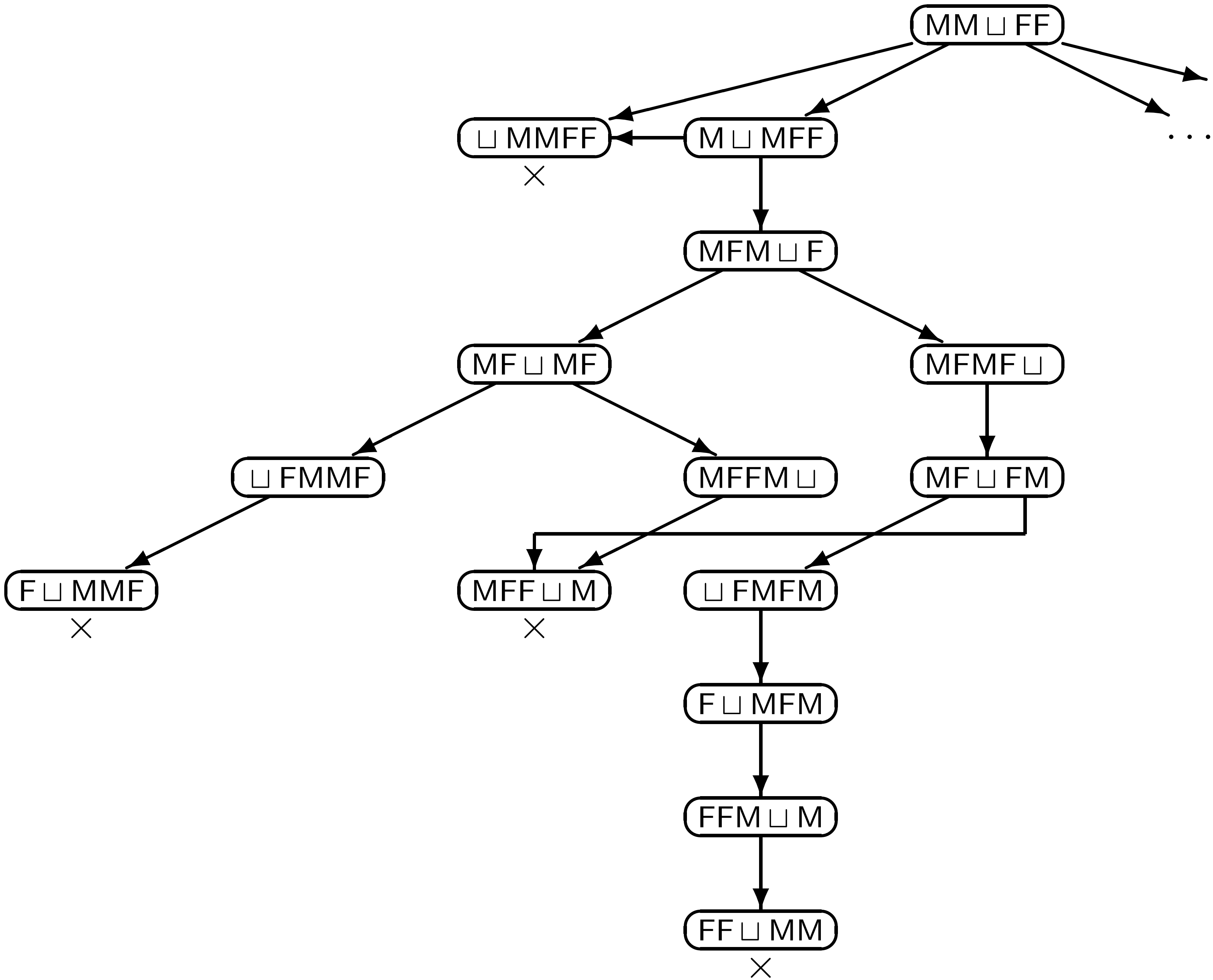
One Step of the Frog Puzzle

M→	M→	M→	M→	←F	←F		←F	←F

Final State of the Frog Puzzle

$\leftarrow$F	$\leftarrow$F	$\leftarrow$F	$\leftarrow$F		M$\rightarrow$	M$\rightarrow$	M$\rightarrow$	M$\rightarrow$

(Partial) State Diagram for the Frog Puzzle



Algorithm 2.10: Incrementing and decrementing	
integer n $\leftarrow$ 0	
p	**q**
integer temp	integer temp
p1: do K times	q1: do K times
p2: temp $\leftarrow$ n	q2: temp $\leftarrow$ n
p3: n $\leftarrow$ temp + 1	q3: n $\leftarrow$ temp − 1

Algorithm 2.11: Zero A	
boolean found	
p	**q**
integer i $\leftarrow$ 0	integer j $\leftarrow$ 1
p1: found $\leftarrow$ false	q1: found $\leftarrow$ false
p2: while not found	q2: while not found
p3: i $\leftarrow$ i + 1	q3: j $\leftarrow$ j $-$ 1
p4: found $\leftarrow$ f(i) = 0	q4: found $\leftarrow$ f(j) = 0

Algorithm 2.12: Zero B	
boolean found $\leftarrow$ false	
p	**q**
integer i $\leftarrow$ 0	integer j $\leftarrow$ 1
p1: while not found	q1: while not found
p2: $\quad$ i $\leftarrow$ i + 1	q2: $\quad$ j $\leftarrow$ j − 1
p3: $\quad$ found $\leftarrow$ f(i) = 0	q3: $\quad$ found $\leftarrow$ f(j) = 0

Algorithm 2.13: Zero C	
boolean found $\leftarrow$ false	
p	**q**
integer i $\leftarrow$ 0	integer j $\leftarrow$ 1
p1: while not found	q1: while not found
p2: i $\leftarrow$ i + 1	q2: j $\leftarrow$ j − 1
p3: if f(i) = 0	q3: if f(j) = 0
p4: found $\leftarrow$ true	q4: found $\leftarrow$ true

Algorithm 2.14: Zero D	
boolean found ← false integer turn ← 1	
p	**q**
integer i ← 0	integer j ← 1
p1: while not found	q1: while not found
p2: await turn = 1 turn ← 2	q2: await turn = 2 turn ← 1
p3: i ← i + 1	q3: j ← j − 1
p4: if f(i) = 0	q4: if f(j) = 0
p5: found ← true	q5: found ← true

Algorithm 2.15: Zero E	
boolean found $\leftarrow$ false integer turn $\leftarrow$ 1	
p	**q**
integer i $\leftarrow$ 0	integer j $\leftarrow$ 1
p1: while not found	q1: while not found
p2: await turn = 1 turn $\leftarrow$ 2	q2: await turn = 2 turn $\leftarrow$ 1
p3: i $\leftarrow$ i + 1	q3: j $\leftarrow$ j − 1
p4: if f(i) = 0	q4: if f(j) = 0
p5: found $\leftarrow$ true	q5: found $\leftarrow$ true
p6: turn $\leftarrow$ 2	q6: turn $\leftarrow$ 1

Algorithm 2.16: Concurrent algorithm A
integer array [1..10] C $\leftarrow$ ten *distinct* initial values integer array [1..10] D
integer myNumber, count p1: myNumber $\leftarrow$ C[i] p2: count $\leftarrow$ number of elements of C less than myNumber p3: D[count + 1] $\leftarrow$ myNumber

Algorithm 2.17: Concurrent algorithm B	
integer n $\leftarrow$ 0	
p	**q**
p1: while n $<$ 2	q1: n $\leftarrow$ n $+$ 1
p2: write(n)	q2: n $\leftarrow$ n $+$ 1

Algorithm 2.18: Concurrent algorithm C	
integer n $\leftarrow$ 1	
p	**q**
p1: while n $<$ 1	q1: while n $>=$ 0
p2: n $\leftarrow$ n $+$ 1	q2: n $\leftarrow$ n $-$ 1

Algorithm 2.19: Stop the loop B			
integer n $\leftarrow$ 0 boolean flag $\leftarrow$ false			
	p		**q**
p1:	while flag = false	q1:	while flag = false
p2:	n $\leftarrow$ 1 $-$ n	q2:	if n = 0
p3:		q3:	flag $\leftarrow$ true

Algorithm 2.20: Stop the loop C	
integer n $\leftarrow$ 0	
boolean flag $\leftarrow$ false	
p	**q**
p1: while flag = false	q1: while n = 0 // Do nothing
p2: n $\leftarrow$ 1 − n	q2: flag $\leftarrow$ true

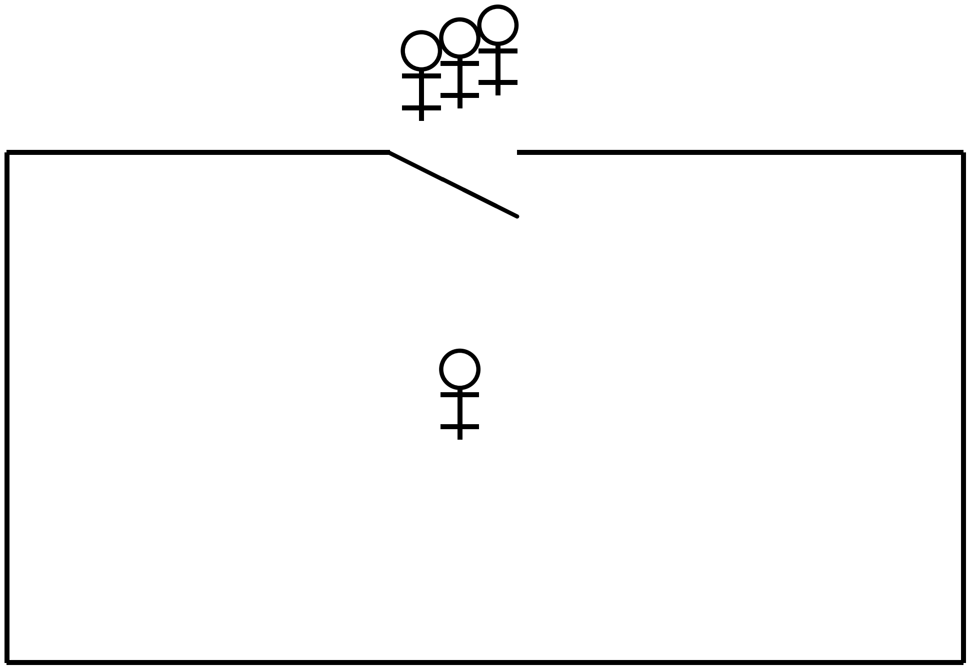
Algorithm 2.21: Welfare crook problem
integer array[0..N] a, b, c ← . . . (as required) integer i ← 0, j ← 0, k ← 0

```
    loop
p1:   if condition-1
p2:     i ← i + 1
p3:   else if condition-2
p4:     j ← j + 1
p5:   else if condition-3
p6:     k ← k + 1
      else exit loop
```

Algorithm 3.1: Critical section problem	
global variables	
p	**q**

local variables
loop forever
    non-critical section
    preprotocol
    critical section
    postprotocol
	local variables
	loop forever
	    non-critical section
	    preprotocol
	    critical section
	    postprotocol

Critical Section

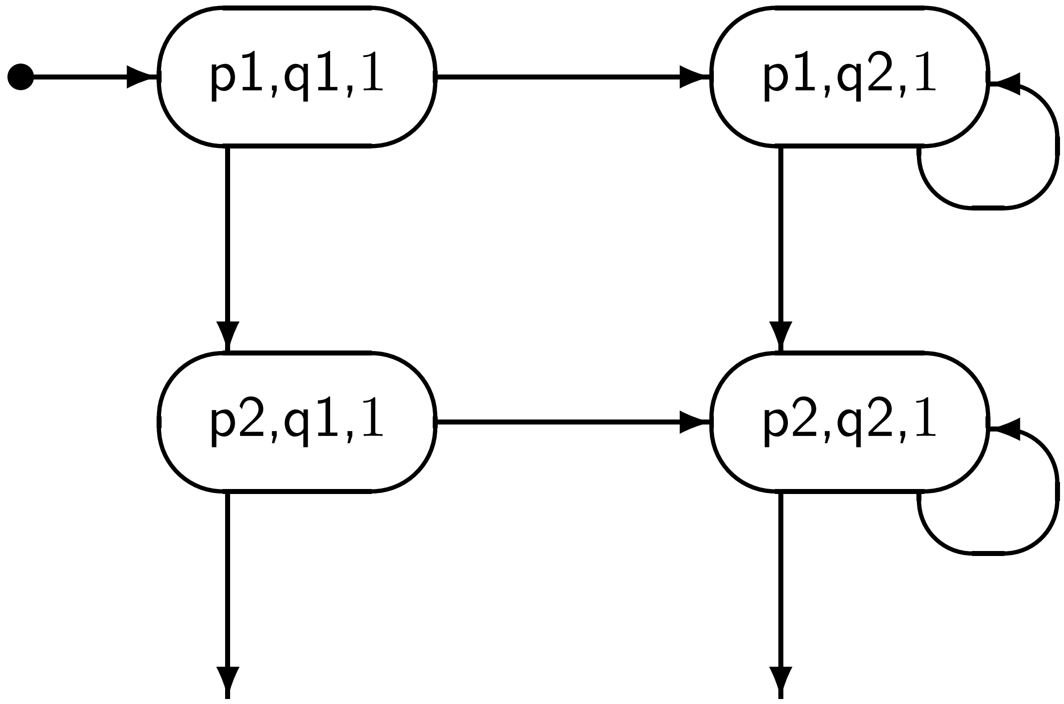


Algorithm 3.2: First attempt			
integer turn $\leftarrow$ 1			
p		**q**	
	loop forever		loop forever
p1:	non-critical section	q1:	non-critical section
p2:	await turn $= 1$	q2:	await turn $= 2$
p3:	critical section	q3:	critical section
p4:	turn $\leftarrow 2$	q4:	turn $\leftarrow 1$

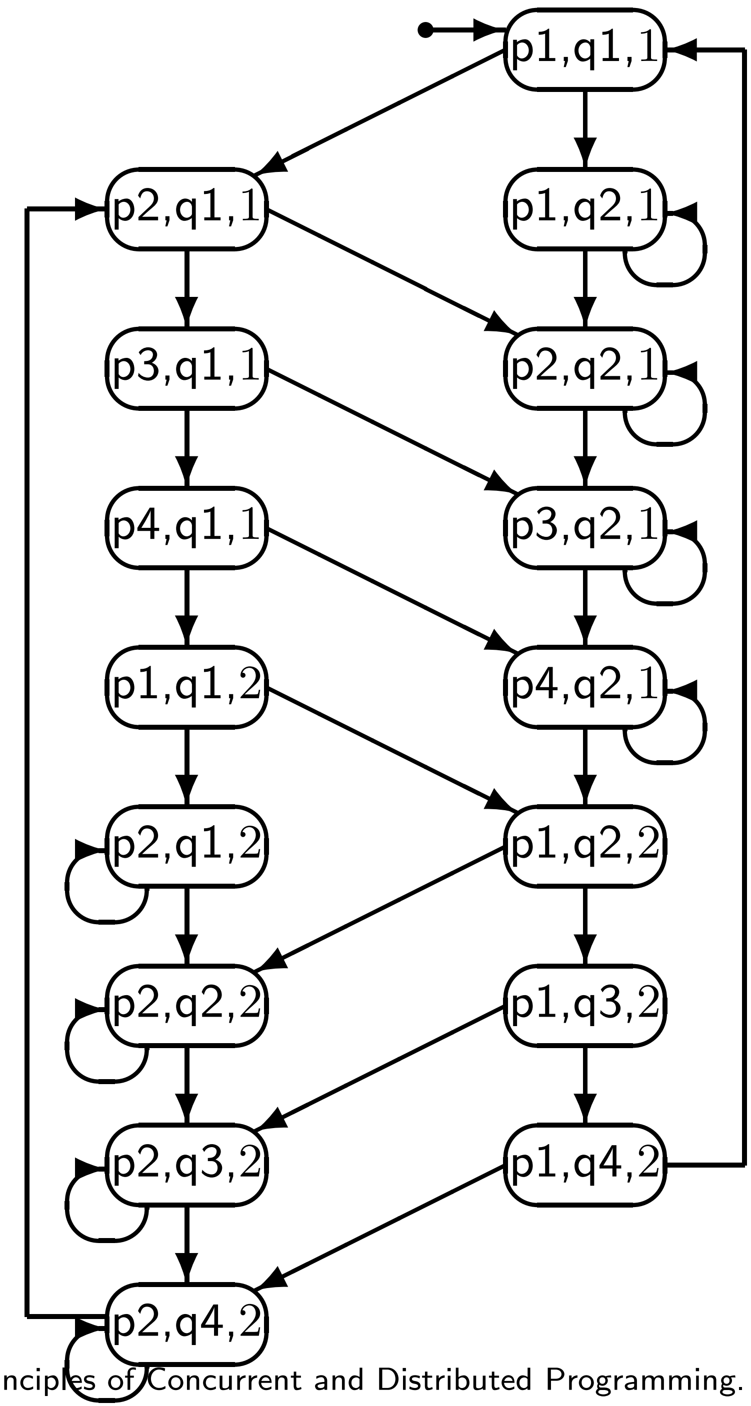
Algorithm 3.3: History in a sequential algorithm	
integer a ← 1, b ← 2	
p1:	Millions of statements
p2:	a ← (a+b)*5
p3:	. . .

Algorithm 3.4: History in a concurrent algorithm	
integer a ← 1, b ← 2	
p	**q**
p1: Millions of statements	q1: Millions of statements
p2: a ← (a+b)*5	q2: b ← (a+b)*5
p3: . . .	q3: . . .

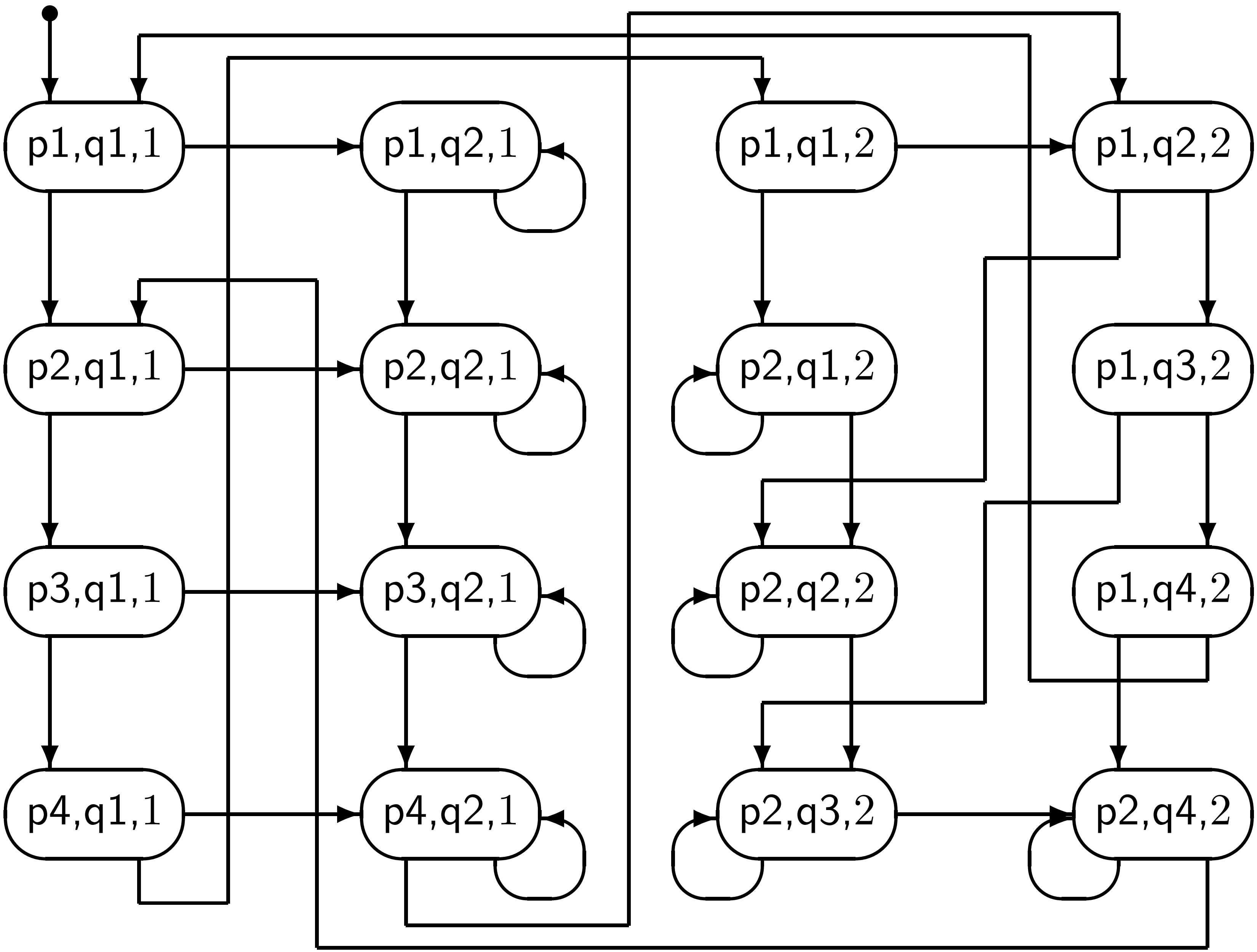
First States of the State Diagram



State Diagram for the First Attempt

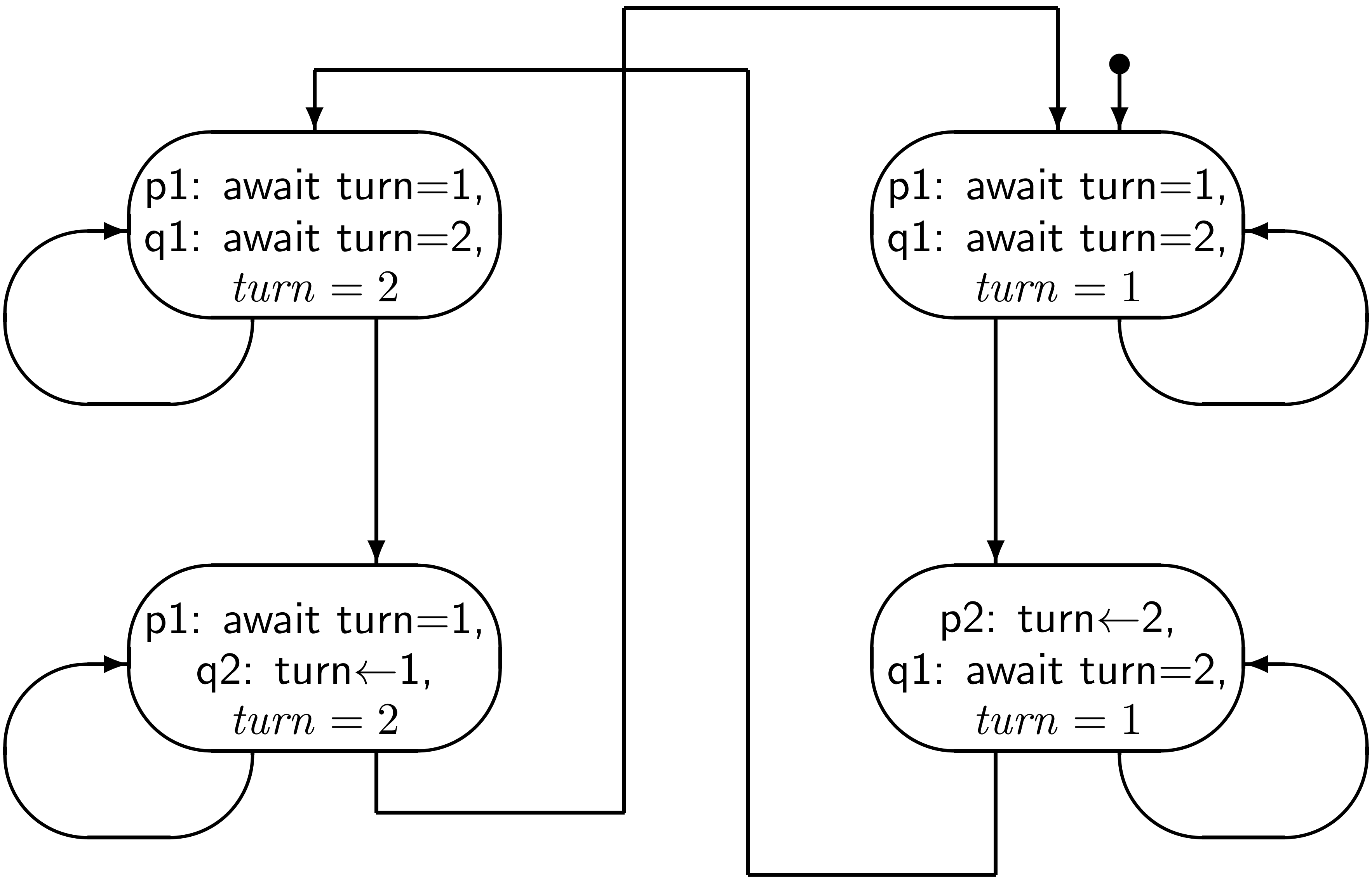


Alternate Layout for First Attempt (Not in book)

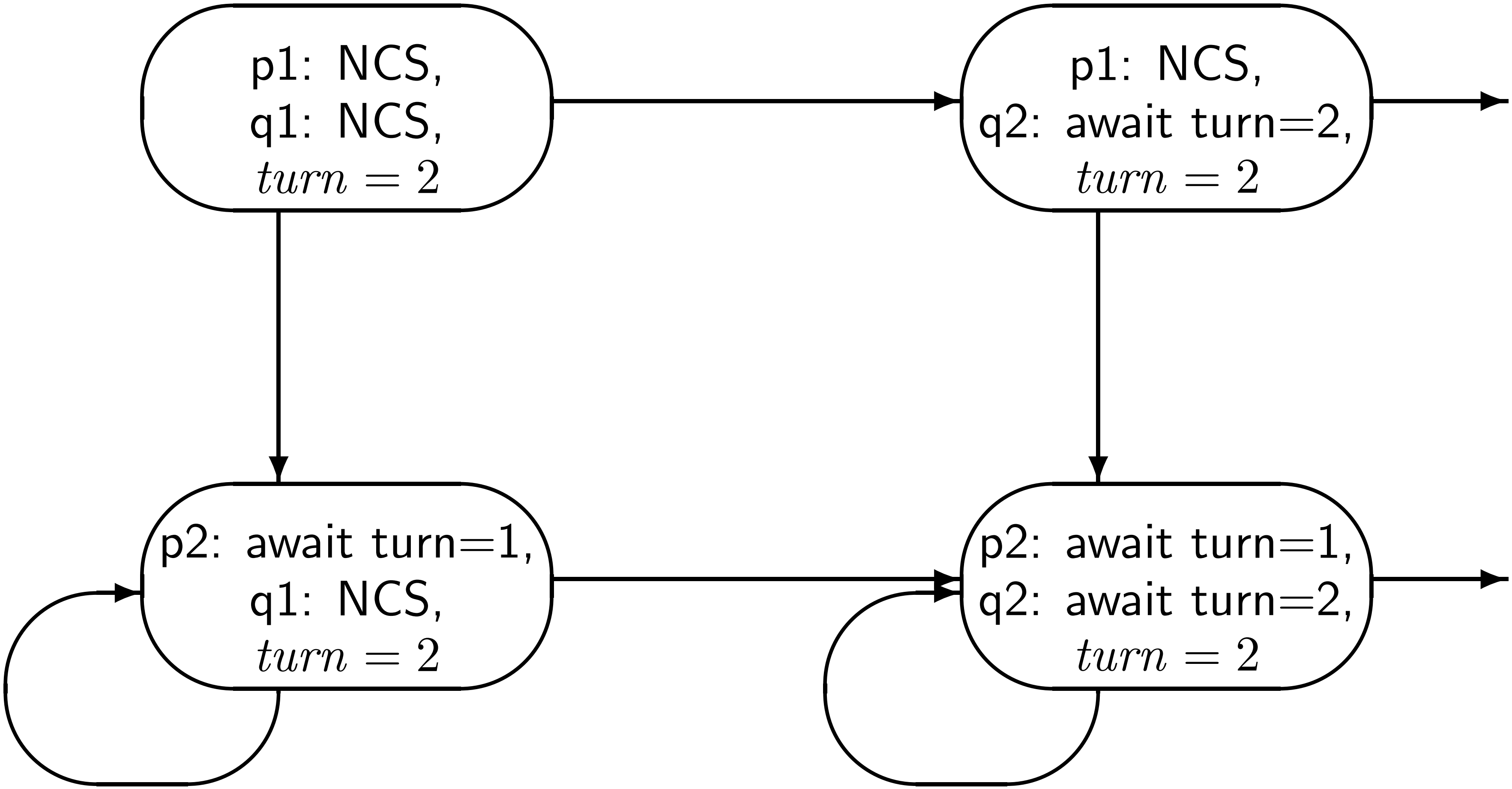


Algorithm 3.5: First attempt (abbreviated)	
integer turn $\leftarrow$ 1	
p	**q**
loop forever	loop forever
p1: await turn = 1	q1: await turn = 2
p2: turn $\leftarrow$ 2	q2: turn $\leftarrow$ 1

State Diagram for the Abbreviated First Attempt



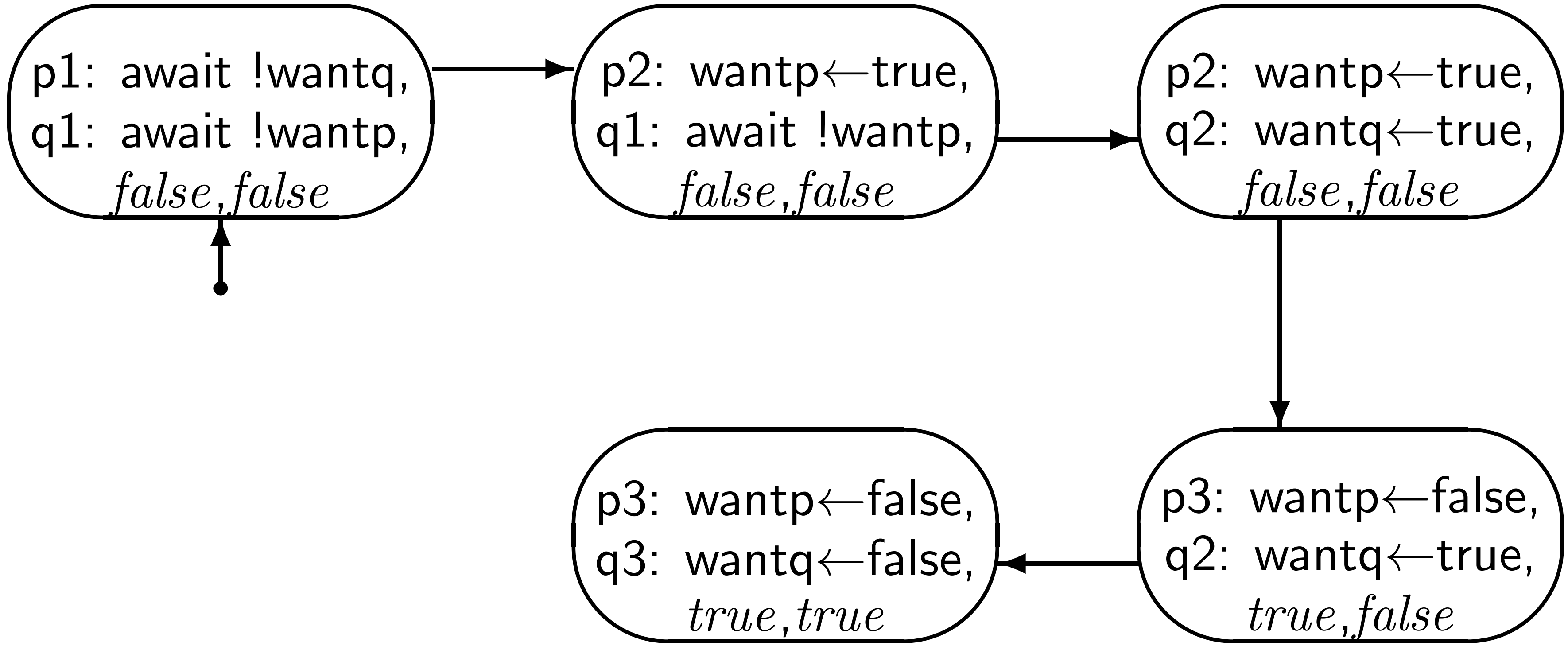
Fragment of the State Diagram for the First Attempt



Algorithm 3.6: Second attempt	
boolean wantp ← false, wantq ← false	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: await wantq = false	q2: await wantp = false
p3: wantp ← true	q3: wantq ← true
p4: critical section	q4: critical section
p5: wantp ← false	q5: wantq ← false

Algorithm 3.7: Second attempt (abbreviated)	
boolean wantp $\leftarrow$ false, wantq $\leftarrow$ false	
p	**q**
loop forever	loop forever
p1: await wantq = false	q1: await wantp = false
p2: wantp $\leftarrow$ true	q2: wantq $\leftarrow$ true
p3: wantp $\leftarrow$ false	q3: wantq $\leftarrow$ false

Fragment of State Diagram for the Second Attempt



Scenario: Mutual Exclusion Does Not Hold

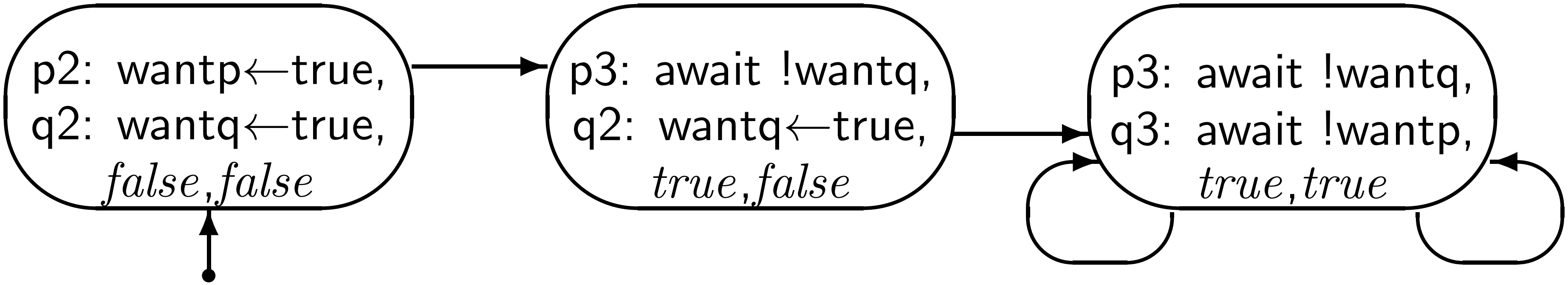
Process p	Process q	wantp	wantq
p1: await wantq=false	q1: await wantp=false	*false*	*false*
p2: wantp←true	**q1: await wantp=false**	*false*	*false*
p2: wantp←true	q2: wantq←true	*false*	*false*
p3: wantp←false	**q3: wantq←true**	*true*	*false*
p3: wantp←false	q3: wantq←false	*true*	*true*

Algorithm 3.8: Third attempt	
boolean wantp ← false, wantq ← false	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp ← true	q2: wantq ← true
p3: await wantq = false	q3: await wantp = false
p4: critical section	q4: critical section
p5: wantp ← false	q5: wantq ← false

Scenario Showing Deadlock in the Third Attempt

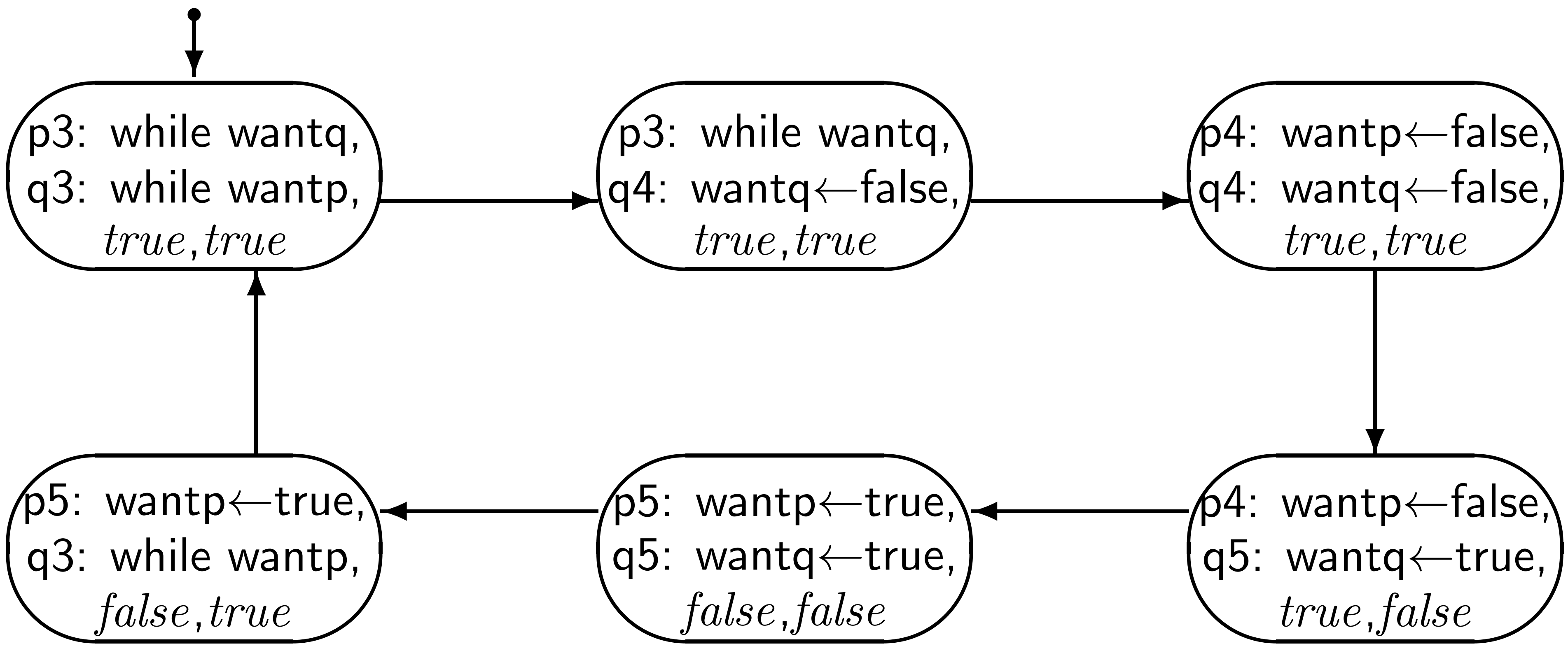
Process p	Process q	wantp	wantq
p1: non-critical section	q1: non-critical section	*false*	*false*
p2: wantp←true	**q1: non-critical section**	*false*	*false*
p2: wantp←true	q2: wantq←true	*false*	*false*
p3: await wantq=false	**q2: wantq←true**	*true*	*false*
p3: await wantq=false	q3: await wantp=false	*true*	*true*

Fragment of the State Diagram Showing Deadlock



Algorithm 3.9: Fourth attempt	
boolean wantp $\leftarrow$ false, wantq $\leftarrow$ false	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp $\leftarrow$ true	q2: wantq $\leftarrow$ true
p3: while wantq	q3: while wantp
p4: wantp $\leftarrow$ false	q4: wantq $\leftarrow$ false
p5: wantp $\leftarrow$ true	q5: wantq $\leftarrow$ true
p6: critical section	q6: critical section
p7: wantp $\leftarrow$ false	q7: wantq $\leftarrow$ false

Cycle in the State Diagram for the Fourth Attempt



Algorithm 3.10: Dekker's algorithm	
boolean wantp $\leftarrow$ false, wantq $\leftarrow$ false integer turn $\leftarrow$ 1	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp $\leftarrow$ true	q2: wantq $\leftarrow$ true
p3: while wantq	q3: while wantp
p4: if turn = 2	q4: if turn = 1
p5: wantp $\leftarrow$ false	q5: wantq $\leftarrow$ false
p6: await turn = 1	q6: await turn = 2
p7: wantp $\leftarrow$ true	q7: wantq $\leftarrow$ true
p8: critical section	q8: critical section
p9: turn $\leftarrow$ 2	q9: turn $\leftarrow$ 1
p10: wantp $\leftarrow$ false	q10: wantq $\leftarrow$ false

Algorithm 3.11: Critical section problem with test-and-set			
integer common $\leftarrow$ 0			
	p		**q**
	integer local1		integer local2
	loop forever		loop forever
p1:	non-critical section	q1:	non-critical section
	repeat		repeat
p2:	test-and-set(	q2:	test-and-set(
	common, local1)		common, local2)
p3:	until local1 $= 0$	q3:	until local2 $= 0$
p4:	critical section	q4:	critical section
p5:	common $\leftarrow$ 0	q5:	common $\leftarrow$ 0

Algorithm 3.12: Critical section problem with exchange	
integer common $\leftarrow$ 1	
p	**q**
integer local1 $\leftarrow$ 0 loop forever p1: non-critical section repeat p2: exchange(common, local1) p3: until local1 = 1 p4: critical section p5: exchange(common, local1)	integer local2 $\leftarrow$ 0 loop forever q1: non-critical section repeat q2: exchange(common, local2) q3: until local2 = 1 q4: critical section q5: exchange(common, local2)

Algorithm 3.13: Peterson's algorithm	
boolean wantp ← false, wantq ← false integer last ← 1	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp ← true	q2: wantq ← true
p3: last ← 1	q3: last ← 2
p4: await wantq = false or last = 2	q4: await wantp = false or last = 1
p5: critical section	q5: critical section
p6: wantp ← false	q6: wantq ← false

Algorithm 3.14: Manna-Pnueli algorithm			
integer wantp $\leftarrow 0$, wantq $\leftarrow 0$			
p		**q**	
	loop forever		loop forever
p1:	non-critical section	q1:	non-critical section
p2:	if wantq $= -1$	q2:	if wantp $= -1$
	wantp $\leftarrow -1$		wantq $\leftarrow 1$
	else wantp $\leftarrow 1$		else wantq $\leftarrow -1$
p3:	await wantq $\neq$ wantp	q3:	await wantp $\neq -$ wantq
p4:	critical section	q4:	critical section
p5:	wantp $\leftarrow 0$	q5:	wantq $\leftarrow 0$

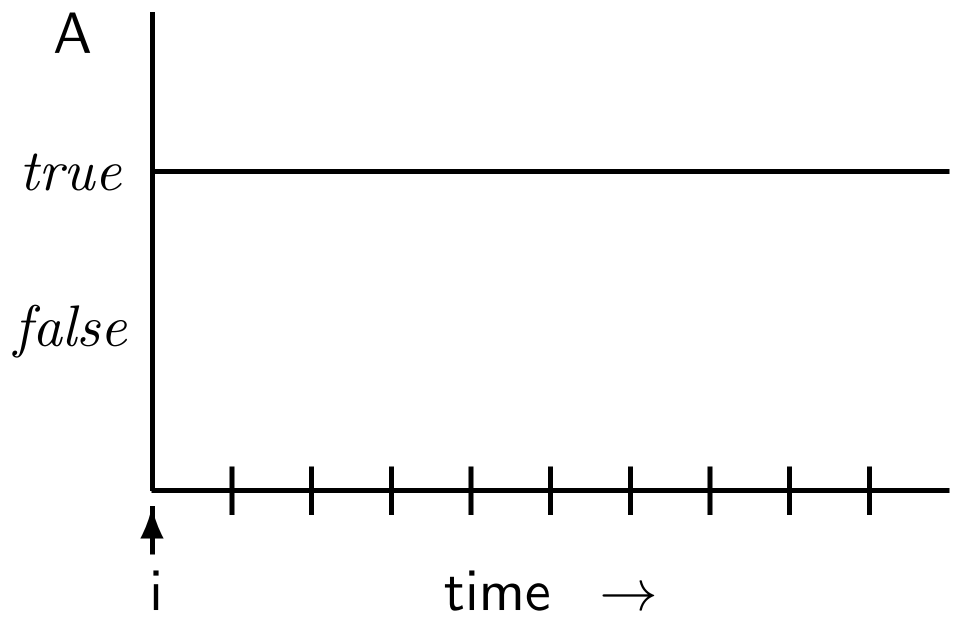
Algorithm 3.15: Doran-Thomas algorithm

```
boolean wantp ← false, wantq ← false
integer turn ← 1
```

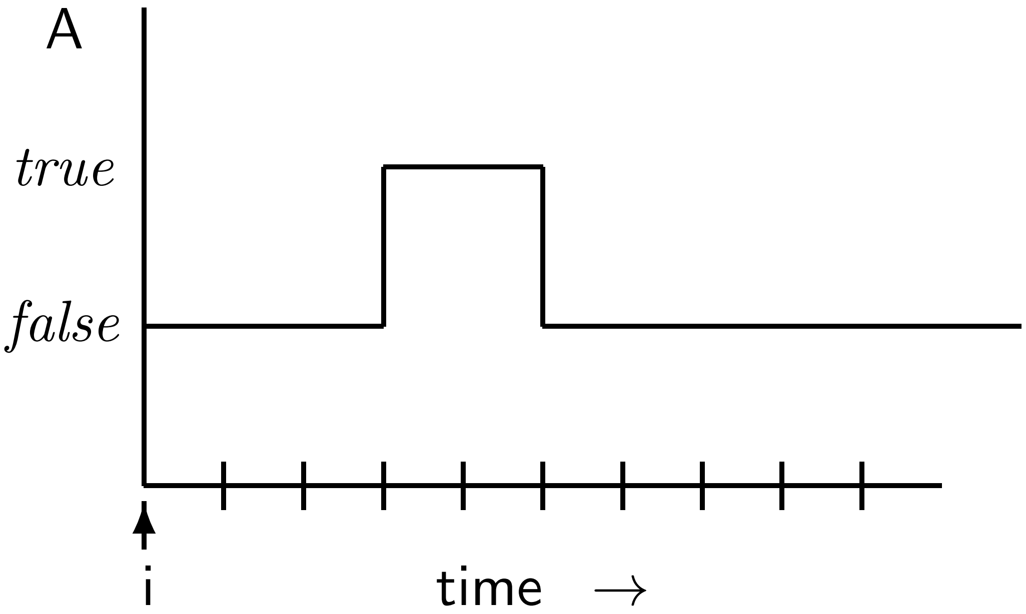
p	q
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp ← true	q2: wantq ← true
p3: if wantq	q3: if wantp
p4: if turn = 2	q4: if turn = 1
p5: wantp ← false	q5: wantq ← false
p6: await turn = 1	q6: await turn = 2
p7: wantp ← true	q7: wantq ← true
p8: await wantq = false	q8: await wantp = false
p9: critical section	q9: critical section
p10: wantp ← false	q10: wantq ← false
p11: turn ← 2	q11: turn ← 1

Algorithm 4.1: Third attempt	
boolean wantp ← false, wantq ← false	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp ← true	q2: wantq ← true
p3: await wantq = false	q3: await wantp = false
p4: critical section	q4: critical section
p5: wantp ← false	q5: wantq ← false

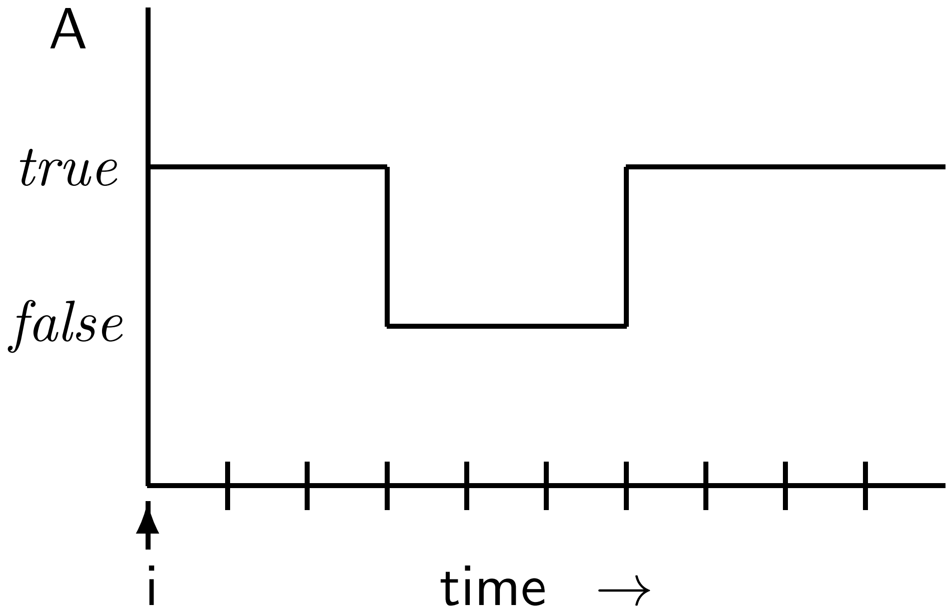
$\Box A$



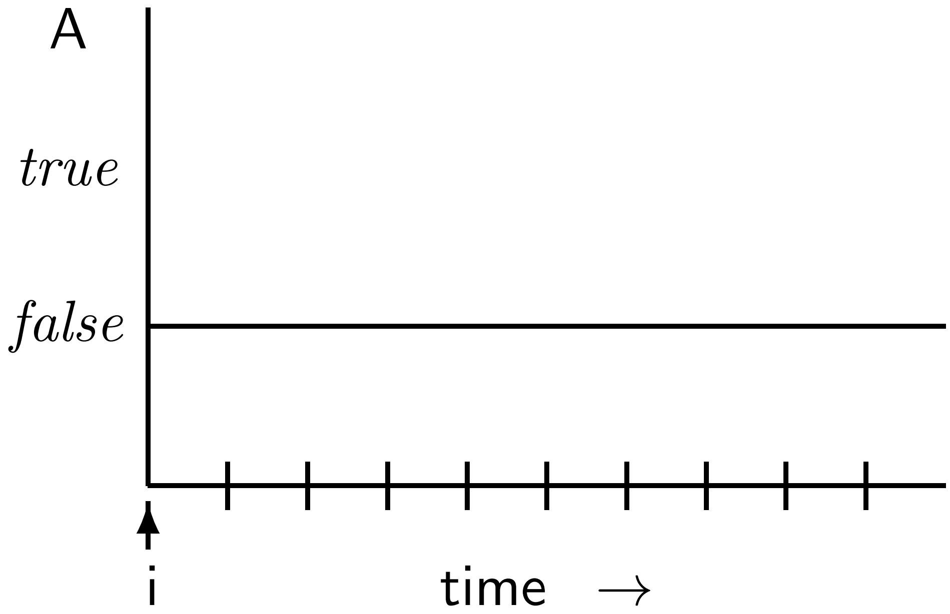
$\Diamond A$



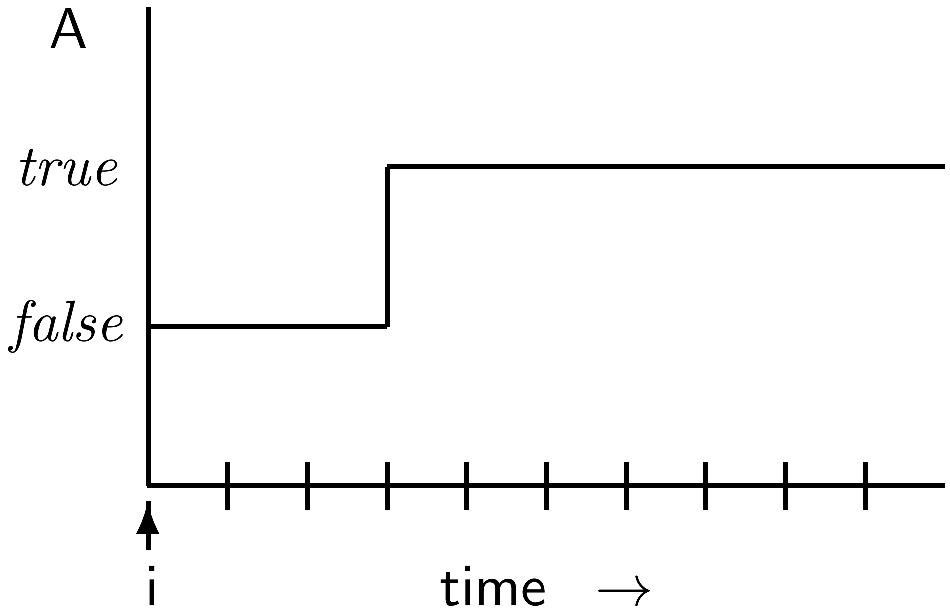
Duality: $\neg \Box A$



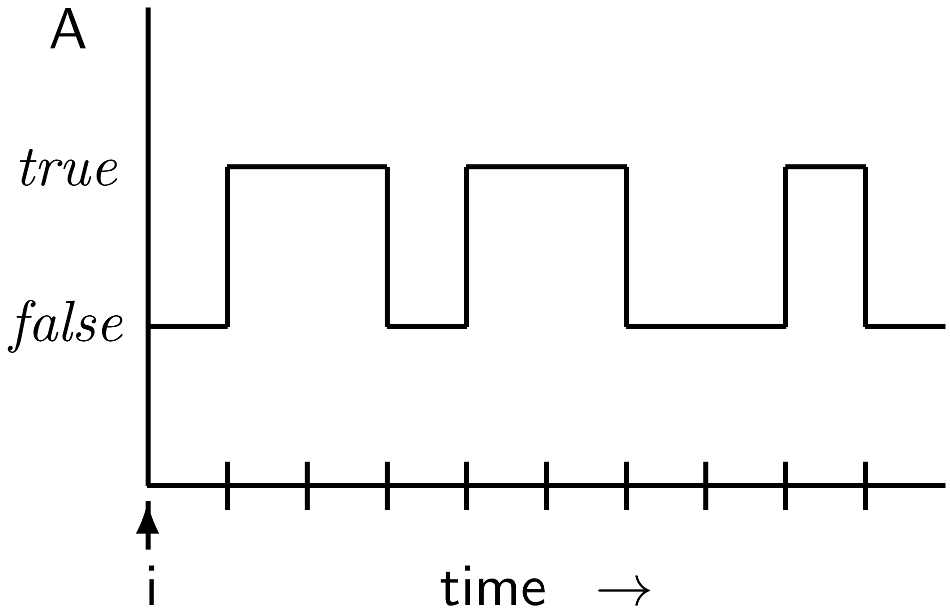
Duality: $\neg \Diamond A$



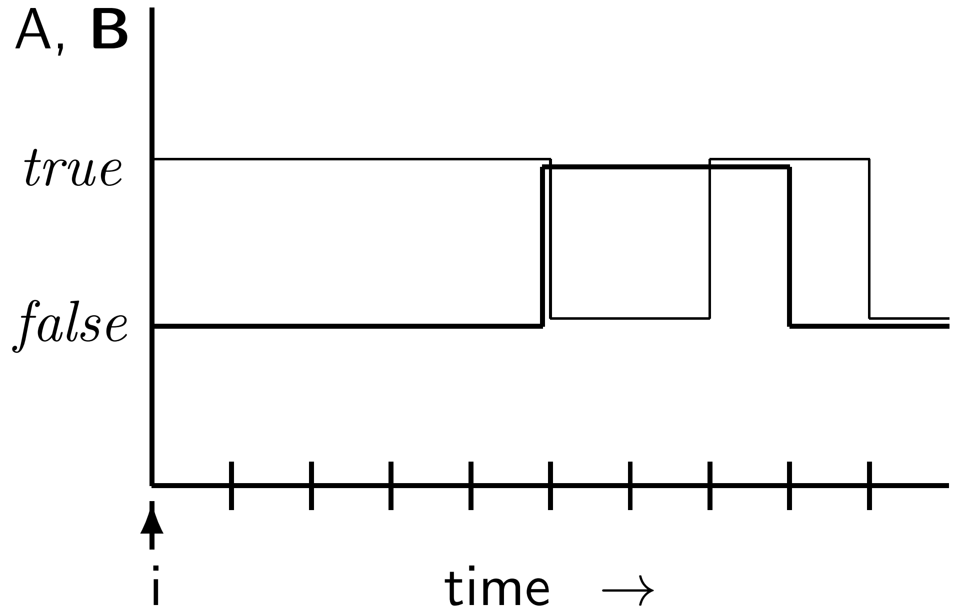
$\Diamond\Box A$



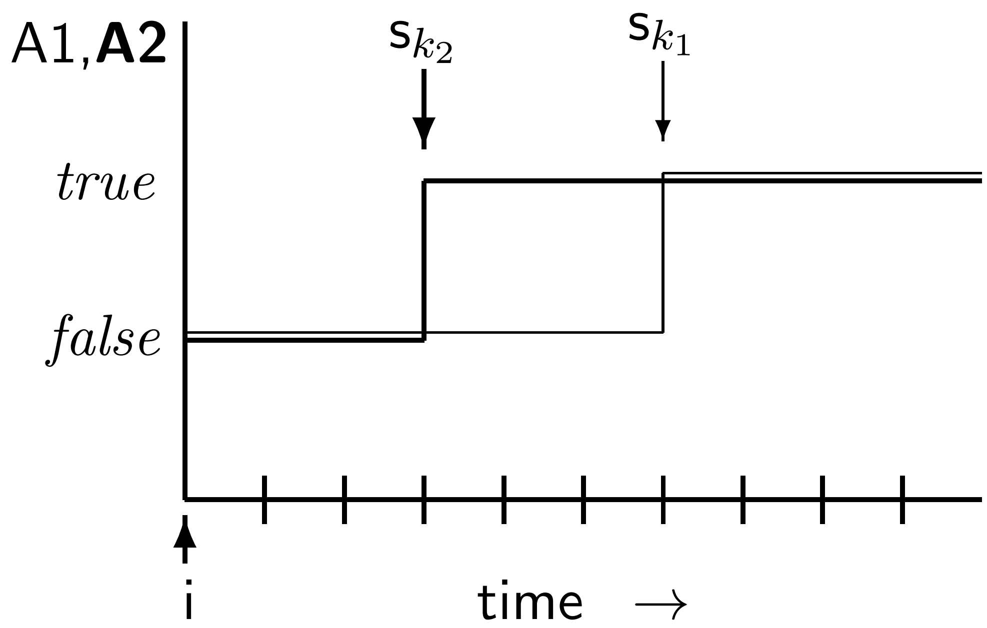
$\Box\Diamond A$



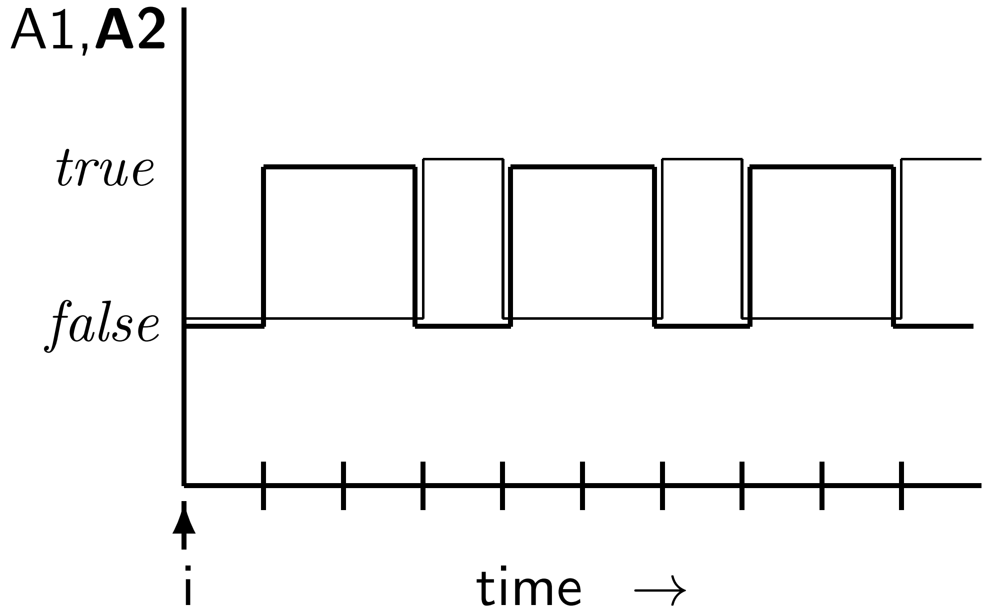
$A \mathcal{U} B$



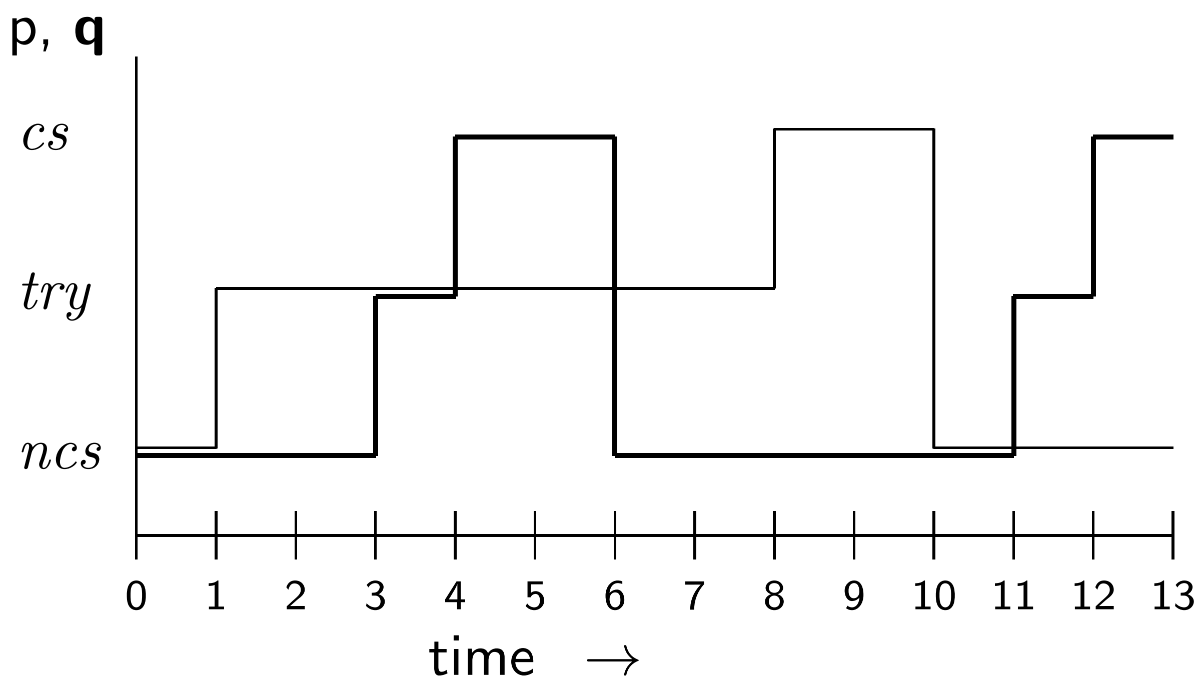
$\Diamond\Box A1 \wedge \Diamond\Box A2$



$\Box\Diamond A1 \wedge \Box\Diamond A2$



Overtaking: $try_p \rightarrow (\neg\, cs_q)\ \mathcal{W}\ (cs_q)\ \mathcal{W}\ (\neg\, cs_q)\ \mathcal{W}\ (cs_p)$



Algorithm 4.2: Dekker's algorithm	
boolean wantp $\leftarrow$ false, wantq $\leftarrow$ false integer turn $\leftarrow$ 1	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp $\leftarrow$ true	q2: wantq $\leftarrow$ true
p3: while wantq	q3: while wantp
p4: if turn $= 2$	q4: if turn $= 1$
p5: wantp $\leftarrow$ false	q5: wantq $\leftarrow$ false
p6: await turn $= 1$	q6: await turn $= 2$
p7: wantp $\leftarrow$ true	q7: wantq $\leftarrow$ true
p8: critical section	q8: critical section
p9: turn $\leftarrow$ 2	q9: turn $\leftarrow$ 1
p10: wantp $\leftarrow$ false	q10: wantq $\leftarrow$ false

Dekker's Algorithm in Promela

```
bool wantp = false, wantq = false;
byte turn = 1;

active proctype p() {
  do :: wantp = true;
    do :: !wantq -> break;
       :: else ->
          if :: (turn == 1)
             :: (turn == 2) ->
                wantp = false; (turn == 1); wantp = true
          fi
    od;
    printf ("MSC: p in CS\n") ;
    turn = 2; wantp = false
 od
}
```

Specifying Correctness in Promela

```
byte critical = 0;

bool PinCS = false;

#define nostarve PinCS /* LTL claim <> nostarve */

active proctype p() {
  do ::
     /* preprotocol */
     critical ++;
     assert(critical <= 1);
     PinCS = true;
     critical --;
     /* postprotocol */
  od
}
```

LTL Translation to Never Claims

```
never {     /* !(<>nostarve) */
accept_init :
T0_init :
     if
     :: (! ((nostarve))) -> goto T0_init
     fi ;
}

never {     /* !([]<>nostarve) */
T0_init :
     if
     :: (! ((nostarve))) -> goto accept_S4
     :: (1) -> goto T0_init
     fi ;
accept_S4:
     if
     :: (! ((nostarve))) -> goto accept_S4
     fi ;
}
```

Algorithm 5.1: Bakery algorithm (two processes)		
integer np $\leftarrow$ 0, nq $\leftarrow$ 0		
p	**q**	
loop forever	loop forever	
p1: non-critical section	q1: non-critical section	
p2: np $\leftarrow$ nq + 1	q2: nq $\leftarrow$ np + 1	
p3: await nq $=$ 0 or np $\leq$ nq	q3: await np $=$ 0 or nq $<$ np	
p4: critical section	q4: critical section	
p5: np $\leftarrow$ 0	q5: nq $\leftarrow$ 0	

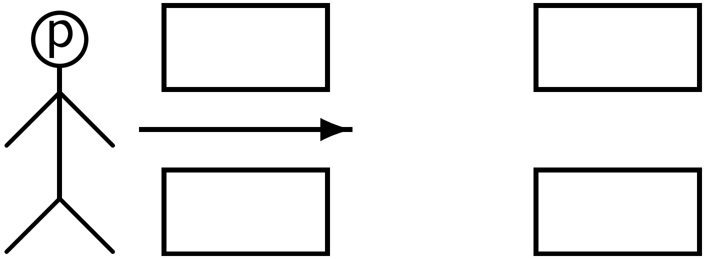
Algorithm 5.2: Bakery algorithm (N processes)	
integer array[1..n] number $\leftarrow$ [0,. . . ,0]	
	loop forever
p1:	non-critical section
p2:	number[i] $\leftarrow 1 +$ max(number)
p3:	for all *other* processes j
p4:	await (number[j] $=$ 0) or (number[i] $\ll$ number[j])
p5:	critical section
p6:	number[i] $\leftarrow$ 0

Algorithm 5.3: Bakery algorithm without atomic assignment
boolean array[1..n] choosing $\leftarrow$ [false,. . . ,false]   integer array[1..n] number $\leftarrow$ [0,. . . ,0]

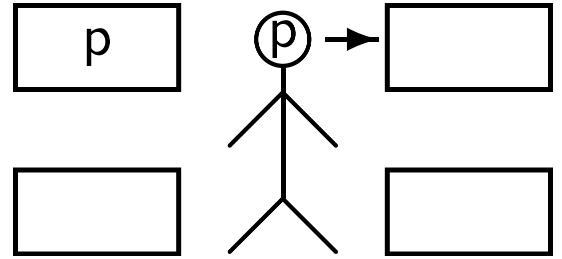
```
    loop forever
p1:    non-critical section
p2:    choosing[i] ← true
p3:    number[i] ← 1 + max(number)
p4:    choosing[i] ← false
p5:    for all other processes j
p6:       await choosing[j] = false
p7:       await (number[j] = 0) or (number[i] ≪ number[j])
p8:    critical section
p9:    number[i] ← 0
```

Algorithm 5.4: Fast algorithm for two processes (outline)			
integer gate1 $\leftarrow$ 0, gate2 $\leftarrow$ 0			
	p		**q**
	loop forever		loop forever
	non-critical section		non-critical section
p1:	gate1 $\leftarrow$ p	q1:	gate1 $\leftarrow$ q
p2:	if gate2 $\neq$ 0 goto p1	q2:	if gate2 $\neq$ 0 goto q1
p3:	gate2 $\leftarrow$ p	q3:	gate2 $\leftarrow$ q
p4:	if gate1 $\neq$ p	q4:	if gate1 $\neq$ q
p5:	if gate2 $\neq$ p goto p1	q5:	if gate2 $\neq$ q goto q1
	critical section		critical section
p6:	gate2 $\leftarrow$ 0	q6:	gate2 $\leftarrow$ 0

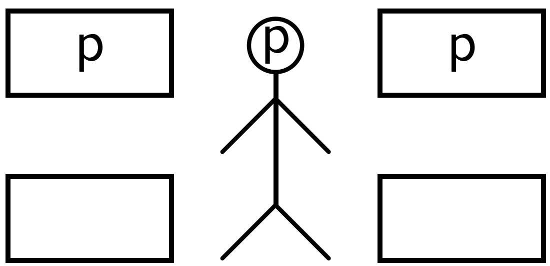
Fast Algorithm - No Contention (1)



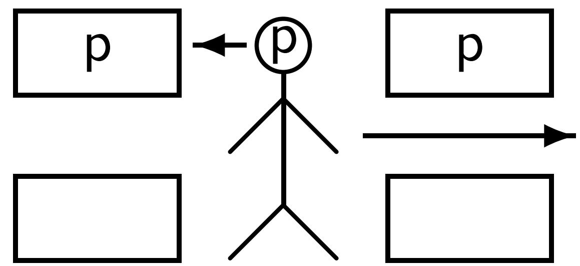
(a)



(b)

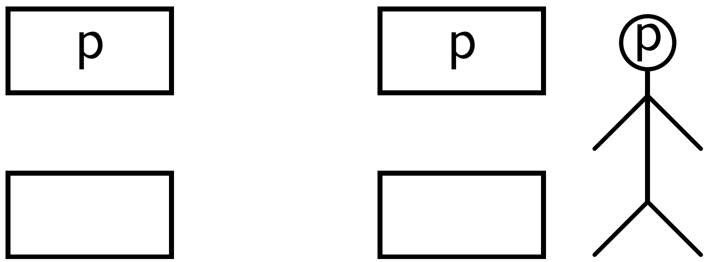


(c)

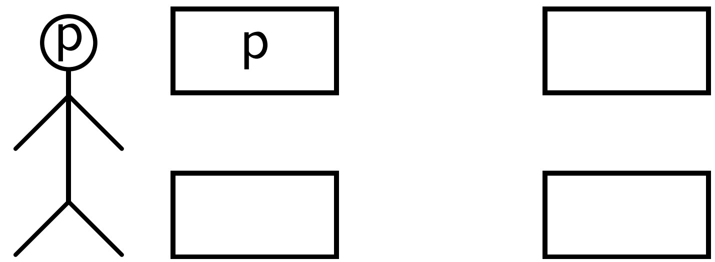


(d)

Fast Algorithm - No Contention (2)

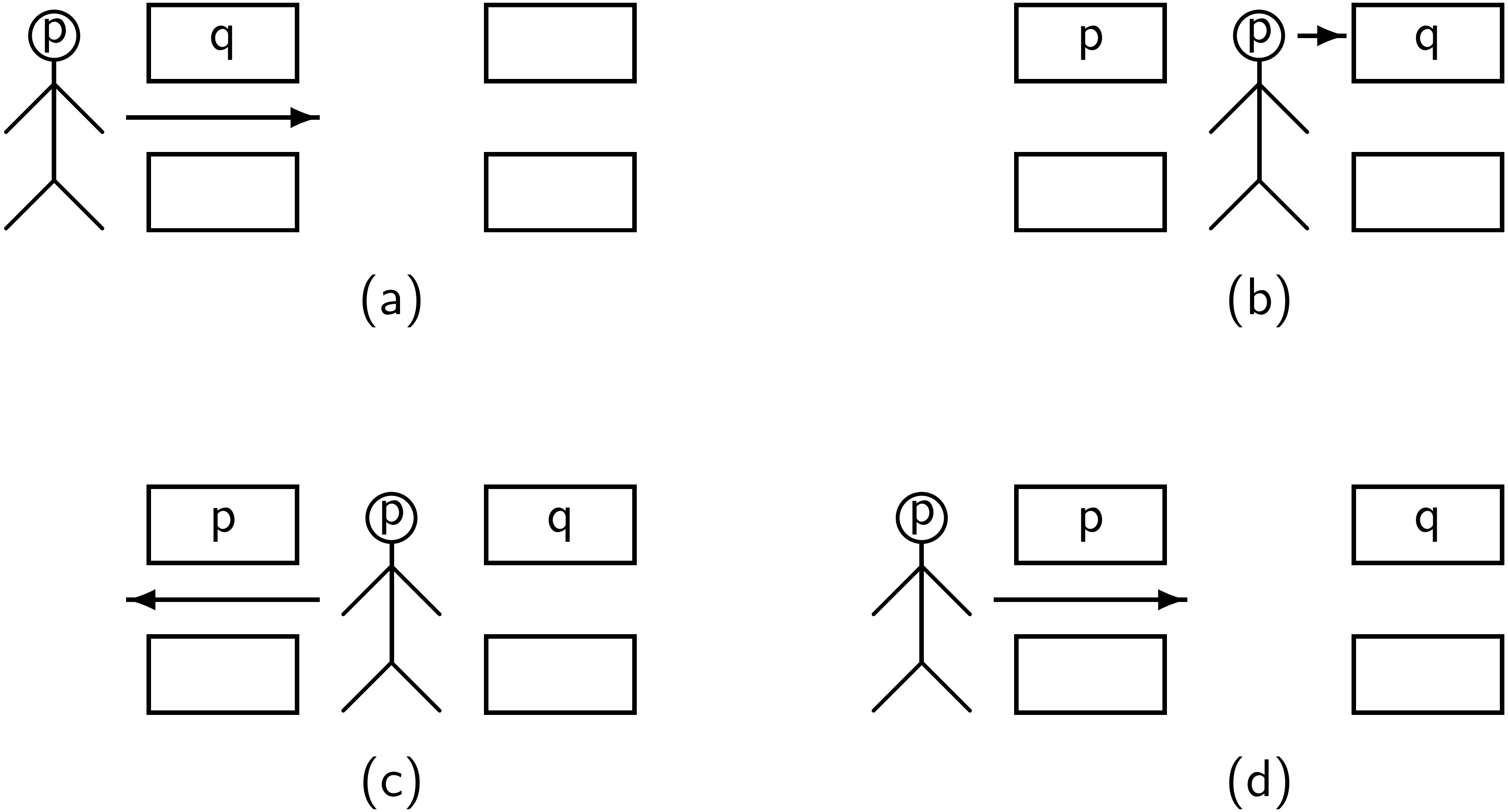


(e)

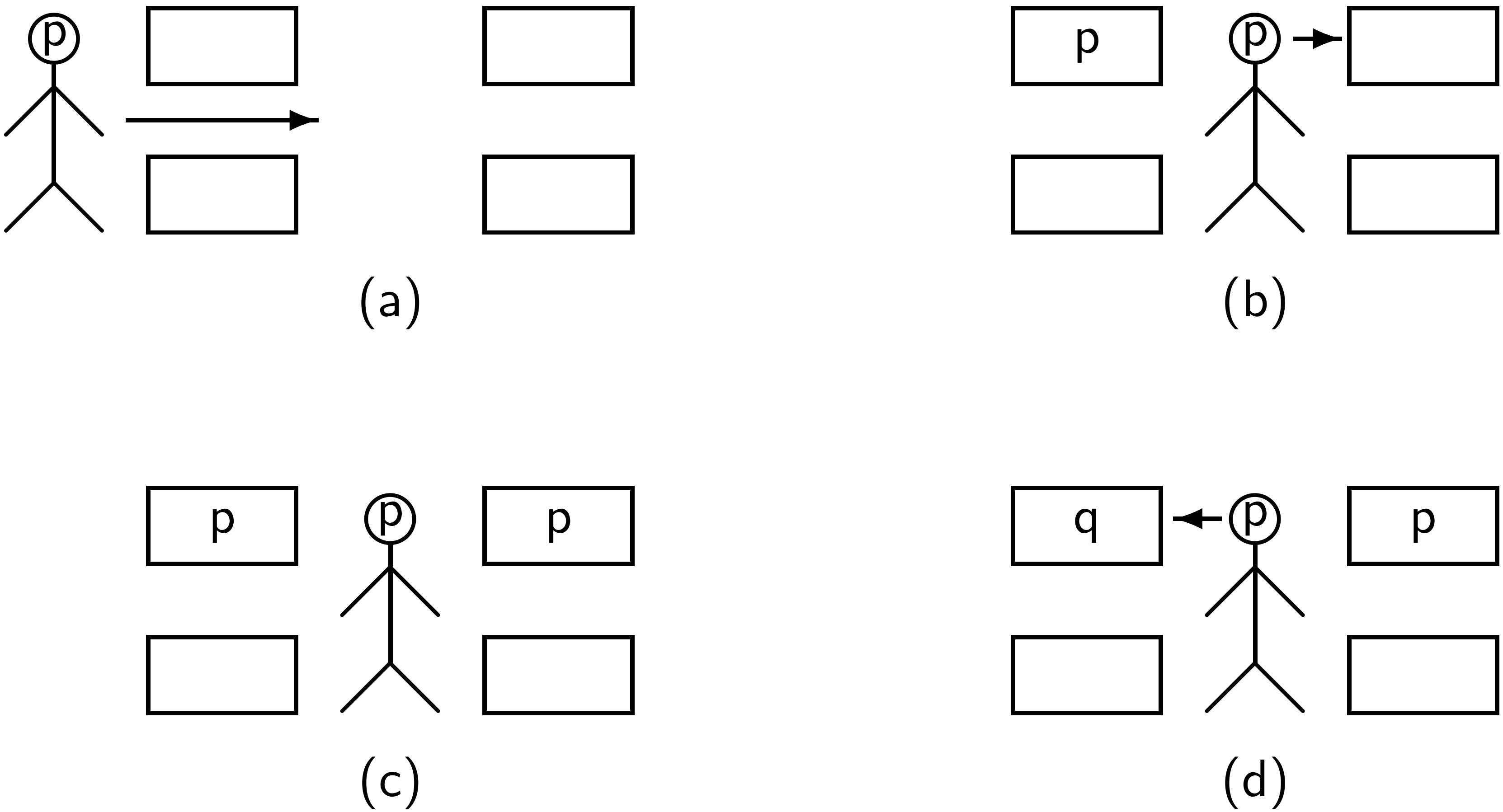


(f)

Fast Algorithm - Contention At Gate 2



Fast Algorithm - Contention At Gate 1 (1)



(a) (b) (c) (d)

Fast Algorithm - Contention At Gate 1 (2)

(e) (f)

Algorithm 5.5: Fast algorithm for two processes (outline)	
integer gate1 $\leftarrow$ 0, gate2 $\leftarrow$ 0	
p	**q**
loop forever	loop forever
non-critical section	non-critical section
p1: gate1 $\leftarrow$ p	q1: gate1 $\leftarrow$ q
p2: if gate2 $\neq$ 0 goto p1	q2: if gate2 $\neq$ 0 goto q1
p3: gate2 $\leftarrow$ p	q3: gate2 $\leftarrow$ q
p4: if gate1 $\neq$ p	q4: if gate1 $\neq$ q
p5: if gate2 $\neq$ p goto p1	q5: if gate2 $\neq$ q goto q1
critical section	critical section
p6: gate2 $\leftarrow$ 0	q6: gate2 $\leftarrow$ 0

Algorithm 5.6: Fast algorithm for two processes			
integer gate1 $\leftarrow$ 0, gate2 $\leftarrow$ 0 boolean wantp $\leftarrow$ false, wantq $\leftarrow$ false			
	p		**q**
p1:	gate1 $\leftarrow$ p	q1:	gate1 $\leftarrow$ q
	wantp $\leftarrow$ true		wantq $\leftarrow$ true
p2:	if gate2 $\neq$ 0	q2:	if gate2 $\neq$ 0
	wantp $\leftarrow$ false		wantq $\leftarrow$ false
	goto p1		goto q1
p3:	gate2 $\leftarrow$ p	q3:	gate2 $\leftarrow$ q
p4:	if gate1 $\neq$ p	q4:	if gate1 $\neq$ q
	wantp $\leftarrow$ false		wantq $\leftarrow$ false
	await wantq = false		await wantp = false
p5:	if gate2 $\neq$ p goto p1	q5:	if gate2 $\neq$ q goto q1
	else wantp $\leftarrow$ true		else wantq $\leftarrow$ true
	critical section		critical section
p6:	gate2 $\leftarrow$ 0	q6:	gate2 $\leftarrow$ 0
	wantp $\leftarrow$ false		wantq $\leftarrow$ false

Algorithm 5.7: Fisher's algorithm

integer gate ← 0

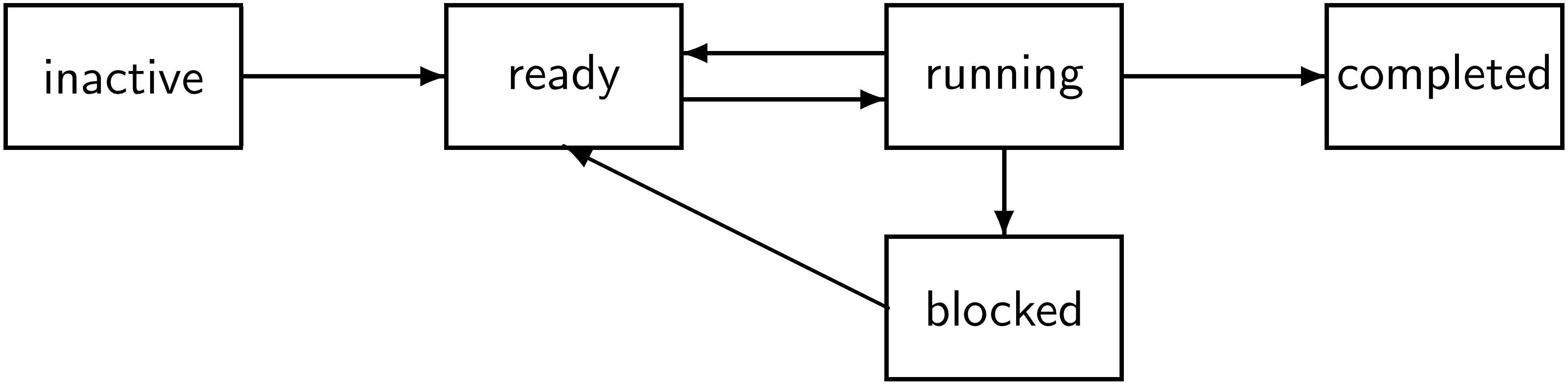
```
    loop forever
      non-critical section
      loop
p1:     await gate = 0
p2:     gate ← i
p3:     delay
p4:   until gate = i
      critical section
p5:   gate ← 0
```

Algorithm 5.8: Lamport's one-bit algorithm
boolean array[1..n] want ← [false,. . . ,false]

```
      loop forever
         non-critical section
p1:      want[i] ← true
p2:      for all processes j ¡ i
p3:         if want[j]
p4:            want[i] ← false
p5:            await not want[j]
               goto p1
p6:      for all processes j ¿ i
p7:         await not want[j]
         critical section
p8:      want[i] ← false
```

Algorithm 5.9: Manna-Pnueli central server algorithm	
integer request ← 0, respond ← 0	
client process i	
	loop forever
	non-critical section
p1:	while respond $\neq$ i
p2:	request ← i
	critical section
p3:	respond ← 0
server process	
	loop forever
p4:	await request $\neq$ 0
p5:	respond ← request
p6:	await respond = 0
p7:	request ← 0

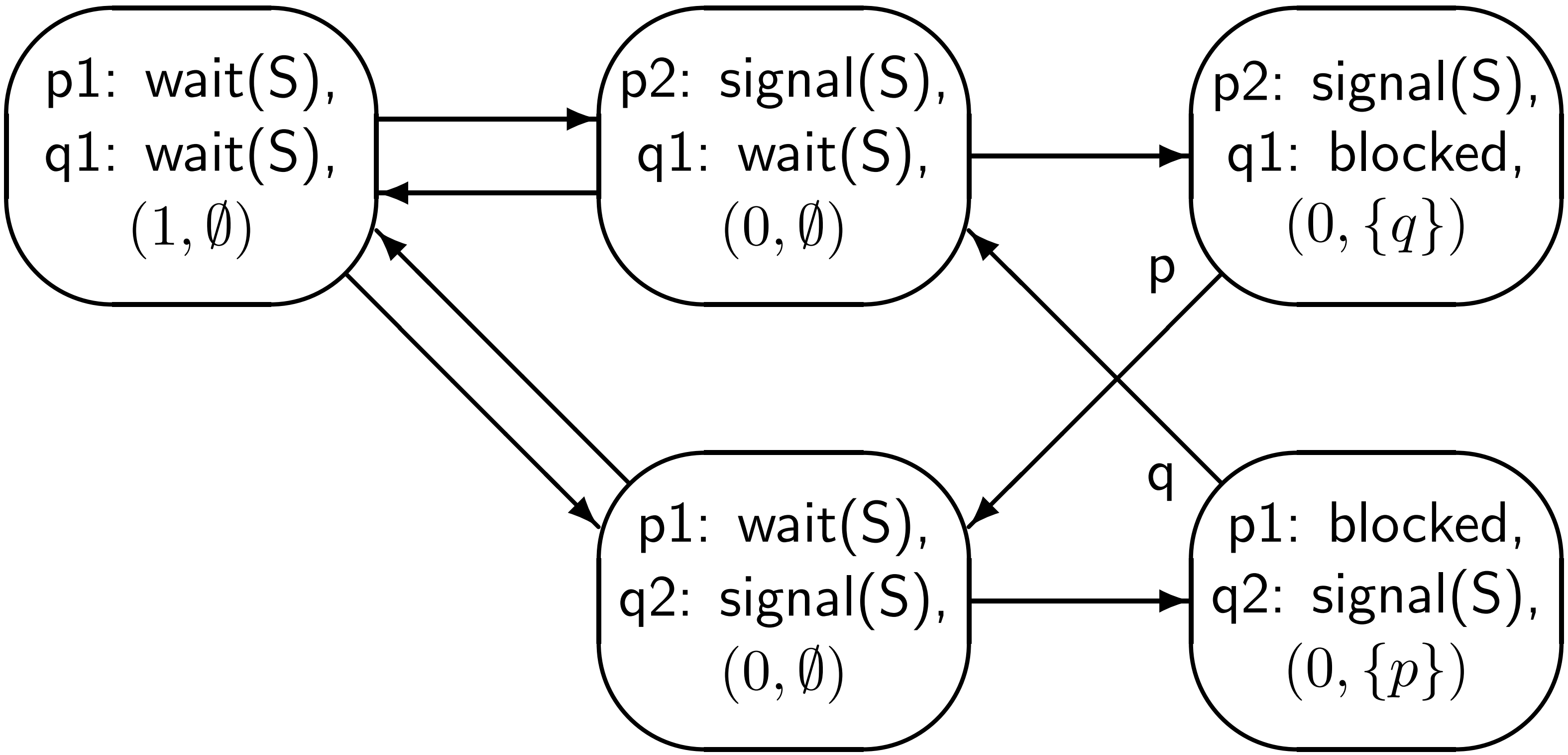
State Changes of a Process



Algorithm 6.1: Critical section with semaphores (two processes)	
binary semaphore S $\leftarrow (1, \emptyset)$	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wait(S)	q2: wait(S)
p3: critical section	q3: critical section
p4: signal(S)	q4: signal(S)

Algorithm 6.2: Critical section with semaphores (two proc., abbrev.)	
binary semaphore S $\leftarrow (1, \emptyset)$	
p	**q**
loop forever	loop forever
p1: wait(S)	q1: wait(S)
p2: signal(S)	q2: signal(S)

State Diagram for the Semaphore Solution



Algorithm 6.3: Critical section with semaphores (N proc.)	
binary semaphore S $\leftarrow (1, \emptyset)$	
	loop forever
p1:	non-critical section
p2:	wait(S)
p3:	critical section
p4:	signal(S)

Algorithm 6.4: Critical section with semaphores (N proc., abbrev.)
binary semaphore S $\leftarrow (1, \emptyset)$
loop forever p1: wait(S) p2: signal(S)

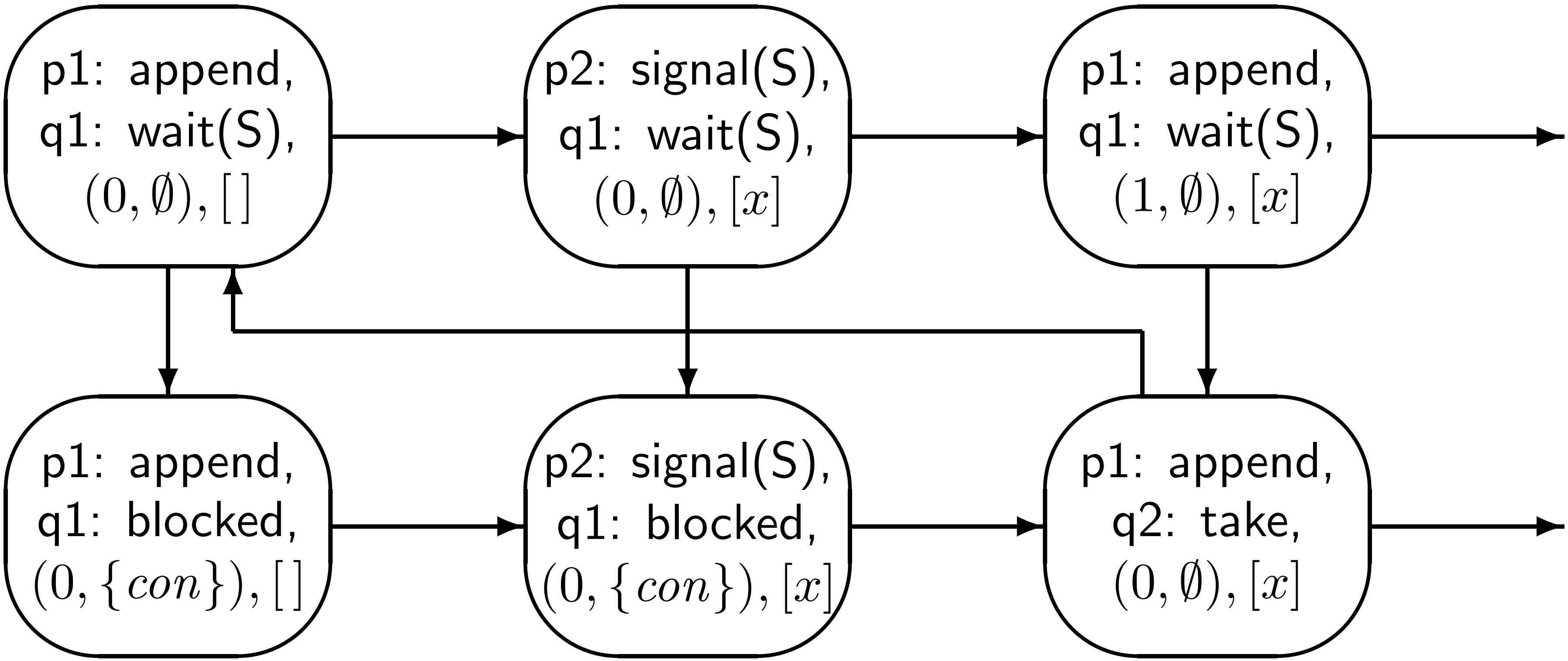
Scenario for Starvation

n	Process p	Process q	Process r	S
1	**p1: wait(S)**	q1: wait(S)	r1: wait(S)	$(1, \emptyset)$
2	p2: signal(S)	**q1: wait(S)**	r1: wait(S)	$(0, \emptyset)$
3	p2: signal(S)	q1: blocked	**r1: wait(S)**	$(0, \{q\})$
4	**p1: signal(S)**	q1: blocked	r1: blocked	$(0, \{q, r\})$
5	**p1: wait(S)**	q1: blocked	r2: signal(S)	$(0, \{q\})$
6	p1: blocked	q1: blocked	**r2: signal(S)**	$(0, \{p, q\})$
7	p2: signal(S)	q1: blocked	**r1: wait(S)**	$(0, \{q\})$

Algorithm 6.5: Mergesort		
integer array A binary semaphore S1 $\leftarrow (0, \emptyset)$ binary semaphore S2 $\leftarrow (0, \emptyset)$		
sort1	**sort2**	**merge**
p1: sort 1st half of A	q1: sort 2nd half of A	r1: wait(S1)
p2: signal(S1)	q2: signal(S2)	r2: wait(S2)
p3:	q3:	r3: merge halves of A

Algorithm 6.6: Producer-consumer (infinite buffer)	
infinite queue of dataType buffer ← empty queue semaphore notEmpty ← $(0, \emptyset)$	
producer	**consumer**
dataType d	dataType d
loop forever	loop forever
p1: d ← produce	q1: wait(notEmpty)
p2: append(d, buffer)	q2: d ← take(buffer)
p3: signal(notEmpty)	q3: consume(d)

Partial State Diagram for Producer-Consumer with Infinite Buffer



Algorithm 6.7: Producer-consumer (infinite buffer, abbreviated)			
infinite queue of dataType buffer $\leftarrow$ empty queue semaphore notEmpty $\leftarrow (0, \emptyset)$			
	producer		**consumer**
	dataType d		dataType d
	loop forever		loop forever
p1:	append(d, buffer)	q1:	wait(notEmpty)
p2:	signal(notEmpty)	q2:	d $\leftarrow$ take(buffer)

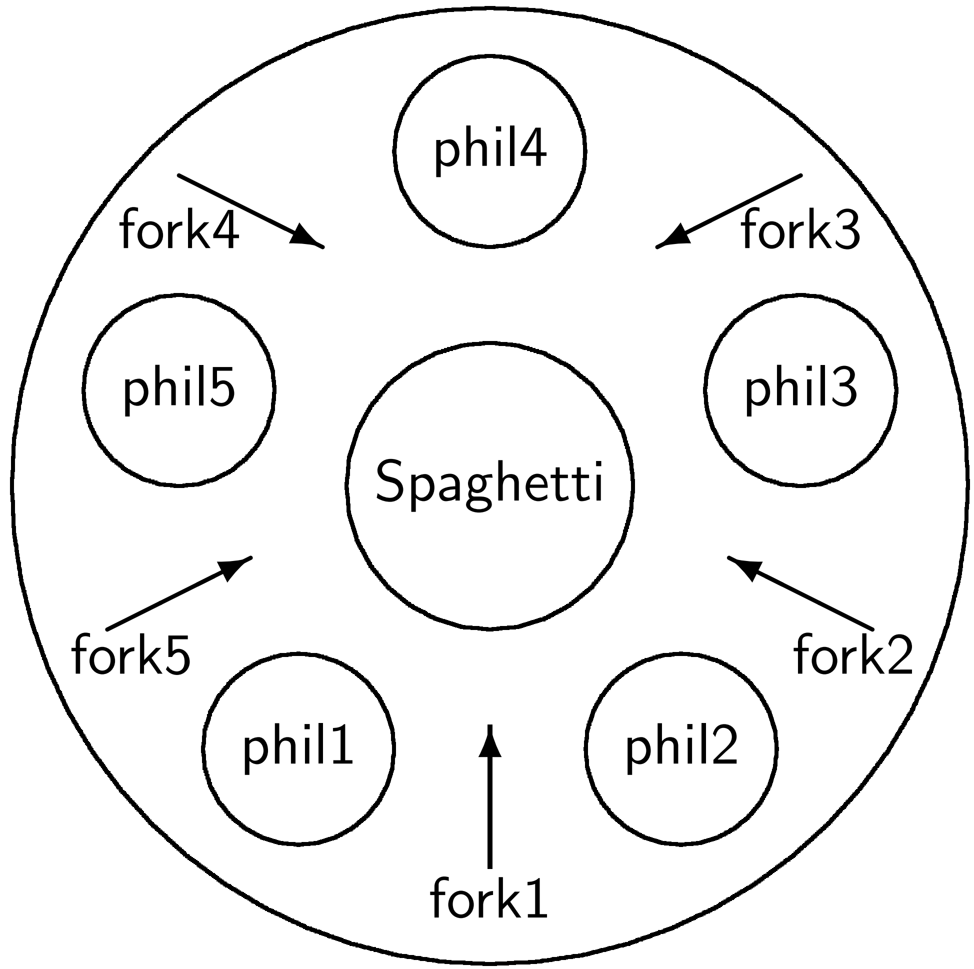
Algorithm 6.8: Producer-consumer (finite buffer, semaphores)	
finite queue of dataType buffer $\leftarrow$ empty queue semaphore notEmpty $\leftarrow (0, \emptyset)$ semaphore notFull $\leftarrow (N, \emptyset)$	
producer	**consumer**
dataType d	dataType d
loop forever	loop forever
p1: d $\leftarrow$ produce	q1: wait(notEmpty)
p2: wait(notFull)	q2: d $\leftarrow$ take(buffer)
p3: append(d, buffer)	q3: signal(notFull)
p4: signal(notEmpty)	q4: consume(d)

Scenario with Busy Waiting

n	Process p	Process q	S
1	**p1: wait(S)**	q1: wait(S)	1
2	p2: signal(S)	**q1: wait(S)**	0
3	**p2: signal(S)**	q1: wait(S)	0
4	**p1: wait(S)**	q1: wait(S)	1

Algorithm 6.9: Dining philosophers (outline)	
	loop forever
p1:	think
p2:	preprotocol
p3:	eat
p4:	postprotocol

The Dining Philosophers



Algorithm 6.10: Dining philosophers (first attempt)	
semaphore array [0..4] fork $\leftarrow$ [1,1,1,1,1]	
	loop forever
p1:	think
p2:	wait(fork[i])
p3:	wait(fork[i+1])
p4:	eat
p5:	signal(fork[i])
p6:	signal(fork[i+1])

Algorithm 6.11: Dining philosophers (second attempt)

```
            semaphore array [0..4] fork ← [1,1,1,1,1]
            semaphore room ← 4

    loop forever
p1:     think
p2:     wait(room)
p3:     wait(fork[i])
p4:     wait(fork[i+1])
p5:     eat
p6:     signal(fork[i])
p7:     signal(fork[i+1])
p8:     signal(room)
```

Algorithm 6.12: Dining philosophers (third attempt)	
semaphore array [0..4] fork ← [1,1,1,1,1]	
philosopher 4	
	loop forever
p1:	think
p2:	wait(fork[0])
p3:	wait(fork[4])
p4:	eat
p5:	signal(fork[0])
p6:	signal(fork[4])

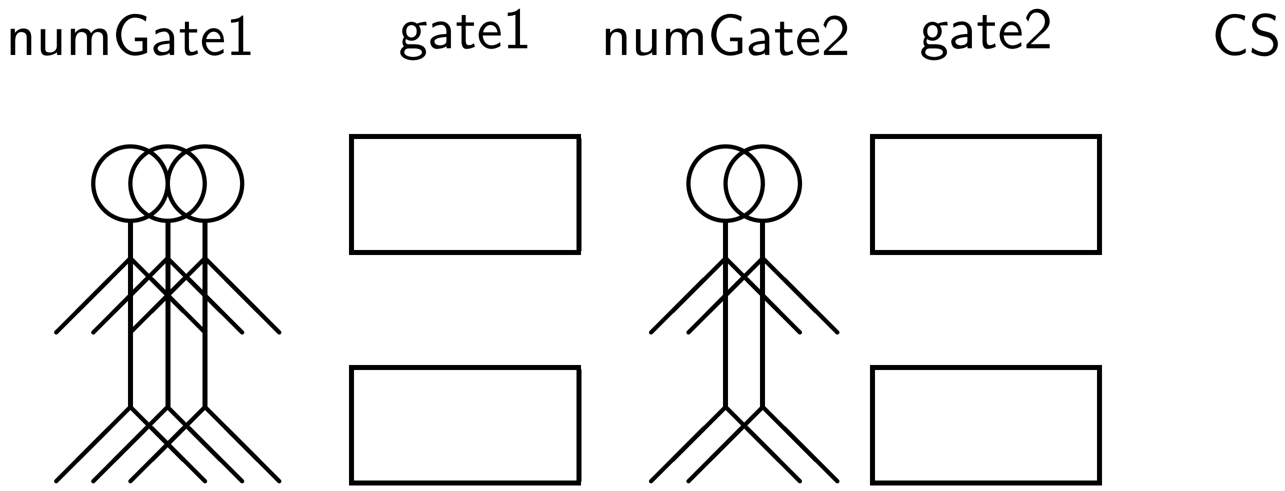
Algorithm 6.13: Barz's algorithm for simulating general semaphores

```
                binary semaphore S ← 1
                binary semaphore gate ← 1
                integer count ← k

     loop forever
        non-critical section
p1:     wait(gate)
p2:     wait(S)                              // Simulated wait
p3:     count ← count − 1
p4:     if count > 0 then
p5:        signal(gate)
p6:     signal(S)
        critical section
p7:     wait(S)                              // Simulated signal
p8:     count ← count + 1
p9:     if count = 1 then
p10:       signal(gate)
p11:    signal(S)
```

Algorithm 6.14: Udding's starvation-free algorithm

```
semaphore gate1 ← 1, gate2 ← 0
integer numGate1 ← 0, numGate2 ← 0

p1:   wait(gate1)
p2:   numGate1 ← numGate1 + 1
p3:   signal(gate1)
p4:   wait(gate1)
p5:   numGate2 ← numGate2 + 1
      numGate1 ← numGate1 − 1        // Statement is missing in the
book
p6:   if numGate1 ¿ 0
p7:      signal(gate1)
p8:   else signal(gate2)
p9:   wait(gate2)
p10:  numGate2 ← numGate2 − 1
      critical section
p11:  if numGate2 ¿ 0
p12:     signal(gate2)
p13:  else signal(gate1)
```

Udding's Starvation-Free Algorithm



Scenario for Starvation in Udding's Algorithm

n	Process p	Process q	gate1	gate2	nGate1	nGate2
1	**p4: wait(g1)**	q4: wait(g1)	1	0	2	0
2	**p9: wait(g2)**	q9: wait(g2)	0	1	0	2
3	**CS**	q9: wait(g2)	0	0	0	1
4	**p12: signal(g2)**	q9: wait(g2)	0	0	0	1
5	p1: wait(g1)	**CS**	0	0	0	0
6	**p1: wait(g1)**	q13: signal(g1)	0	0	0	0
7	p1: blocked	**q13: signal(g1)**	0	0	0	0
8	p4: wait(g1)	**q1: wait(g1)**	1	0	1	0
9	**p4: wait(g1)**	q4: wait(g1)	1	0	2	0

Semaphores in Java

```java
import java.util.concurrent.Semaphore;
class CountSem extends Thread {
  static volatile int n = 0;
  static Semaphore s = new Semaphore(1);

  public void run() {
    int temp;
    for (int i = 0; i < 10; i++) {
      try {
        s.acquire();
      }
      catch (InterruptedException e) {}
      temp = n;
      n = temp + 1;
      s.release();
    }
  }

  public static void main(String[] args) {
      /* As before */
  }
}
```

Semaphores in Ada

```
protected type Semaphore(Initial : Natural) is
   entry Wait;
   procedure Signal;
private
   Count: Natural := Initial ;
end Semaphore;

protected body Semaphore is
   entry Wait when Count > 0 is
   begin
      Count := Count − 1;
   end Wait;

   procedure Signal is
   begin
      Count := Count + 1;
      end Signal;
end Semaphore;
```

Busy-Wait Semaphores in Promela

```
inline wait( s ) {
        atomic { s > 0 ; s-- }
}

inline signal ( s ) { s++ }
```

Weak Semaphores in Promela (3 processes) (1)

```
typedef Semaphore {
        byte count;
        bool blocked[NPROCS];
};

inline  initSem(S, n) {
  S.count = n
}
```

Weak Semaphores in Promela (3 processes) (2)

```
inline wait(S) {
    atomic {
      if
      :: S.count >= 1 -> S.count--
      :: else ->
          S.blocked[_pid-1] = true;
          !S.blocked[_pid-1]
      fi
    }
}

inline signal (S) {
    atomic {
      if
      :: S.blocked[0] -> S.blocked[0] = false
      :: S.blocked[1] -> S.blocked[1] = false
      :: S.blocked[2] -> S.blocked[2] = false
      :: else -> S.count++
      fi
    }
}
```

Weak Semaphores in Promela (N processes) (1)

```
typedef Semaphore {
        byte count;
        bool blocked[NPROCS];
  byte i, choice;
};

inline initSem(S, n) {
  S.count = n
}

inline wait(S) {
   atomic {
     if
     :: S.count >= 1 -> S.count--
     :: else ->
          S.blocked[_pid-1] = true;
          !S.blocked[_pid-1]
     fi
   }
}
```

Weak Semaphores in Promela (N processes) (2)

```
inline signal (S) {
    atomic {
        S.i = 0;
        S.choice = 255;
        do
        :: (S.i == NPROCS) -> break
        :: (S.i < NPROCS) && !S.blocked[S.i] -> S.i++
        :: else ->
            if
            :: (S.choice == 255) -> S.choice = S.i
            :: (S.choice != 255) -> S.choice = S.i
            :: (S.choice != 255) ->
            fi ;
            S.i ++
        od;
        if
        :: S.choice == 255 -> S.count++
        :: else -> S.blocked[S.choice] = false
        fi
    }
}
```

Barz's Algorithm in Promela (N processes, K in CS)

```
byte gate = 1;
int  count = K;

active [N] proctype P () {
  do ::
    atomic { gate > 0; gate--; }
    d_step {
      count--;
      if
      :: count > 0 -> gate++
      :: else
      fi
    }
    /* Critical section */
    d_step {
      count++;
      if
      :: count == 1 -> gate++
      :: else
      fi
    }
  od
}
```

Algorithm 6.15: Semaphore algorithm A	
semaphore S ← 1, semaphore T ← 0	
p	**q**
p1: wait(S)	q1: wait(T)
p2: write("p")	q2: write("q")
p3: signal(T)	q3: signal(S)

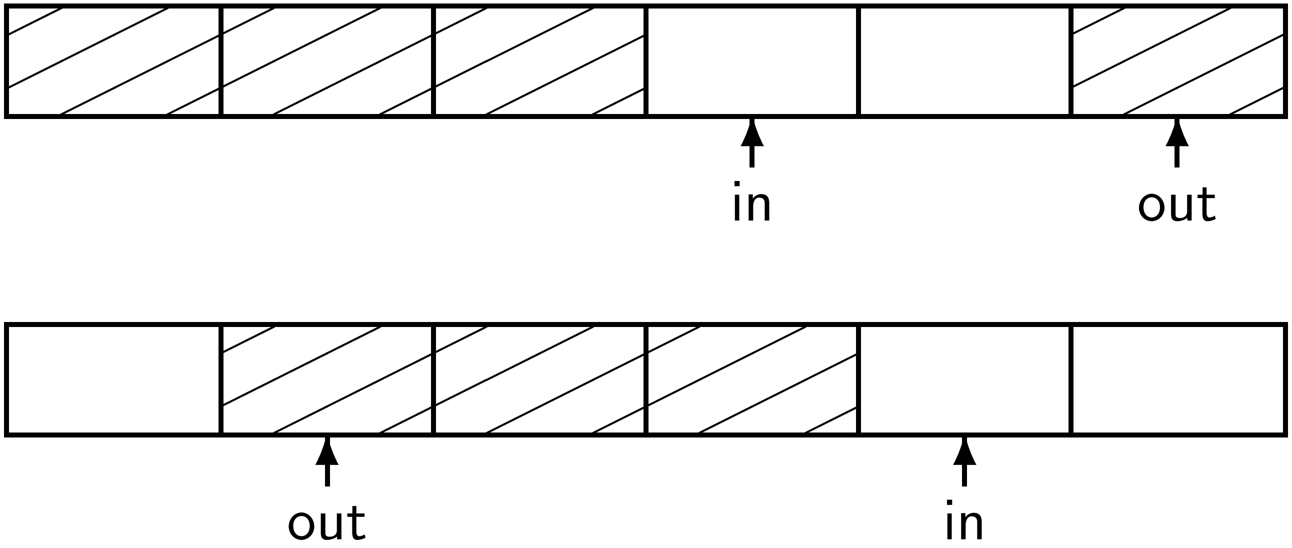
Algorithm 6.16: Semaphore algorithm B		
semaphore S1 ← 0, S2 ← 0		
p	**q**	**r**
p1: write("p")	q1: wait(S1)	r1: wait(S2)
p2: signal(S1)	q2: write("q")	r2: write("r")
p3: signal(S2)	q3:	r3:

Algorithm 6.17: Semaphore algorithm with a loop	
semaphore S $\leftarrow$ 1 boolean B $\leftarrow$ false	
p	**q**
p1: wait(S)	q1: wait(S)
p2: B $\leftarrow$ true	q2: while not B
p3: signal(S)	q3: write("*")
p4:	q4: signal(S)

Algorithm 6.18: Critical section problem (k out of N processes)

	binary semaphore S $\leftarrow$ 1, delay $\leftarrow$ 0
	integer count $\leftarrow$ k
	integer m
	loop forever
p1:	non-critical section
p2:	wait(S)
p3:	count $\leftarrow$ count $-$ 1
p4:	m $\leftarrow$ count
p5:	signal(S)
p6:	if m $\leq$ -1 wait(delay)
p7:	critical section
p8:	wait(S)
p9:	count $\leftarrow$ count $+$ 1
p10:	if count $\leq$ 0 signal(delay)
p11:	signal(S)

Circular Buffer



Algorithm 6.19: Producer-consumer (circular buffer)	
dataType array [0..N] buffer integer in, out $\leftarrow$ 0 semaphore notEmpty $\leftarrow (0, \emptyset)$ semaphore notFull $\leftarrow (N, \emptyset)$	
producer	**consumer**
dataType d loop forever p1: d $\leftarrow$ produce p2: wait(notFull) p3: buffer[in] $\leftarrow$ d p4: in $\leftarrow$ (in+1) modulo N p5: signal(notEmpty)	dataType d loop forever q1: wait(notEmpty) q2: d $\leftarrow$ buffer[out] q3: out $\leftarrow$ (out+1) modulo N q4: signal(notFull) q5: consume(d)

Algorithm 6.20: Simulating general semaphores

```
        binary semaphore S ← 1, gate ← 0
        integer count ← 0

    wait
p1:  wait(S)
p2:  count ← count − 1
p3:  if count < 0
p4:     signal(S)
p5:     wait(gate)
p6:  else signal(S)

    signal
p7:  wait(S)
p8:  count ← count + 1
p9:  if count ≤ 0
p10:    signal(gate)
p11: signal(S)
```

Weak Semaphores in Promela with Channels

```
typedef Semaphore {
            byte count;
            chan ch = [NPROCS] of { pid };
            byte temp, i ;
};
inline  initSem(S, n) { S.count = n }
inline  wait(S) {
    atomic {
       if
       :: S.count >= 1 -> S.count--;
       :: else -> S.ch ! _pid; !( S.ch ?? [eval( _pid )])
       fi
    }
}
inline  signal (S) {
    atomic {
       S.i = len(S.ch);
       if
       :: S.i == 0 -> S.count++ /*No blocked process, increment count*/
       :: else ->
                   do
          :: S.i == 1 -> S.ch ? _; break /*Remove only blocked process*/
          :: else -> S.i--;
             S.ch ? S.temp;
```

Algorithm 6.21: Readers and writers with semaphores
semaphore readerSem $\leftarrow$ 0, writerSem $\leftarrow$ 0 integer delayedReaders $\leftarrow$ 0, delayedWriters $\leftarrow$ 0 semaphore entry $\leftarrow$ 1 integer readers $\leftarrow$ 0, writers $\leftarrow$ 0
SignalProcess

if writers = 0 or delayedReaders > 0
    delayedReaders $\leftarrow$ delayedReaders $-$ 1
    signal(readerSem)
else if readers = 0 and writers = 0 and delayedWriters > 0
    delayedWriters $\leftarrow$ delayedWriters $-$ 1
    signal(writerSem)
else signal(entry)

Algorithm 6.21: Readers and writers with semaphores

StartRead

```
p1:  wait(entry)
p2:  if writers > 0
p3:     delayedReaders ← delayedReaders + 1
p4:     signal(entry)
p5:     wait(readerSem)
p6:  readers ← readers + 1
p7:  SignalProcess
```

EndRead

```
p8:  wait(entry)
p9:  readers ← readers − 1
p10: SignalProcess
```

Algorithm 6.21: Readers and writers with semaphores

StartWrite

```
p11: wait(entry)
p12: if writers > 0 or readers > 0
p13:     delayedWriters ← delayedWriters + 1
p14:     signal(entry)
p15:     wait(writerSem)
p16: writers ← writers + 1
p17: SignalProcess
```

EndWrite

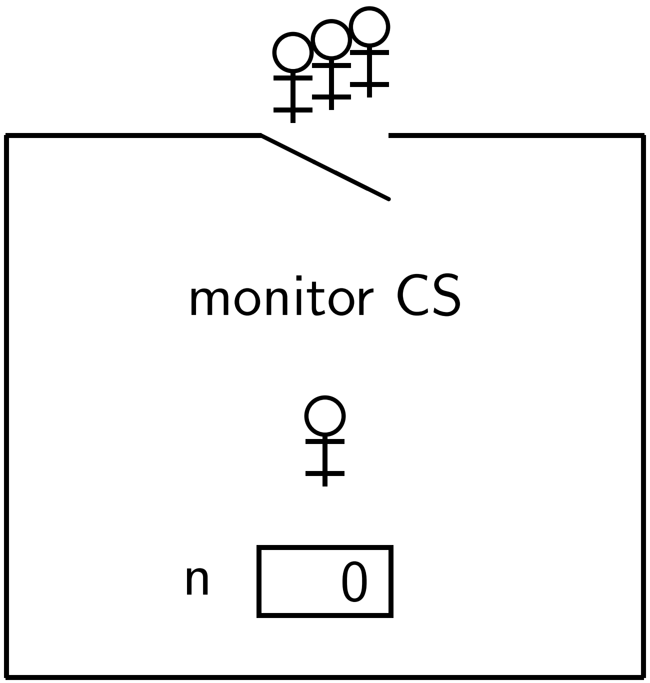
```
p18: wait(entry)
p19: writers ← writers − 1
p20: SignalProcess
```

Algorithm 7.1: Atomicity of monitor operations	

monitor CS
   integer n ← 0
 	
   operation increment
      integer temp
      temp ← n
      n ← temp + 1	

p	**q**
p1: CS.increment	q1: CS.increment

Executing a Monitor Operation

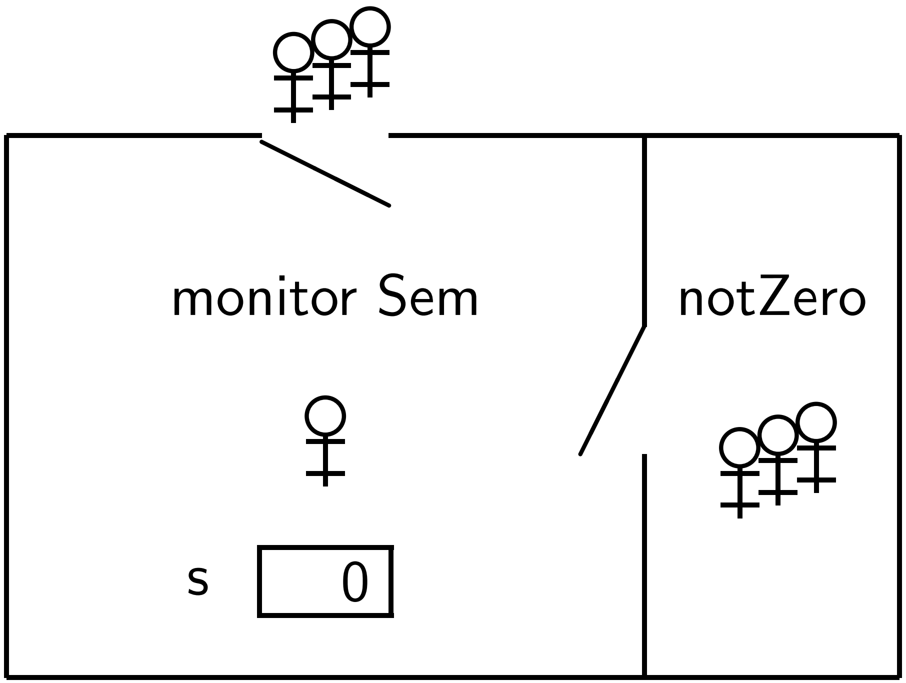


Algorithm 7.2: Semaphore simulated with a monitor

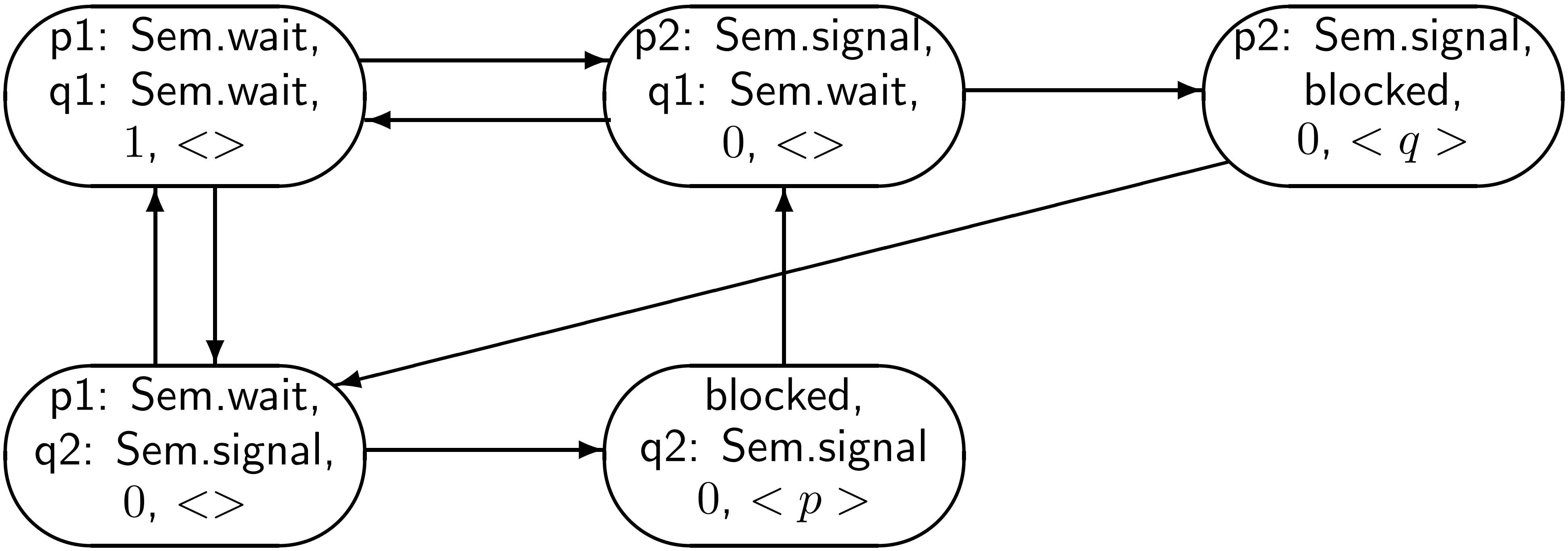
```
monitor Sem
   integer s ← k
   condition notZero
   operation wait
      if s = 0
         waitC(notZero)
      s ← s − 1
   operation signal
      s ← s + 1
      signalC(notZero)
```

p	q
loop forever	loop forever
non-critical section	non-critical section
p1: Sem.wait	q1: Sem.wait
critical section	critical section
p2: Sem.signal	q2: Sem.signal

Condition Variable in a Monitor



State Diagram for the Semaphore Simulation

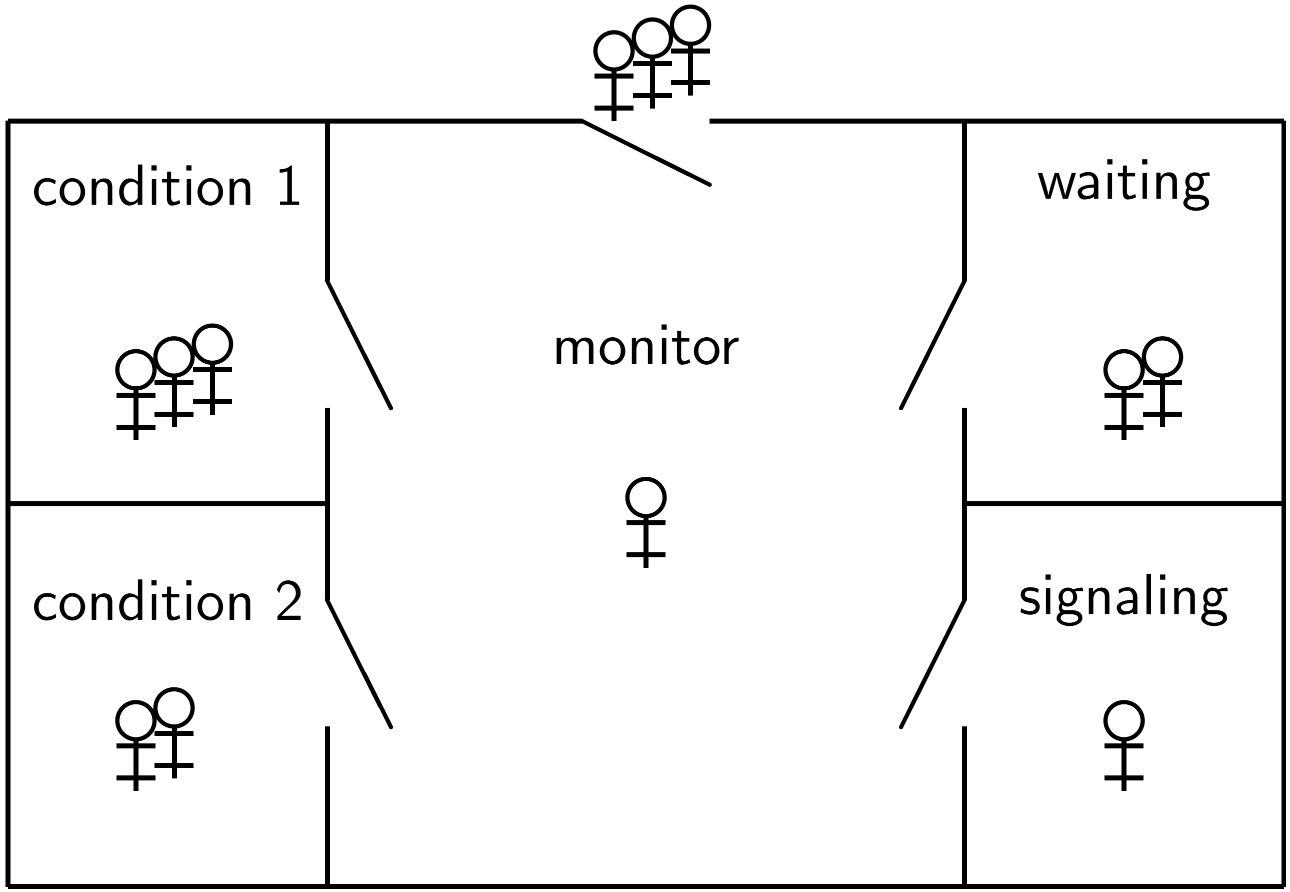


Algorithm 7.3: Producer-consumer (finite buffer, monitor)

```
monitor PC
   bufferType buffer ← empty
   condition notEmpty
   condition notFull
   operation append(datatype V)
      if buffer is full
         waitC(notFull)
      append(V, buffer)
      signalC(notEmpty)
   operation take()
      datatype W
      if buffer is empty
         waitC(notEmpty)
      W ← head(buffer)
      signalC(notFull)
      return W
```

Algorithm 7.3: Producer-consumer (finite buffer, monitor) (continued)			
producer		**consumer**	
	datatype D		datatype D
	loop forever		loop forever
p1:	D ← produce	q1:	D ← PC.take
p2:	PC.append(D)	q2:	consume(D)

The Immediate Resumption Requirement



Algorithm 7.4: Readers and writers with a monitor

```
monitor RW
   integer readers ← 0
   integer writers ← 0
   condition OKtoRead, OKtoWrite
   operation StartRead
      if writers ≠ 0 or not empty(OKtoWrite)
         waitC(OKtoRead)
      readers ← readers + 1
      signalC(OKtoRead)
   operation EndRead
      readers ← readers − 1
      if readers = 0
         signalC(OKtoWrite)
```

Algorithm 7.4: Readers and writers with a monitor (continued)

operation StartWrite
    if writers $\neq$ 0 or readers $\neq$ 0
        waitC(OKtoWrite)
    writers $\leftarrow$ writers + 1

operation EndWrite
    writers $\leftarrow$ writers $-$ 1
    if empty(OKtoRead)
        then signalC(OKtoWrite)
        else signalC(OKtoRead)

reader	writer
p1: RW.StartRead	q1: RW.StartWrite
p2: read the database	q2: write to the database
p3: RW.EndRead	q3: RW.EndWrite

Algorithm 7.5: Dining philosophers with a monitor

```
monitor ForkMonitor
   integer array[0..4] fork ← [2, . . . , 2]
   condition array[0..4] OKtoEat
   operation takeForks(integer i)
      if fork[i] ≠ 2
         waitC(OKtoEat[i])
      fork[i+1] ← fork[i+1] − 1
      fork[i−1] ← fork[i−1] − 1

   operation releaseForks(integer i)
      fork[i+1] ← fork[i+1] + 1
      fork[i−1] ← fork[i−1] + 1
      if fork[i+1] = 2
         signalC(OKtoEat[i+1])
      if fork[i−1] = 2
         signalC(OKtoEat[i−1])
```

Algorithm 7.5: Dining philosophers with a monitor (continued)	
	philosopher i
	loop forever
p1:	think
p2:	takeForks(i)
p3:	eat
p4:	releaseForks(i)

Scenario for Starvation of Philosopher 2

n	phil1	phil2	phil3	$f0$	$f1$	$f2$	$f3$	$f4$
1	**take(1)**	take(2)	take(3)	2	2	2	2	2
2	release(1)	take(2)	**take(3)**	1	2	1	2	2
3	release(1)	**take(2) and waitC(OK[2])**	release(3)	1	2	0	2	1
4	**release(1)**	(blocked)	release(3)	1	2	0	2	1
5	**take(1)**	(blocked)	release(3)	2	2	1	2	1
6	release(1)	(blocked)	**release(3)**	1	2	0	2	1
7	release(1)	(blocked)	**take(3)**	1	2	1	2	2

Readers and Writers in C

```
monitor RW {
  int readers = 0, writing = 1;
  condition OKtoRead, OKtoWrite;

  void StartRead() {
    if (writing || !empty(OKtoWrite)) waitc(OKtoRead);
    readers = readers + 1;
    signalc (OKtoRead);
  }
  void EndRead() {
    readers = readers − 1;
    if (readers == 0) signalc (OKtoWrite);
  }

  void StartWrite () {
    if (writing || (readers != 0)) waitc(OKtoWrite);
    writing = 1;
  }
  void EndWrite() {
    writing = 0;
    if (empty(OKtoRead)) signalc(OKtoWrite);
    else                 signalc (OKtoRead);
  }
}
```

Algorithm 7.6: Readers and writers with a protected object

```
protected object RW
   integer readers ← 0
   boolean writing ← false
   operation StartRead when not writing
      readers ← readers + 1
   operation EndRead
      readers ← readers − 1
   operation StartWrite when not writing and readers = 0
      writing ← true

   operation EndWrite
      writing ← false
```

reader	writer
loop forever	loop forever
p1: RW.StartRead	q1: RW.StartWrite
p2: read the database	q2: write to the database
p3: RW.EndRead	q3: RW.EndWrite

Context Switches in a Monitor

Process reader	Process writer
waitC(OKtoRead)	operation EndWrite
(blocked)	**writing $\leftarrow$ false**
(blocked)	**signalC(OKtoRead)**
readers $\leftarrow$ readers + 1	return from EndWrite
signalC(OKtoRead)	return from EndWrite
read the data	**return from EndWrite**
read the data	...

Context Switches in a Protected Object

Process reader	Process writer
when not writing	operation EndWrite
(blocked)	**writing** $\leftarrow$ **false**
(blocked)	**when not writing**
(blocked)	**readers** $\leftarrow$ **readers** $+$ **1**
read the data	...

Simple Readers and Writers in Ada

```
protected RW is
   procedure Write(I: Integer );
   function Read return Integer ;
private
   N: Integer := 0;
end RW;

protected body RW is
   procedure Write(I: Integer ) is
   begin
      N := I;
   end Write;
   function Read return Integer is
   begin
      return N;
   end Read;
end RW;
```

Readers and Writers in Ada (1)

```
protected RW is
    entry StartRead;
    procedure EndRead;
    entry Startwrite ;
    procedure EndWrite;
private
    Readers: Natural :=0;
    Writing: Boolean := false ;
end RW;
```

Readers and Writers in Ada (2)

```
protected body RW is
   entry StartRead
     when not Writing is
   begin
      Readers := Readers + 1;
   end StartRead;

   procedure EndRead is
   begin
      Readers := Readers − 1;
   end EndRead;

   entry StartWrite
     when not Writing and Readers = 0 is
   begin
      Writing := true;
   end StartWrite;

   procedure EndWrite is
   begin
      Writing := false ;
   end EndWrite;
end RW;
```

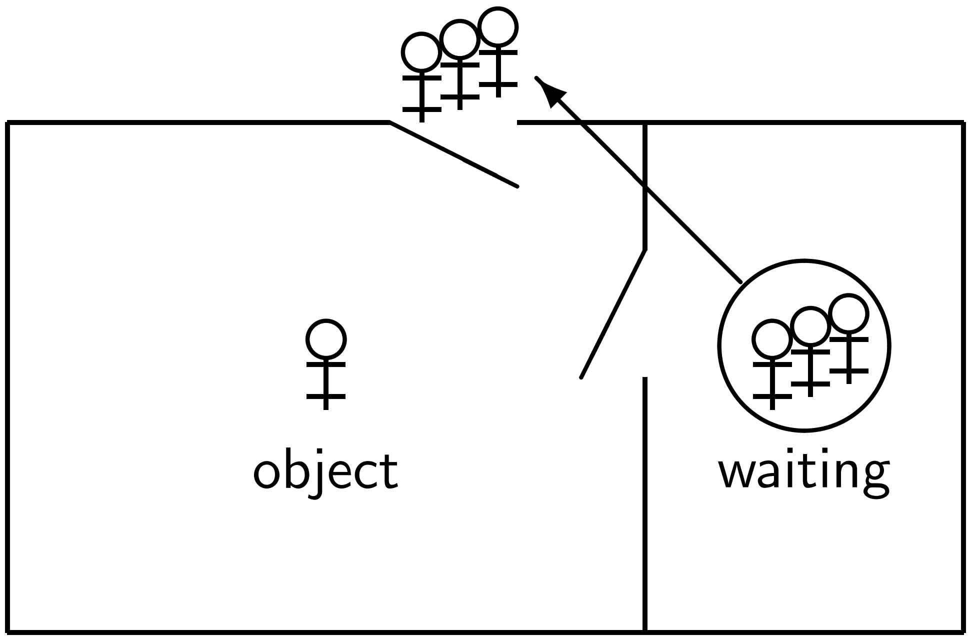
Producer-Consumer in Java (1)

```
class PCMonitor {
  final int N = 5;
  int Oldest = 0, Newest = 0;
  volatile int Count = 0;
  int Buffer[] = new int[N];

  synchronized void Append(int V) {
    while (Count == N)
      try {
          wait();
      } catch (InterruptedException e) {}
    Buffer[Newest] = V;
    Newest = (Newest + 1) % N;
    Count = Count + 1;
    notifyAll ();
  }
```

Producer-Consumer in Java (2)

```
    synchronized int Take() {
      int temp;
      while (Count == 0)
        try {
            wait();
        } catch (InterruptedException e) {}
      temp = Buffer[Oldest];
      Oldest = (Oldest + 1) % N;
      Count = Count - 1;
      notifyAll ();
      return temp;
    }
}
```

A Monitor in Java With notifyAll



Java Monitor for RW (`try-catch` omitted)

```
class RWMonitor {
  volatile int readers = 0;
  volatile boolean writing = false;

  synchronized void StartRead() {
    while (writing ) wait();
    readers = readers + 1;
    notifyAll ();
  }
  synchronized void EndRead() {
    readers = readers − 1;
    if (readers == 0) notifyAll();
  }

  synchronized void StartWrite() {
    while (writing || (readers != 0)) wait();
    writing = true;
  }

  synchronized void EndWrite() {
    writing = false;
    notifyAll ();
  }
}
```

Simulating Monitors in Promela (1)

```
bool lock = false;

typedef Condition {
    bool gate;
    byte waiting;
}

#define emptyC(C) (C.waiting == 0)

inline enterMon() {
    atomic {
        !lock;
        lock = true;
    }
}

inline leaveMon() {
  lock = false;
}
```

Simulating Monitors in Promela (2)

```
inline waitC(C) {
    atomic {
        C.waiting++;
        lock = false;      /* Exit monitor */
        C.gate;            /* Wait for gate */
        lock = true;       /* IRR */
        C.gate = false;    /* Reset gate */
        C.waiting--;
    }
}

inline signalC(C) {
    atomic {
        if
            /* Signal only if waiting */
        :: (C.waiting > 0) ->
          C.gate = true;
          !lock;           /* IRR - wait for released lock */
          lock = true; /* Take lock again */
        :: else
        fi;
    }
}
```

Readers and Writers in Ada (1)

```
protected RW is
   entry Start_Read;
   procedure End_Read;
   entry Start_Write ;
   procedure End_Write;
private
   Waiting_To_Read : integer  := 0;
   Readers :  Natural  := 0;
   Writing  :  Boolean := false ;
end RW;
```

Readers and Writers in Ada (2)

```
protected RW is
   entry StartRead;
   procedure EndRead;
   entry Startwrite ;
   procedure EndWrite;
   function NumberReaders return Natural;
private
   entry ReadGate;
   entry WriteGate;
   Readers: Natural :=0;
   Writing: Boolean := false ;
end RW;
```

Algorithm 8.1: Producer-consumer (channels)			
channel of integer ch			
producer		**consumer**	
	integer x		integer y
	loop forever		loop forever
p1:	$x \leftarrow$ produce	q1:	ch $\Rightarrow$ y
p2:	ch $\Leftarrow$ x	q2:	consume(y)

Algorithm 8.2: Conway's problem

```
constant integer MAX ← 9
constant integer K ← 4
channel of integer inC, pipe, outC
```

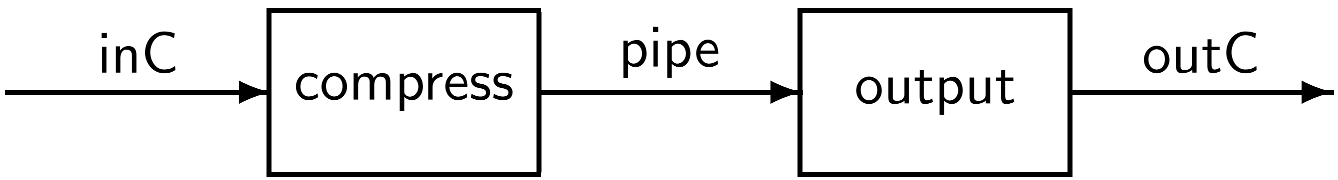
compress

```
char c, previous ← 0
integer n ← 0
inC ⇒ previous
loop forever
   inC ⇒ c
   if (c = previous) and
         (n < MAX − 1)
      n ← n + 1
   else
      if n > 0
         pipe ⇐ intToChar(n+1)
         n ← 0
      pipe ⇐ previous
      previous ← c
```

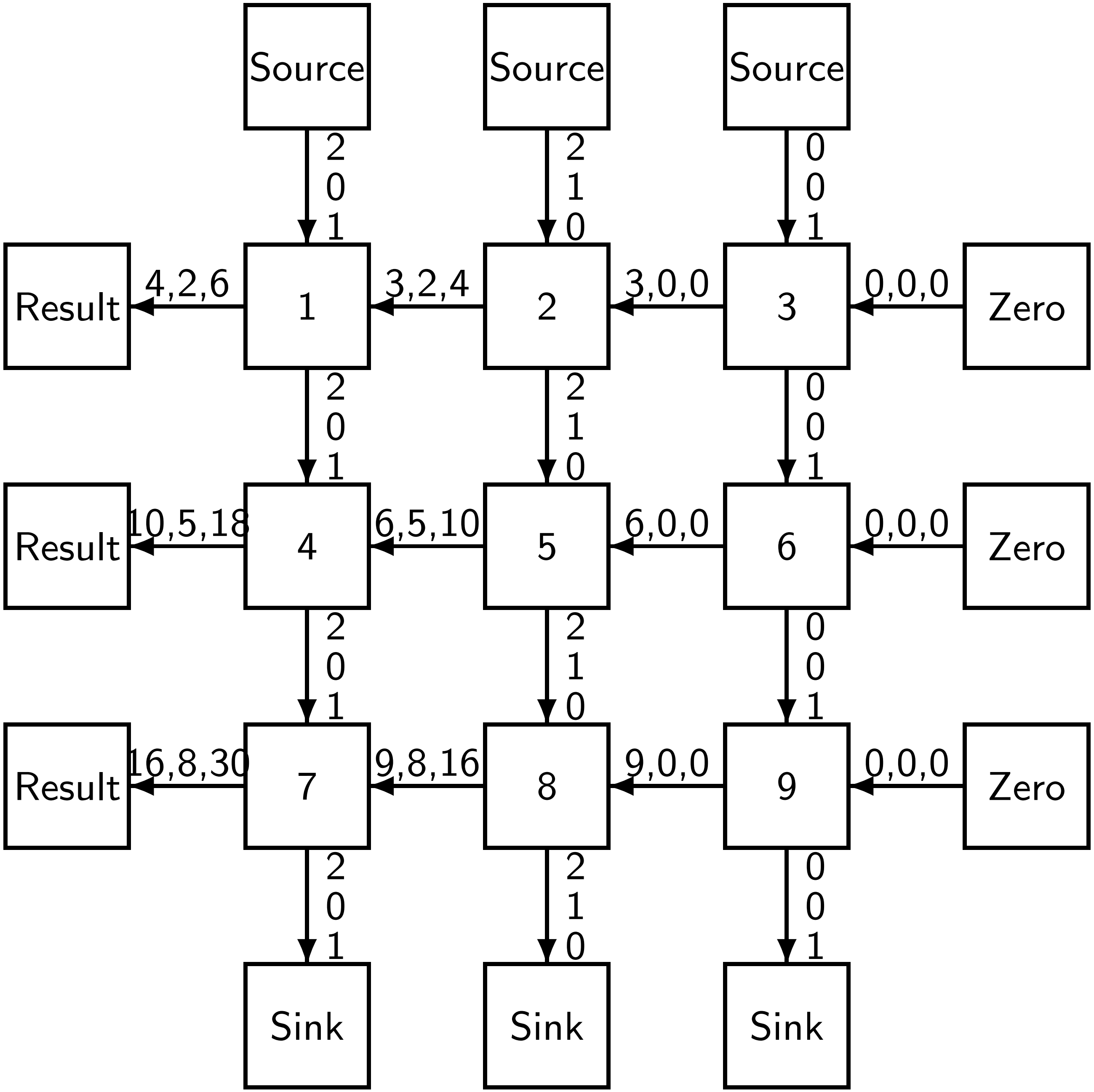
output

```
char c
integer m ← 0

loop forever
   pipe ⇒ c
   outC ⇐ c

   m ← m + 1

   if m >= K
      outC ⇐ newline
      m ← 0
```

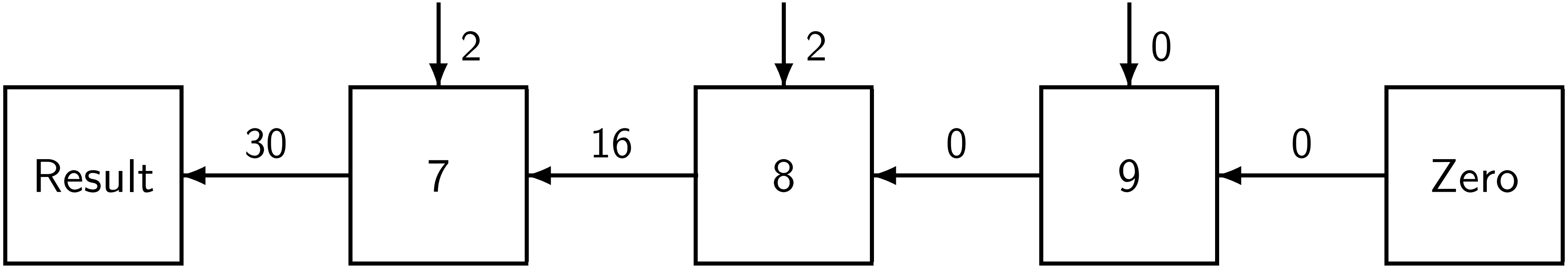
Conway's Problem



Process Array for Matrix Multiplication



Computation of One Element



Algorithm 8.3: Multiplier process with channels	
	integer FirstElement channel of integer North, East, South, West integer Sum, integer SecondElement
	loop forever
p1:	North $\Rightarrow$ SecondElement
p2:	East $\Rightarrow$ Sum
p3:	Sum $\leftarrow$ Sum + FirstElement $\cdot$ SecondElement
p4:	South $\Leftarrow$ SecondElement
p5:	West $\Leftarrow$ Sum

Algorithm 8.4: Multiplier with channels and selective input

integer FirstElement
channel of integer North, East, South, West
integer Sum, integer SecondElement

```
    loop forever
      either
p1:     North ⇒ SecondElement
p2:     East ⇒ Sum
      or
p3:     East ⇒ Sum
p4:     North ⇒ SecondElement
p5:   South ⇐ SecondElement
p6:   Sum ← Sum + FirstElement · SecondElement
p7:   West ⇐ Sum
```

Algorithm 8.5: Dining philosophers with channels			
channel of boolean forks[5]			
	philosopher i		**fork i**
	boolean dummy		boolean dummy
	loop forever		loop forever
p1:	think	q1:	forks[i] $\Leftarrow$ true
p2:	forks[i] $\Rightarrow$ dummy	q2:	forks[i] $\Rightarrow$ dummy
p3:	forks[i+1] $\Rightarrow$ dummy	q3:	
p4:	eat	q4:	
p5:	forks[i] $\Leftarrow$ true	q5:	
p6:	forks[i+1] $\Leftarrow$ true	q6:	

Conway's Problem in Promela (1)

```
#define N 9
#define K 4

chan inC, pipe, outC = [0] of { byte };

active proctype Compress() {
  byte previous, c, count = 0;
  inC ? previous;
  do
  :: inC ? c ->
     if
     :: (c == previous) && (count < N-1) -> count++
     :: else ->
        if
        :: count > 0 ->
           pipe ! count+1;
           count = 0
        :: else
        fi;
        pipe ! previous;
        previous = c;
     fi
  od
}
```

Conway's Problem in Promela (2)

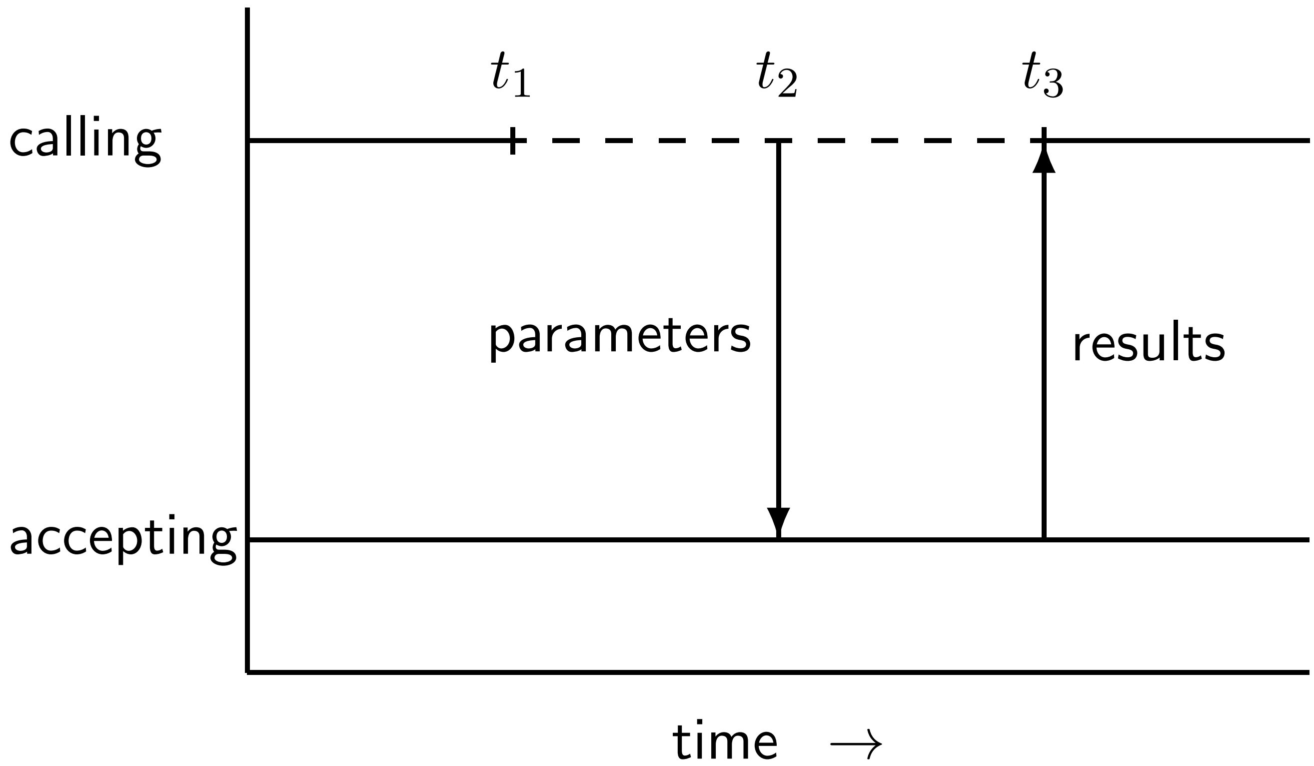
```
active proctype Output() {
  byte c, count = 0;
  do
  :: pipe ? c;
     outC ! c;
     count++;
     if
     :: count >= K ->
        outC ! '\n';
        count = 0
     :: else
     fi
  od
}
```

Multiplier Process in Promela

```
proctype Multiplier (byte Coeff;
      chan North; chan East; chan South; chan West) {
   byte Sum, X;
   for (i ,0, SIZE−1)
      if  ::  North ? X −> East ? Sum;
          ::  East  ? Sum −> North ? X;
      fi ;
      South ! X;
      Sum = Sum + X∗Coeff;
      West ! Sum;
   rof (i )
}
```

Algorithm 8.6: Rendezvous	
client	**server**
integer parm, result	integer p, r
loop forever	loop forever
p1: parm $\leftarrow \ldots$	q1:
p2: server.service(parm, result)	q2: accept service(p, r)
p3: use(result)	q3: r $\leftarrow$ do the service(p)

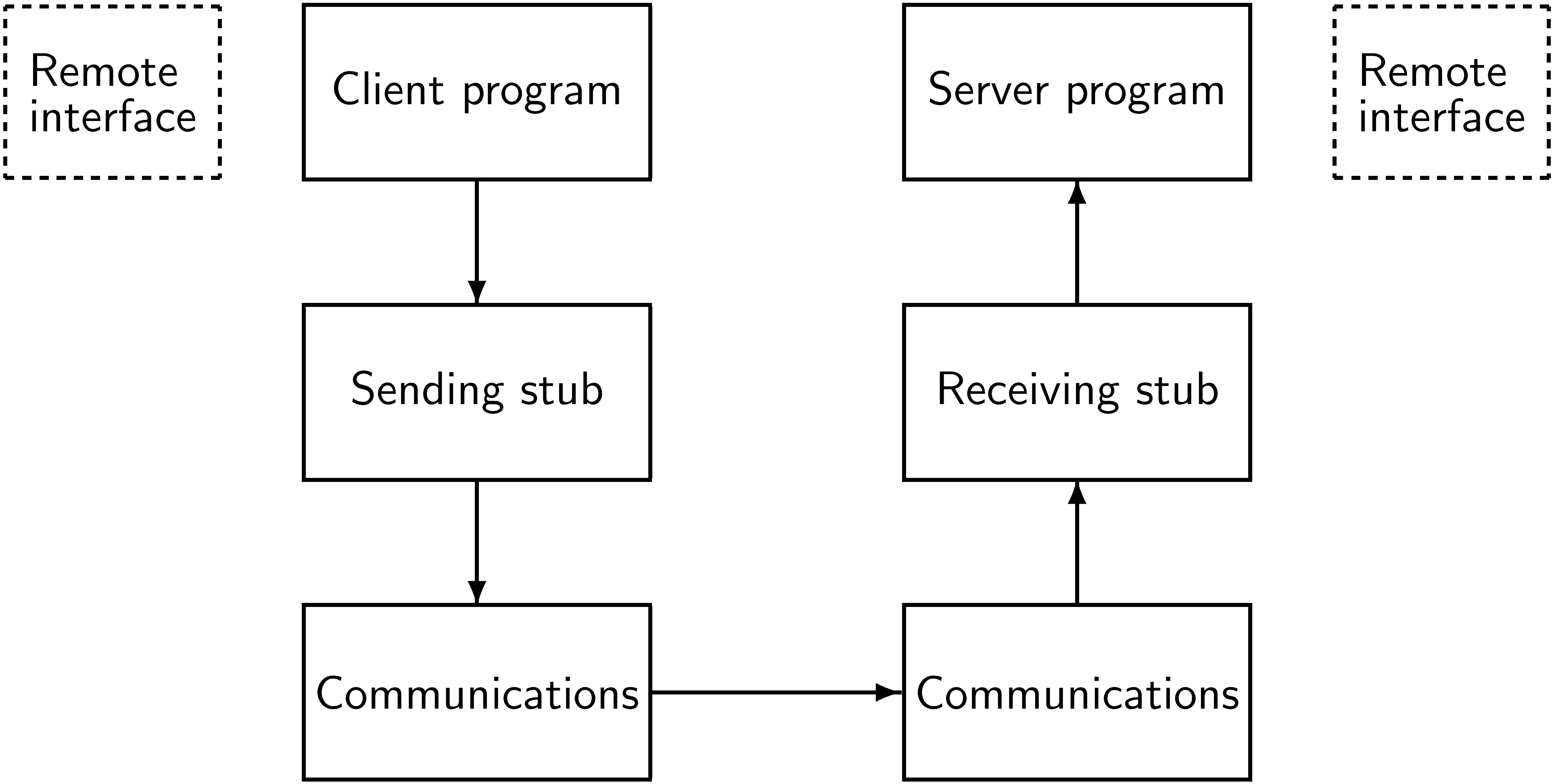
Timing Diagram for a Rendezvous



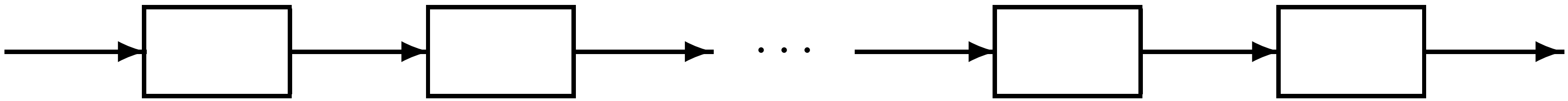
Bounded Buffer in Ada

```
task body Buffer is
  B: Buffer_Array ;
  In_Ptr , Out_Ptr, Count: Index := 0;

begin
  loop
    select
      when Count < Index'Last =>
        accept Append(I: in Integer ) do
            B(In_Ptr) := I ;
        end Append;
      Count := Count + 1; In_Ptr := In_Ptr + 1;
    or
      when Count > 0 =>
        accept Take(I: out Integer ) do
            I := B(Out_Ptr);
        end Take;
      Count := Count − 1; Out_Ptr := Out_Ptr + 1;
    or
        terminate;
    end select;
  end loop;
end Buffer;
```

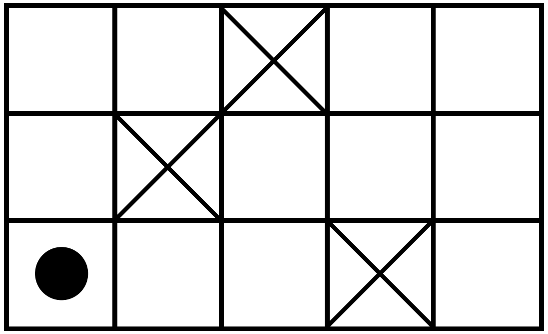
Remote Procedure Call



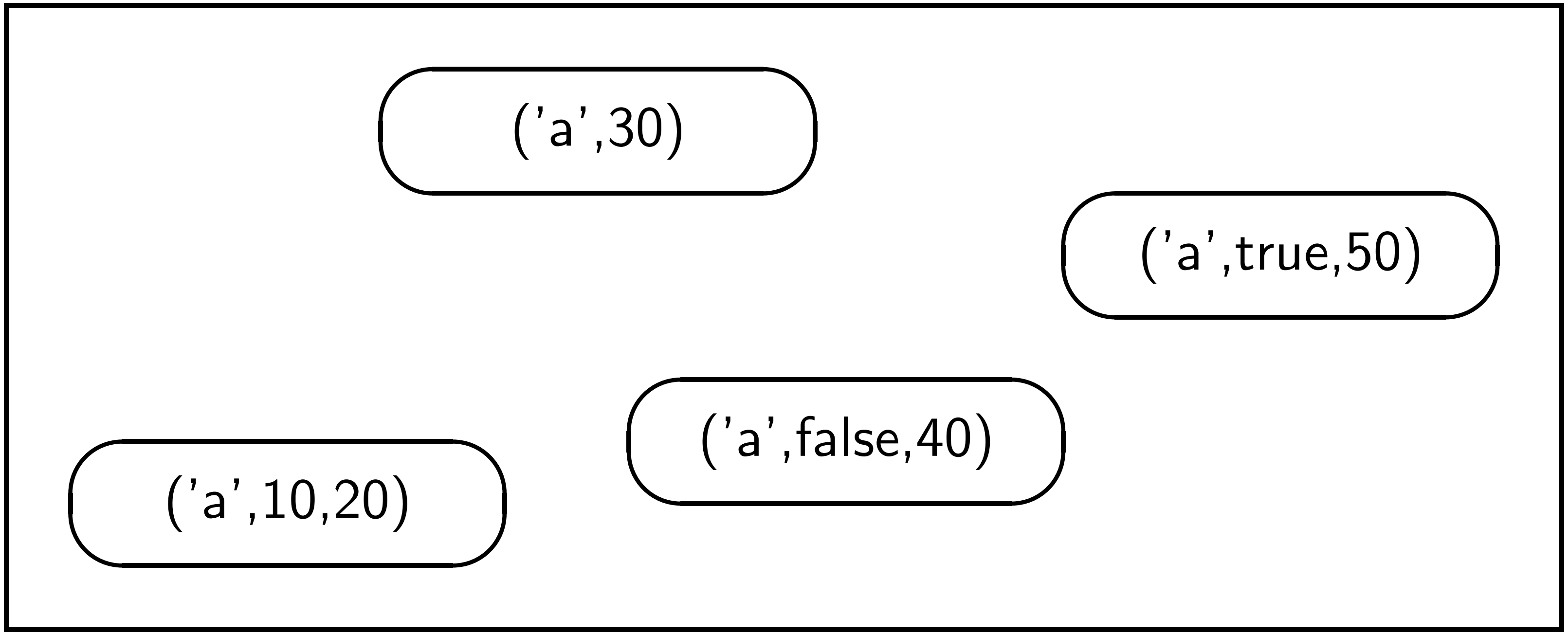
Pipeline Sort



Hoare's Game



A Space



Algorithm 9.1: Critical section problem in Linda	
	loop forever
p1:	non-critical section
p2:	removenote('s')
p3:	critical section
p4:	postnote('s')

Algorithm 9.2: Client-server algorithm in Linda			
	client		**server**
	constant integer me ← . . .		integer client
	serviceType service		serviceType s
	dataType result, parm		dataType r, p
p1:	service ← // Service requested	q1:	removenote('S', client, s, p)
p2:	postnote('S', me, service, parm)	q2:	r ← do (s, p)
p3:	removenote('R', me, result)	q3:	postnote('R', client, r)

Algorithm 9.3: Specific service			
client		**server**	
	constant integer me $\leftarrow$ . . .		integer client
	serviceType service		serviceType s
	dataType result, parm		dataType r, p
p1:	service $\leftarrow$ // Service requested	q1:	s $\leftarrow$ // Service provided
p2:	postnote('S', me, service, parm)	q2:	removenote('S', client, s=, p)
p3:		q3:	r $\leftarrow$ do (s, p)
p4:	removenote('R', me, result)	q4:	postnote('R', client, r)

Algorithm 9.4: Buffering in a space	
producer	**consumer**
integer count ← 0	integer count ← 0
integer v	integer v
loop forever	loop forever
p1: v ← produce	q1: removenote('B', count=, v)
p2: postnote('B', count, v)	q2: consume(v)
p3: count ← count + 1	q3: count ← count + 1

Algorithm 9.5: Multiplier process with channels in Linda
parameters: integer FirstElement parameters: integer North, East, South, West integer Sum, integer SecondElement
integer Sum, integer SecondElement
loop forever p1: removenote('E', North=, SecondElement) p2: removenote('S', East=, Sum) p3: Sum $\leftarrow$ Sum + FirstElement $\cdot$ SecondElement p4: postnote('E', South, SecondElement) p5: postnote('S', West, Sum)

Algorithm 9.6: Matrix multiplication in Linda	
constant integer n ← . . .	
master	**worker**
integer i, j, result	integer r, c, result
integer r, c	integer array[1..n] vec1, vec2
	loop forever
p1: for i from 1 to n	q1: removenote('T', r, c)
p2: for j from 1 to n	q2: readnote('A', r=, vec1)
p3: postnote('T', i, j)	q3: readnote('B', c=, vec2)
p4: for i from 1 to n	q4: result ← vec1 · vec2
p5: for j from 1 to n	q5: postnote('R', r, c, result)
p6: removenote('R', r, c, result)	q6:
p7: print r, c, result	q7:

Algorithm 9.7: Matrix multiplication in Linda with granularity	
constant integer n ← . . .	
constant integer chunk ← . . .	
master	**worker**
integer i, j, result	integer r, c, k, result
integer r, c	integer array[1..n] vec1, vec2
	loop forever
for i from 1 to n	removenote('T', r, k)
for j from 1 to n step by chunk	readnote('A', r=, vec1)
postnote('T', i, j)	for c from k to k+chunk-1
for i from 1 to n	readnote('B', c=, vec2)
for j from 1 to n	result ← vec1 · vec2
removenote('R', r, c, result)	postnote('R', r, c, result)
print r, c, result	

Definition of Notes in Java

```
public class Note {
    public String id;
    public Object[] p;

    // Constructor for an array of objects
    public Note (String id, Object[] p) {
        this.id = id;
        if (p != null) this.p = p.clone();
    }

    // Constructor for a single integer
    public Note (String id, int p1) {
        this(id, new Object[]{new Integer(p1)});
    }

    // Accessor for a single integer value
    public int get(int i) {
        return ((Integer)p[i]).intValue();
    }
}
```

Matrix Multiplication in Java

```
private class Worker extends Thread {
    public void run() {
        Note task = new Note("task");
        while (true) {
            Note t = space.removenote(task);
            int row = t.get(0), col = t.get(1);
            Note r = space.readnote(match("a", row));
            Note c = space.readnote(match("b", col));
            int ip = 0;
            for (int i = 1; i <= SIZE; i++)
                ip = ip + r.get(i)*c.get(i);
            space.postnote(new Note("result", row, col, ip));
        }
    }
}
```

Matrix Multiplication in Promela

```
chan space = [25] of { byte, short, short, short, short };

active[WORKERS] proctype Worker() {
    short row, col, ip, r1, r2, r3, c1, c2, c3;
    do
    :: space ?? 't', row, col, _, _;
       space ?? <'a', eval(row), r1, r2, r3>;
       space ?? <'b', eval(col), c1, c2, c3>;
       ip = r1*c1 + r2*c2 + r3*c3;
       space ! 'r', row, col, ip, 0;
    od;
}
```

Algorithm 9.8: Matrix multiplication in Linda (exercise)

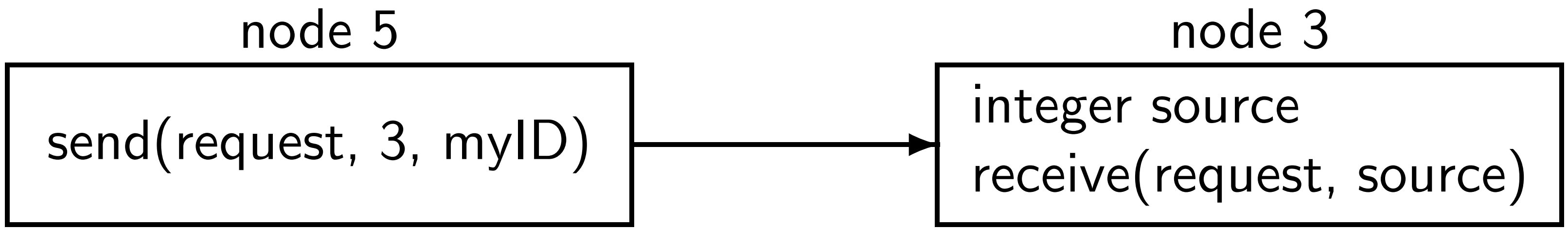
constant integer n ← . . .

master	worker
integer i, j, result	integer i, r, c, result
integer r, c	integer array[1..n] vec1, vec2
	loop forever
postnote('T', 0)	removenote('T' i)
	if i ¡ (n · n) − 1
	postnote('T', i+1)
	r ← (i / n) + 1
	c ← (i modulo n) + 1
for i from 1 to n	readnote('A', r=, vec1)
for j from 1 to n	readnote('B', c=, vec2)
removenote('R', r, c, result)	result ← vec1 · vec2
print r, c, result	postnote('R', r, c, result)

Sending and Receiving Messages

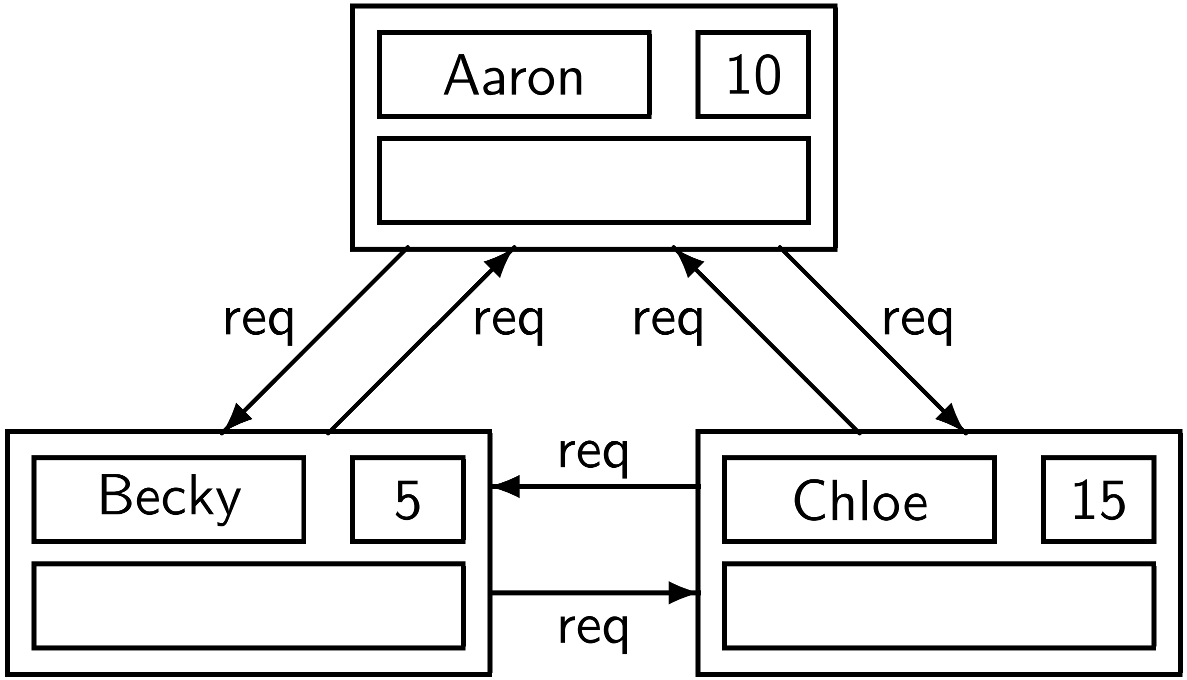
node 5		node 3
integer k ← 20 send(request, 3, k, 30)	→	integer m, n receive(request, m, n)

Sending a Message and Expecting a Reply

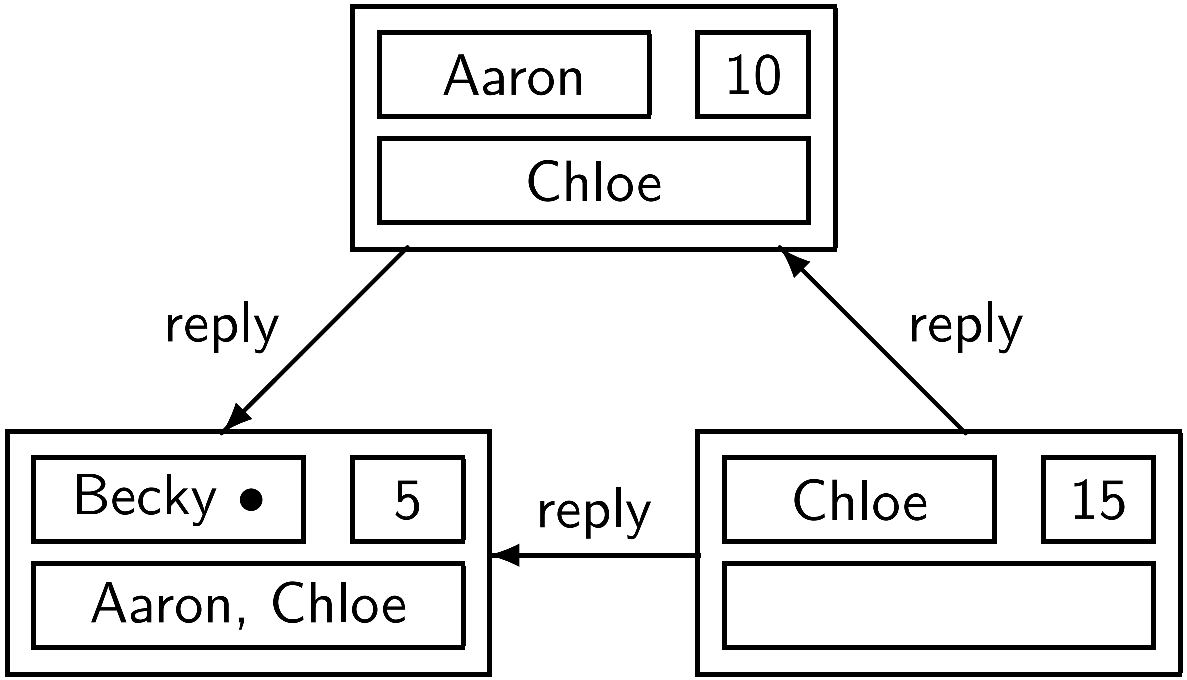


Algorithm 10.1: Ricart-Agrawala algorithm (outline)	
	integer myNum ← 0
	set of node IDs deferred ← empty set
	main
p1:	non-critical section
p2:	myNum ← chooseNumber
p3:	for all *other* nodes N
p4:	send(request, N, myID, myNum)
p5:	await reply's from all *other* nodes
p6:	critical section
p7:	for all nodes N in deferred
p8:	remove N from deferred
p9:	send(reply, N, myID)
	receive
	integer source, reqNum
p10:	receive(request, source, reqNum)
p11:	if reqNum < myNum
p12:	send(reply,source,myID)
p13:	else add source to deferred

RA Algorithm (1)

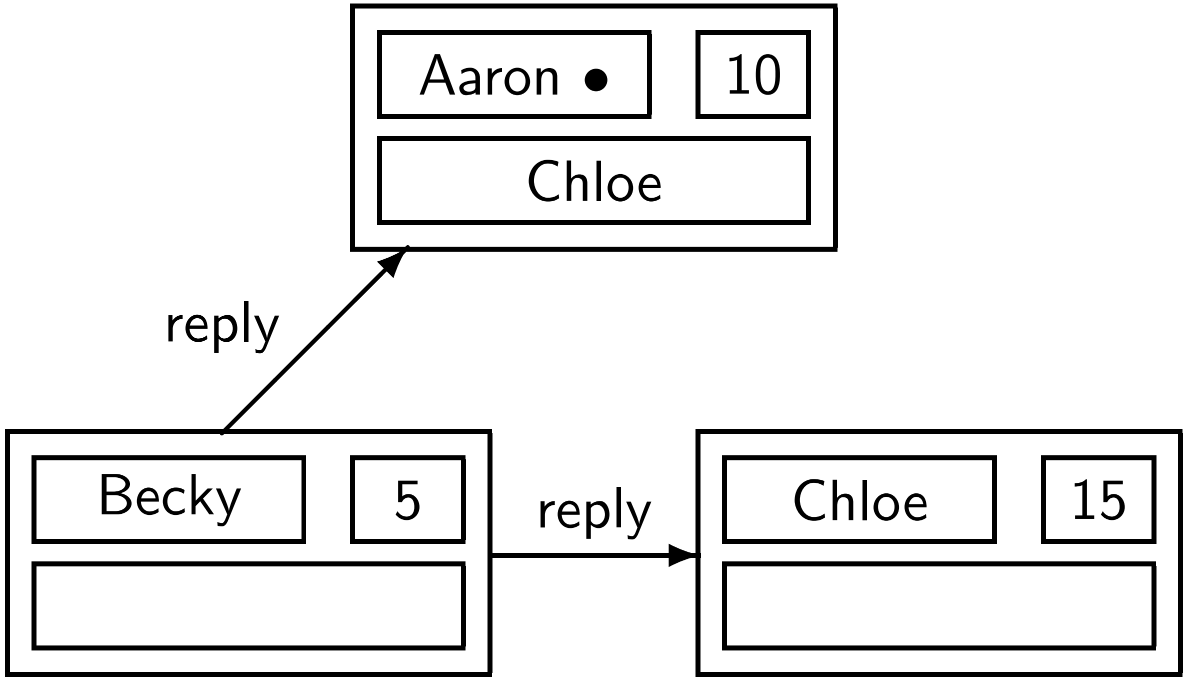


RA Algorithm (2)

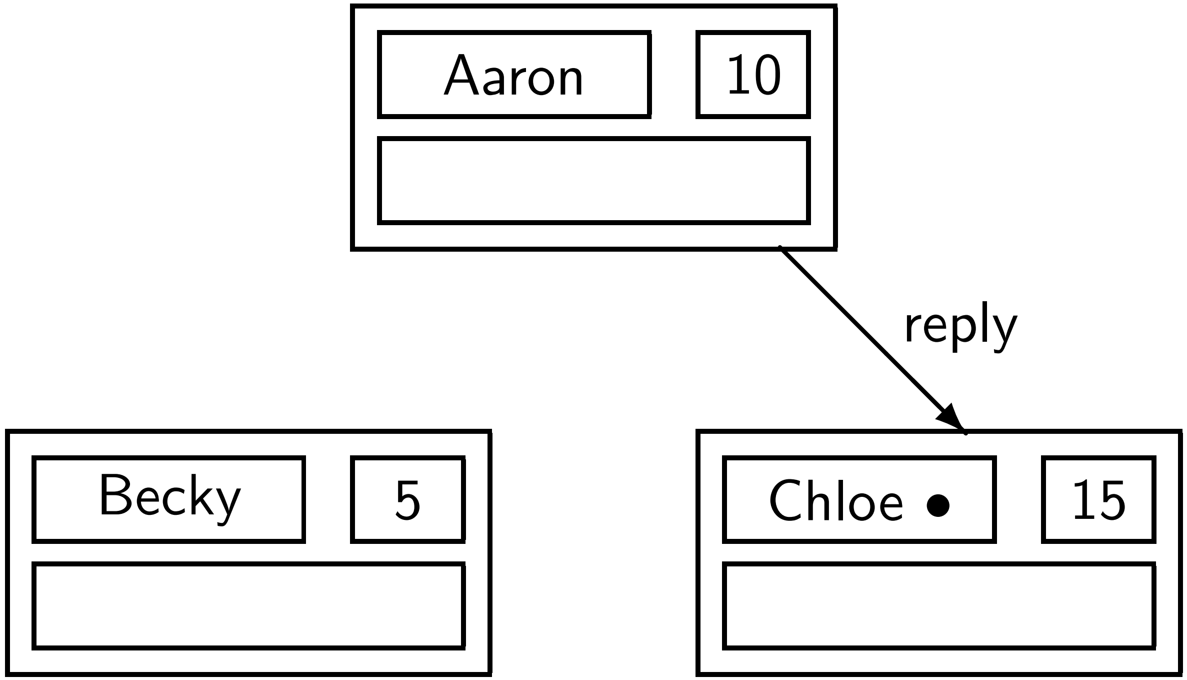


Virtual Queue in the RA Algorithm

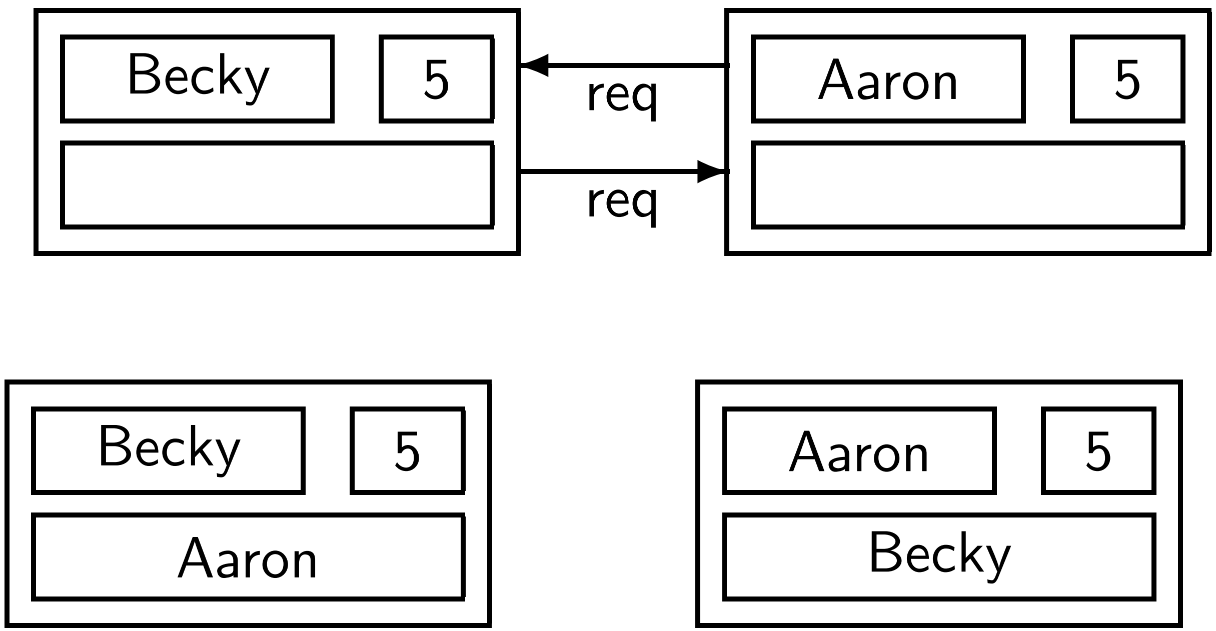
RA Algorithm (3)



RA Algorithm (4)

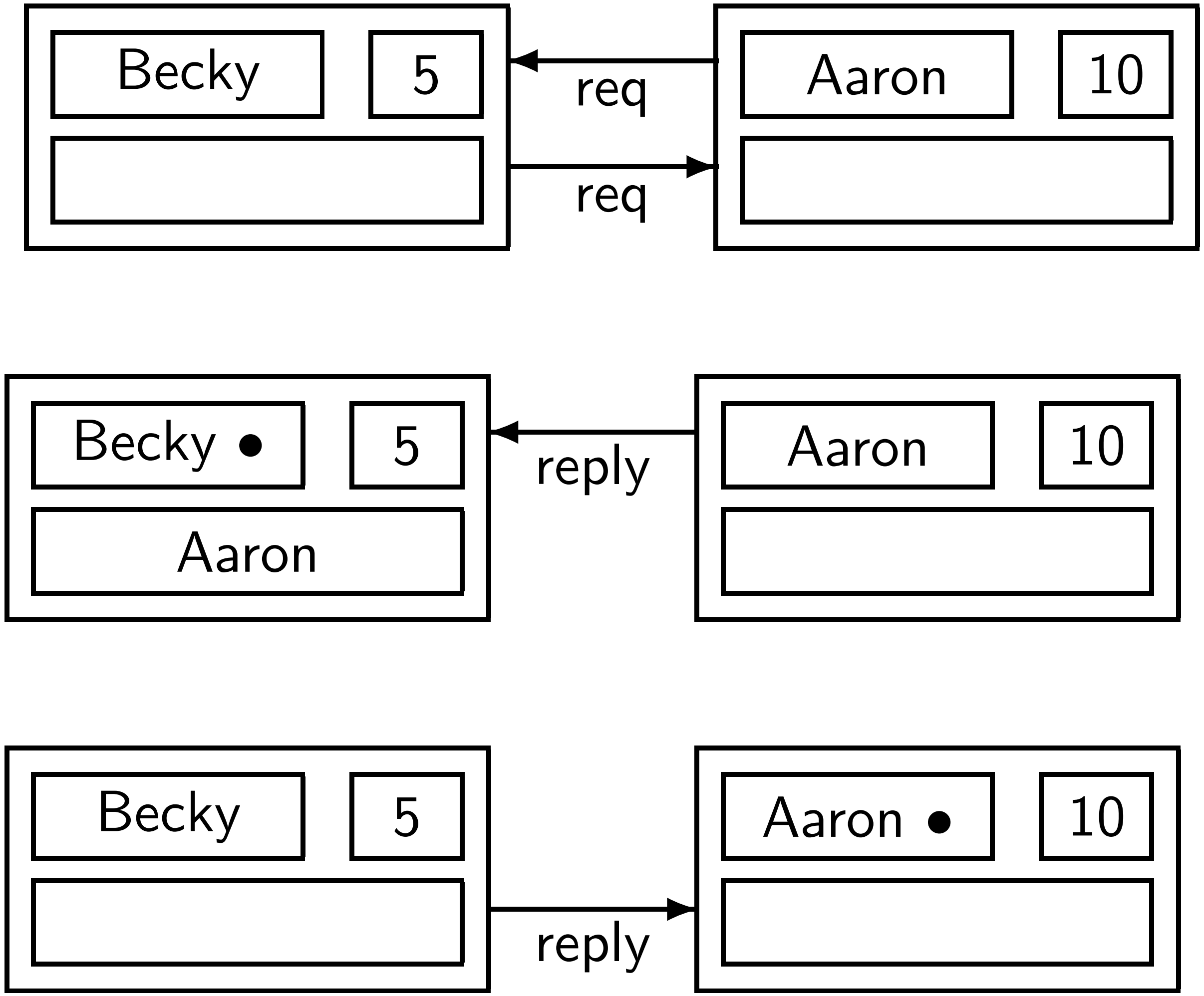


Equal Ticket Numbers

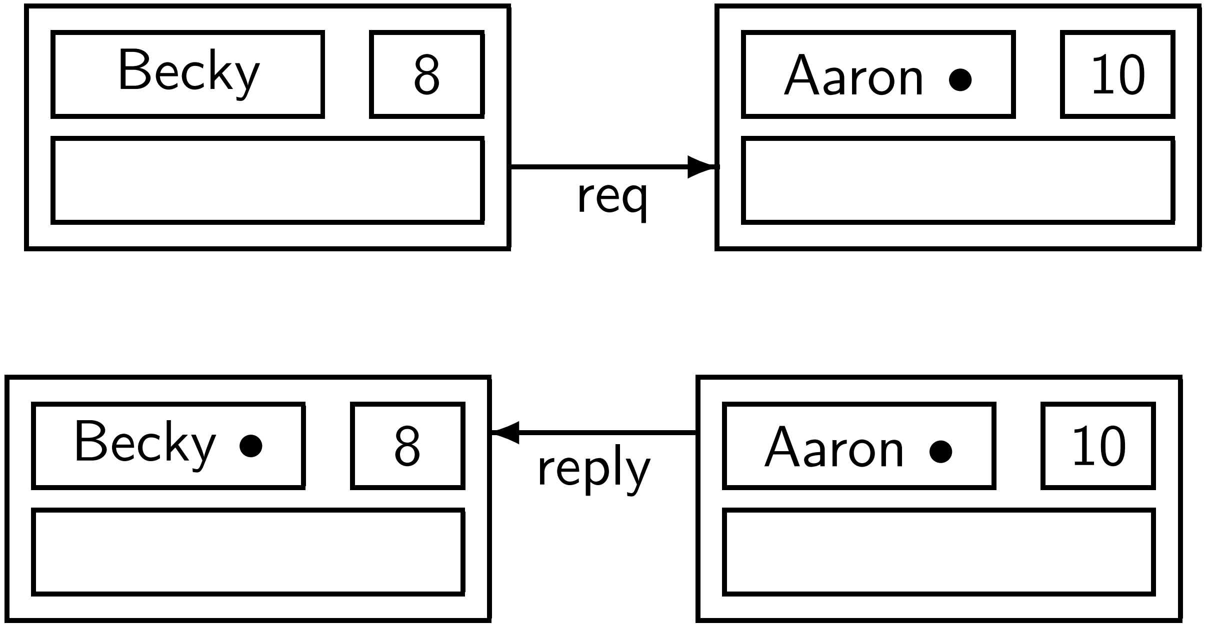


Note: This figure is not in the book.

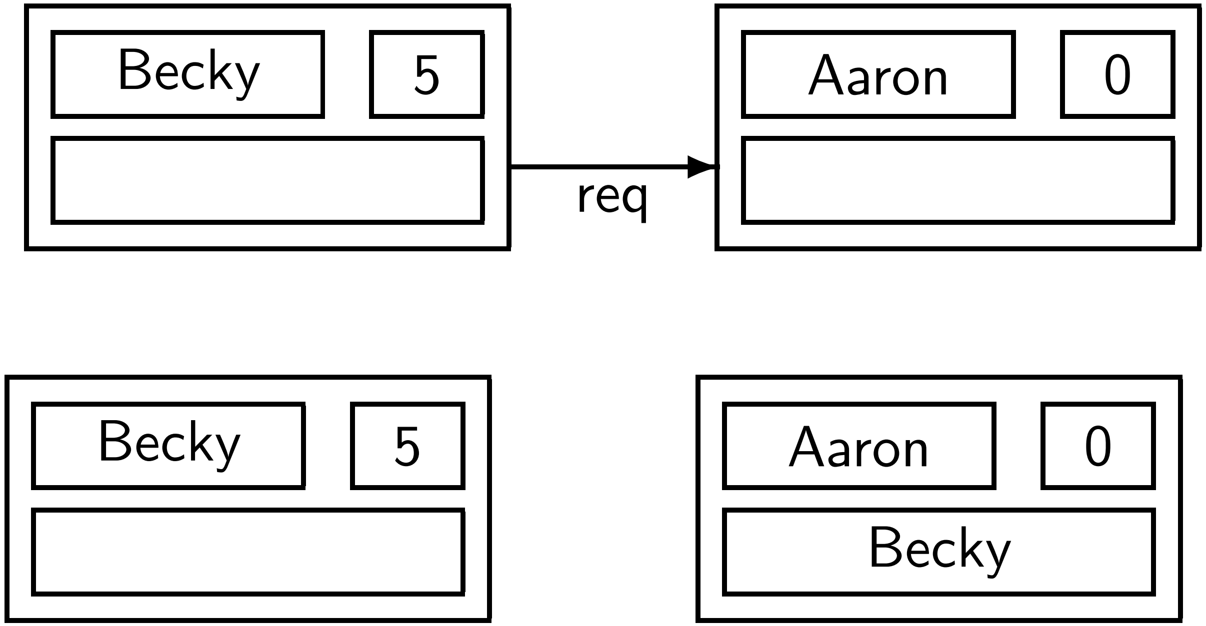
Choosing Ticket Numbers (1)



Choosing Ticket Numbers (2)



Quiescent Nodes



Algorithm 10.2: Ricart-Agrawala algorithm
integer myNum ← 0 set of node IDs deferred ← empty set integer highestNum ← 0 boolean requestCS ← false
Main

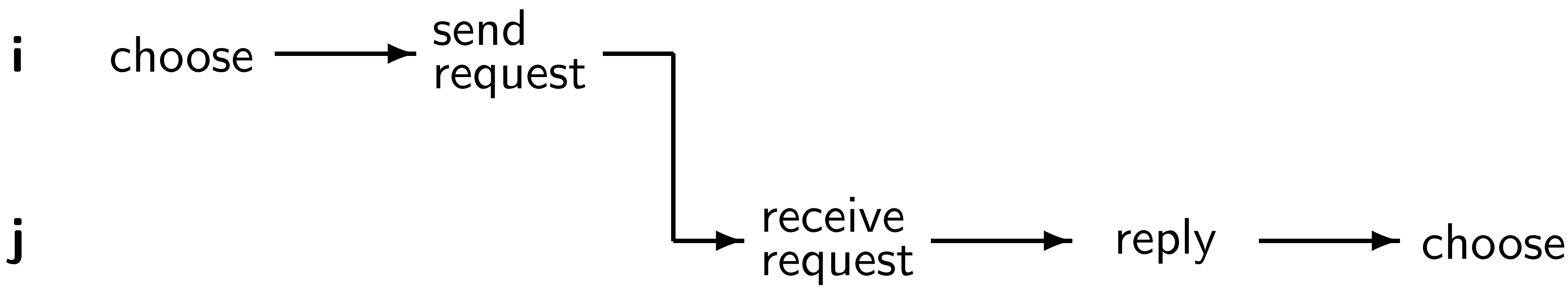
```
      loop forever
p1:      non-critical section
p2:      requestCS ← true
p3:      myNum ← highestNum + 1
p4:      for all other nodes N
p5:         send(request, N, myID, myNum)
p6:      await reply's from all other nodes
p7:      critical section
p8:      requestCS ← false
p9:      for all nodes N in deferred
p10:        remove N from deferred
p11:        send(reply, N, myID)
```

Algorithm 10.2: Ricart-Agrawala algorithm (continued)

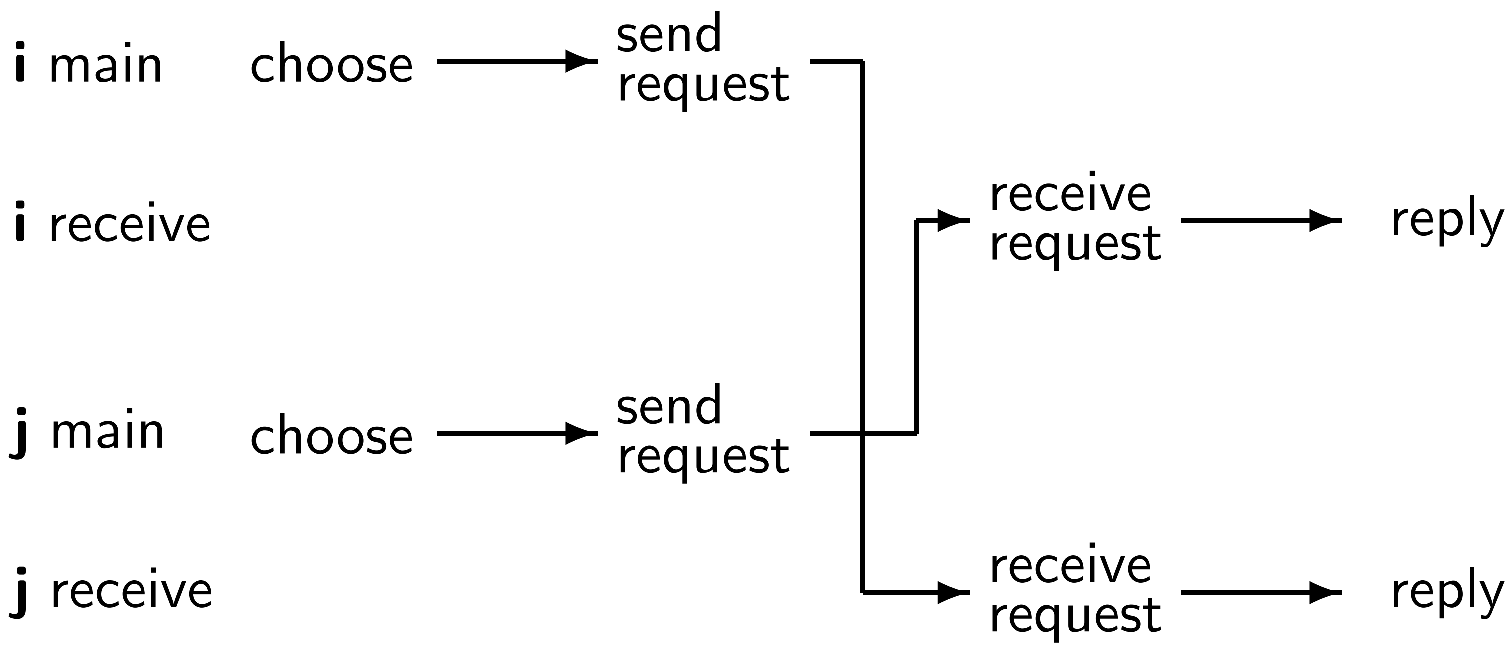
Receive

```
    integer source, requestedNum
    loop forever
p1:    receive(request, source, requestedNum)
p2:    highestNum ← max(highestNum, requestedNum)
p3:    if not requestCS or requestedNum ≪ myNum
p4:       send(reply, source, myID)
p5:    else add source to deferred
```

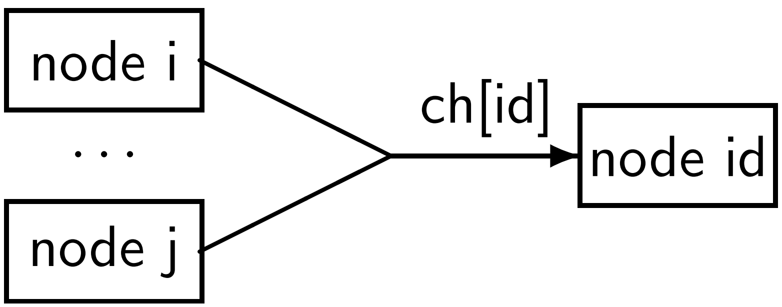
Correct of the RA Algorithm (Case 1)



Correct of the RA Algorithm (Case 2)



Channels in RA Algorithm in Promela



RA Algorithm in Promela – Main Process

```
proctype Main( byte myID ) {
  do ::
    atomic {
      requestCS[myID] = true ;
      myNum[myID] = highestNum[myID] + 1 ;
    }
    for (J,0, NPROCS-1)
      if
      :: J != myID ->
          ch[J] ! request, myID, myNum[myID];
      :: else
      fi
    rof (J);
    for (K,0,NPROCS-2)
      ch[myID] ?? reply, _ , _;
    rof (K);
    critical_section ();
    requestCS[myID] = false;
    byte N;
    do
      :: empty(deferred[myID]) -> break;
      :: deferred [ myID] ? N -> ch[N] ! reply, 0, 0
    od
  od
}
```

RA Algorithm in Promela – Receive Process

```
proctype Receive( byte myID ) {
    byte reqNum, source;
    do ::
        ch[myID] ?? request, source, reqNum;
        highestNum[myID] =
            ((reqNum > highestNum[myID]) ->
                reqNum : highestNum[myID]);
        atomic {
            if
            :: requestCS[myID] &&
                 ( (myNum[myID] < reqNum) ||
                 ( (myNum[myID] == reqNum) &&
                       (myID < source)
                 ) ) ->
                   deferred [myID] ! source
            :: else ->
                 ch[source] ! reply , 0, 0
            fi
        }
    od
}
```

Algorithm 10.3: Ricart-Agrawala token-passing algorithm

```
      boolean haveToken ← true in node 0, false in others
      integer array[NODES] requested ← [0,. . . ,0]
      integer array[NODES] granted ← [0,. . . ,0]
      integer myNum ← 0
      boolean inCS ← false
```

sendToken

```
  if exists N such that requested[N] > granted[N]
     for some such N
        send(token, N, granted)
        haveToken ← false
```

Algorithm 10.3: Ricart-Agrawala token-passing algorithm (continued)

Main

```
     loop forever
p1:     non-critical section
p2:     if not haveToken
p3:        myNum ← myNum + 1
p4:        for all other nodes N
p5:           send(request, N, myID, myNum)
p6:        receive(token, granted)
p7:        haveToken ← true
p8:     inCS ← true
p9:     critical section
p10:    granted[myID] ← myNum
p11:    inCS ← false
p12:    sendToken
```

Algorithm 10.3: Ricart-Agrawala token-passing algorithm (continued)

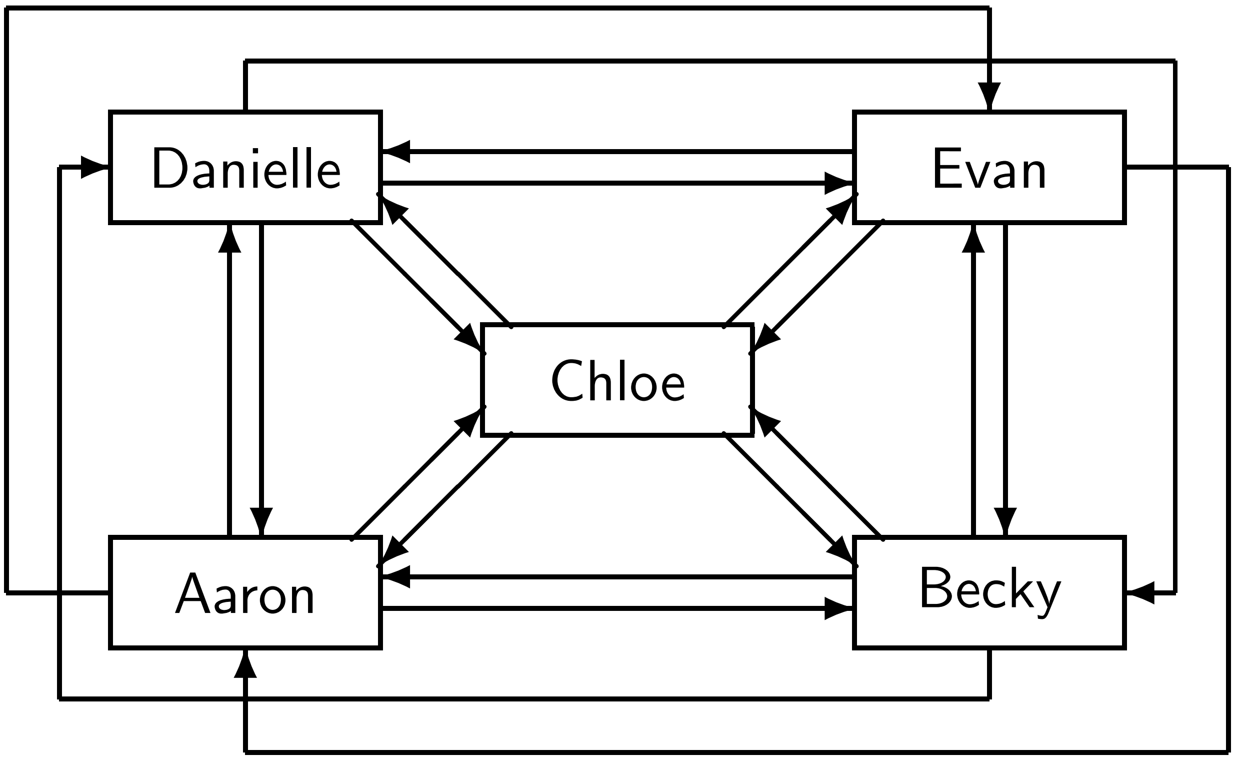
Receive

```
     integer source, reqNum
     loop forever
p13:    receive(request, source, reqNum)
p14:    requested[source] ← max(requested[source], reqNum)
p15:    if haveToken and not inCS
p16:       sendToken
```

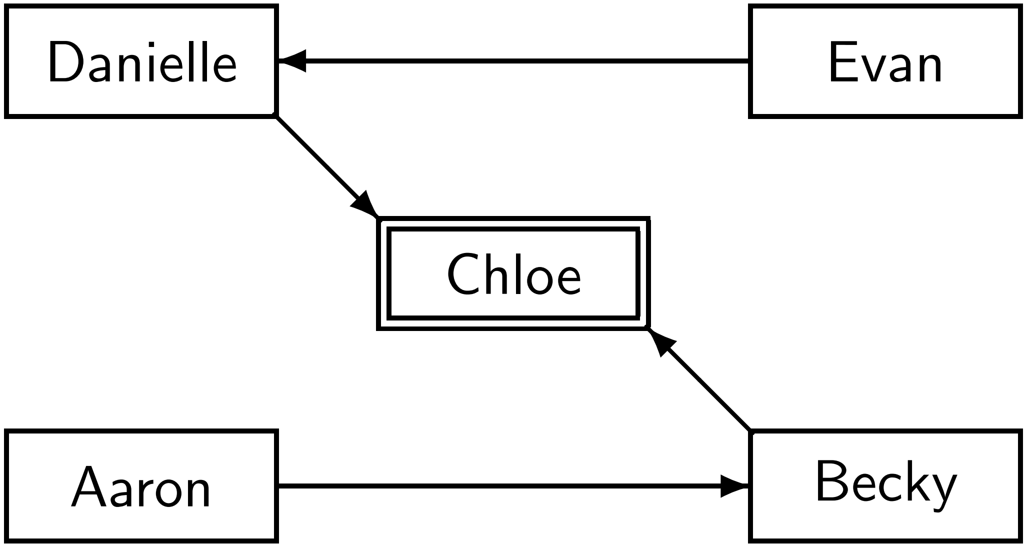
Data Structures for RA Token-Passing Algorithm

	Aaron	Becky	Chloe	Danielle	Evan
requested	4	3	0	5	1
granted	4	2	2	4	1

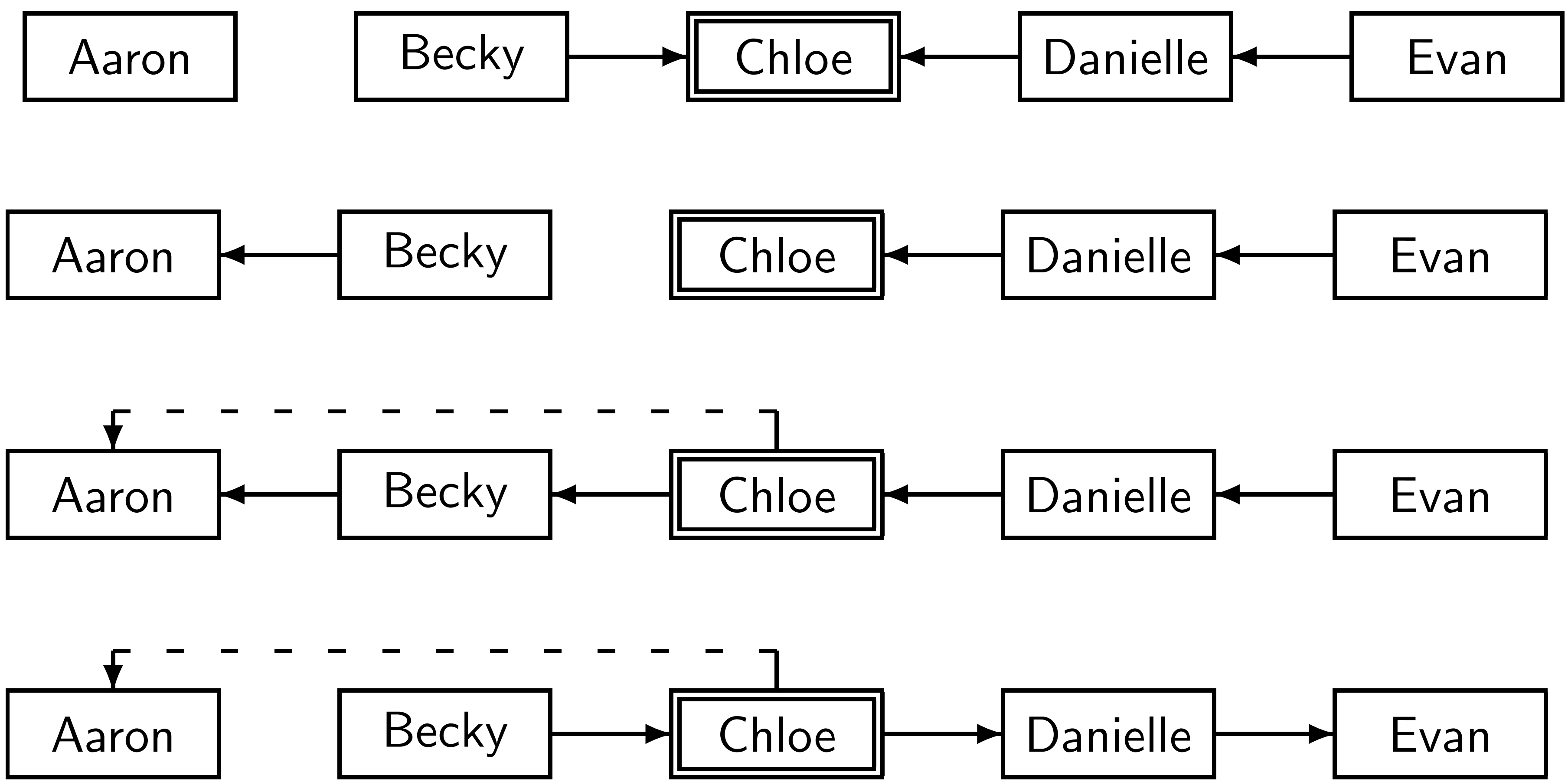
Distributed System for Neilsen-Mizuno Algorithm



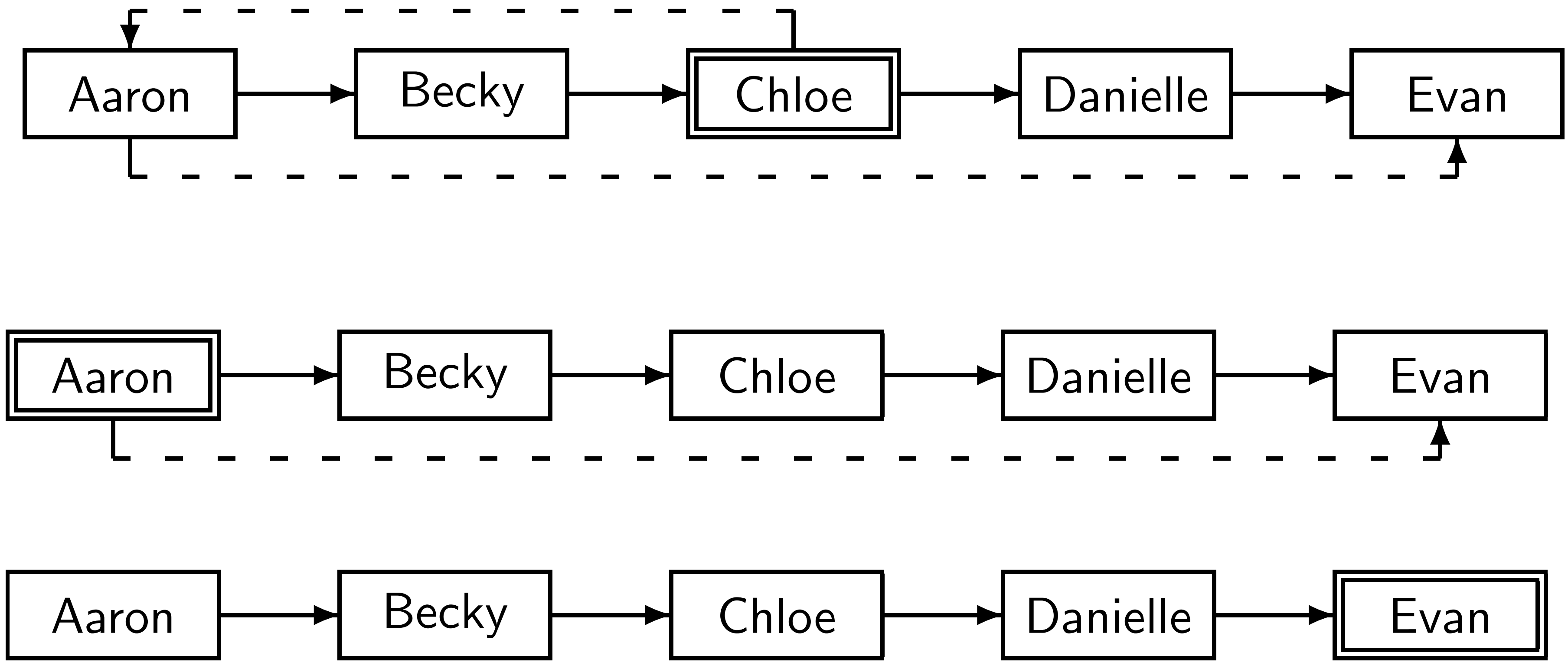
Spanning Tree in Neilsen-Mizuno Algorithm



Neilsen-Mizuno Algorithm (1)



Neilsen-Mizuno Algorithm (2)



Algorithm 10.4: Neilsen-Mizuno token-passing algorithm

```
        integer parent ← (initialized to form a tree)
        integer deferred ← 0
        boolean holding ← true in the root, false in others
```

Main

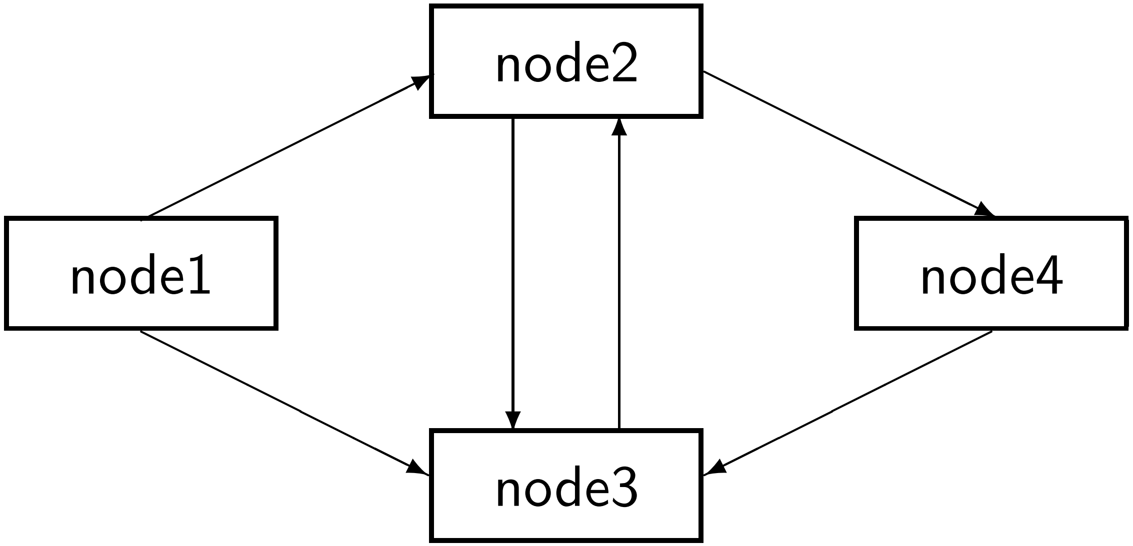
```
      loop forever
p1:     non-critical section
p2:     if not holding
p3:        send(request, parent, myID, myID)
p4:        parent ← 0
p5:        receive(token)
p6:     holding ← false
p7:     critical section
p8:     if deferred ≠ 0
p9:        send(token, deferred)
p10:       deferred ← 0
p11:    else holding ← true
```

Algorithm 10.4: Neilsen-Mizuno token-passing algorithm (continued)

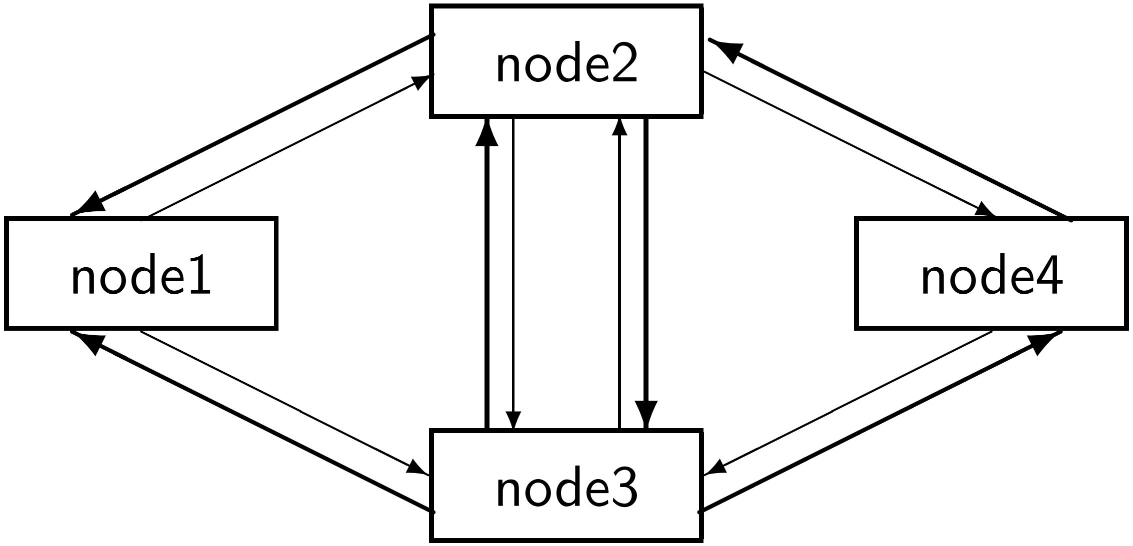
Receive

```
       integer source, originator
       loop forever
p12:     receive(request, source, originator)
p13:     if parent = 0
p14:       if holding
p15:         send(token, originator)
p16:         holding ← false
p17:       else deferred ← originator
p18:     else send(request, parent, myID, originator)
p19:     parent ← source
```

Distributed System with an Environment Node



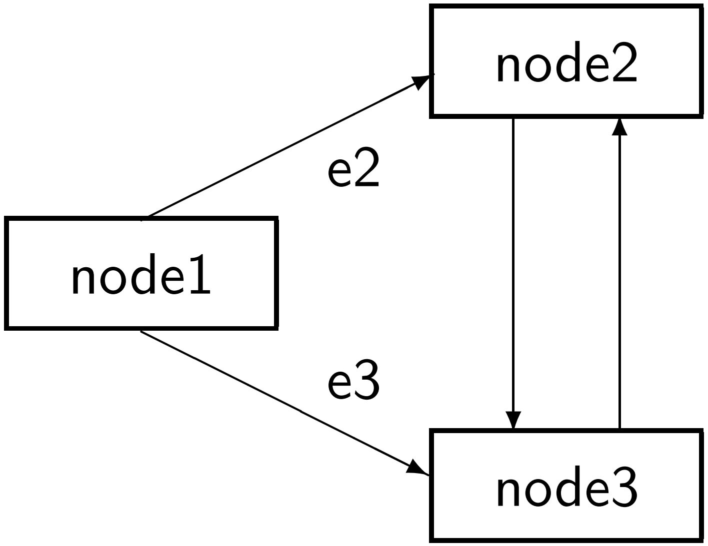
Back Edges



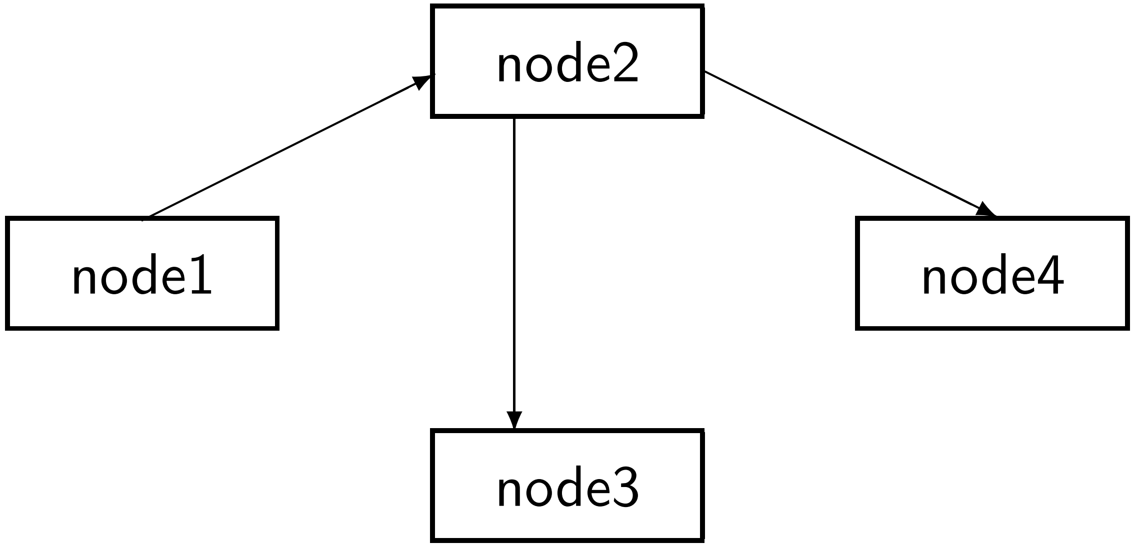
Algorithm 11.1: Dijkstra-Scholten algorithm (preliminary)	
	integer array[incoming] inDeficit $\leftarrow$ [0,. . . ,0] integer inDeficit $\leftarrow$ 0, integer outDeficit $\leftarrow$ 0
	send message
p1:	send(message, destination, myID)
p2:	increment outDeficit
	receive message
p3:	receive(message, source)
p4:	increment inDeficit[source] and inDeficit
	send signal
p5:	when inDeficit $>$ 1 or (inDeficit $=$ 1 and isTerminated and outDeficit $=$ 0)
p6:	E $\leftarrow$ some edge E with inDeficit[E] $\neq$ 0
p7:	send(signal, E, myID)
p8:	decrement inDeficit[E] and inDeficit
	receive signal
p9:	receive(signal, _)
p10:	decrement outDeficit

Algorithm 11.2: Dijkstra-Scholten algorithm (env., preliminary)	
integer outDeficit ← 0	
	computation
p1:	for all outgoing edges E
p2:	send(message, E, myID)
p3:	increment outDeficit
p4:	await outDeficit = 0
p5:	announce system termination
	receive signal
p6:	receive(signal, source)
p7:	decrement outDeficit

The Preliminary DS Algorithm is Unsafe



Spanning Tree



Algorithm 11.3: Dijkstra-Scholten algorithm	
	integer array[incoming] inDeficit $\leftarrow$ [0,. . . ,0]
	integer inDeficit $\leftarrow$ 0
	integer outDeficit $\leftarrow$ 0
	integer parent $\leftarrow -1$
	send message
p1:	when parent $\neq -1$ // Only active nodes send messages
p2:	send(message, destination, myID)
p3:	increment outDeficit
	receive message
p4:	receive(message,source)
p5:	if parent $= -1$
p6:	parent $\leftarrow$ source
p7:	increment inDeficit[source] and inDeficit

Algorithm 11.3: Dijkstra-Scholten algorithm (continued)

send signal

```
p8:  when inDeficit > 1
p9:     E ← some edge E for which
           (inDeficit[E] > 1) or (inDeficit[E] = 1 and E ≠ parent)
p10:    send(signal, E, myID)
p11:    decrement inDeficit[E] and inDeficit
p12: or when inDeficit = 1 and isTerminated and outDeficit = 0
p13:    send(signal, parent, myID)
p14:    inDeficit[parent] ← 0
p15:    inDeficit ← 0
p16:    parent ← −1
```

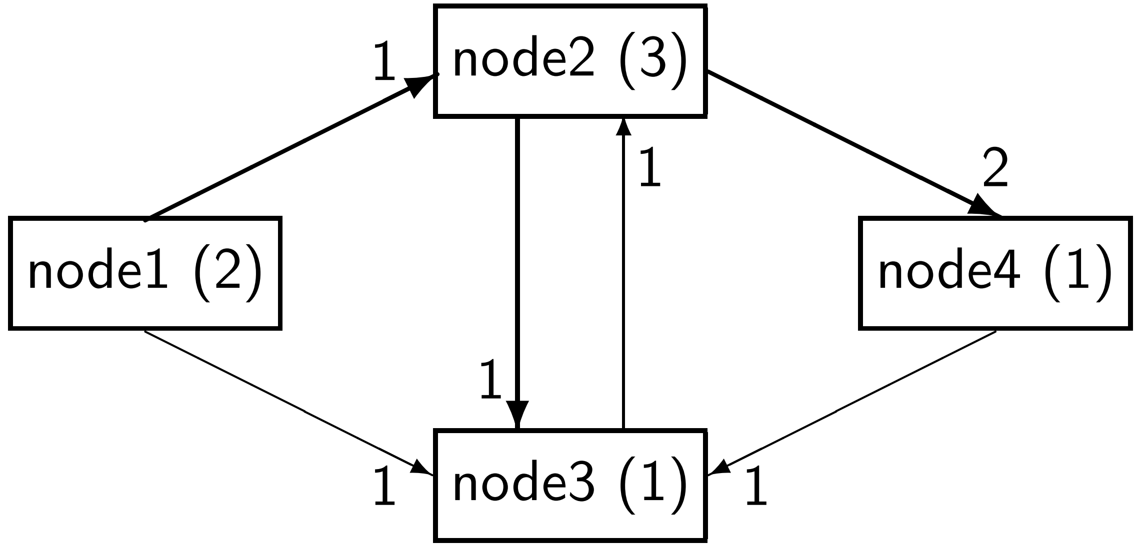
receive signal

```
p17: receive(signal, _)
p18: decrement outDeficit
```

Partial Scenario for DS Algorithm

Action	node1	node2	node3	node4
1 $\Rightarrow$ 2	(-1,[ ],0)	(-1,[0,0],0)	(-1,[0,0,0],0)	(-1,[0],0)
2 $\Rightarrow$ 4	(-1,[ ],1)	(1,[1,0],0)	(-1,[0,0,0],0)	(-1,[0],0)
2 $\Rightarrow$ 3	(-1,[ ],1)	(1,[1,0],1)	(-1,[0,0,0],0)	(2,[1],0)
2 $\Rightarrow$ 4	(-1,[ ],1)	(1,[1,0],2)	(2,[0,1,0],0)	(2,[1],0)
1 $\Rightarrow$ 3	(-1,[ ],1)	(1,[1,0],3)	(2,[0,1,0],0)	(2,[2],0)
3 $\Rightarrow$ 2	(-1,[ ],2)	(1,[1,0],3)	(2,[1,1,0],0)	(2,[2],0)
4 $\Rightarrow$ 3	(-1,[ ],2)	(1,[1,1],3)	(2,[1,1,0],1)	(2,[2],0)
	(-1,[ ],2)	(1,[1,1],3)	(2,[1,1,1],1)	(2,[2],1)

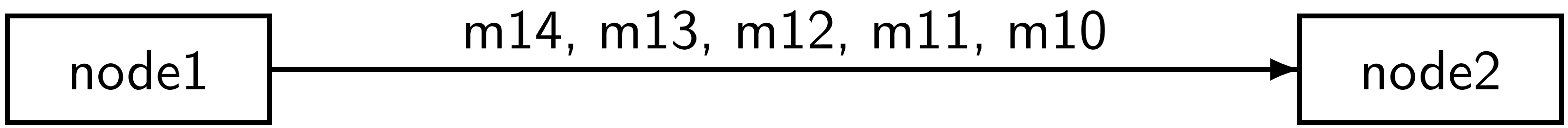
Data Structures After Partial Scenario



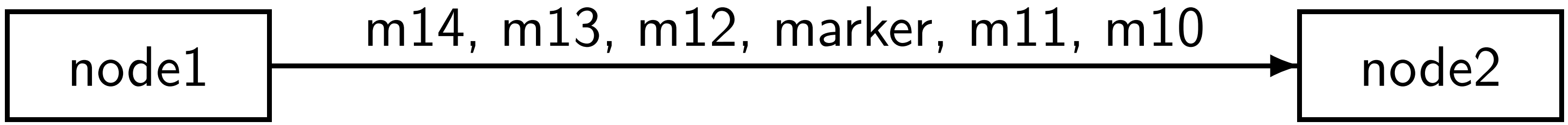
Algorithm 11.4: Credit-recovery algorithm (environment node)	
	float weight $\leftarrow$ 1.0
	computation
p1:	for all outgoing edges E
p2:	weight $\leftarrow$ weight / 2.0
p3:	send(message, E, weight)
p4:	await weight = 1.0
p5:	announce system termination
	receive signal
p6:	receive(signal, w)
p7:	weight $\leftarrow$ weight + w

Algorithm 11.5: Credit-recovery algorithm (non-environment node)	
	constant integer parent ← 0 // Environment node boolean active ← false float weight ← 0.0
	send message
p1:	if active // Only active nodes send messages
p2:	weight ← weight / 2.0
p3:	send(message, destination, myID, weight)
	receive message
p4:	receive(message, source, w)
p5:	active ← true
p6:	weight ← weight + w
	send signal
p7:	when terminated
p8:	send(signal, parent, weight)
p9:	weight ← 0.0
p10:	active ← false

Messages on a Channel



Sending a Marker



Algorithm 11.6: Chandy-Lamport algorithm for global snapshots
integer array[outgoing] lastSent ← [0, . . . , 0] integer array[incoming] lastReceived ← [0, . . . , 0] integer array[outgoing] stateAtRecord ← [-1, . . . , -1] integer array[incoming] messageAtRecord ← [-1, . . . , -1] integer array[incoming] messageAtMarker ← [-1, . . . , -1]
send message
p1: send(message, destination, myID) p2: lastSent[destination] ← message
receive message
p3: receive(message,source) p4: lastReceived[source] ← message

Algorithm 11.6: Chandy-Lamport algorithm (continued)

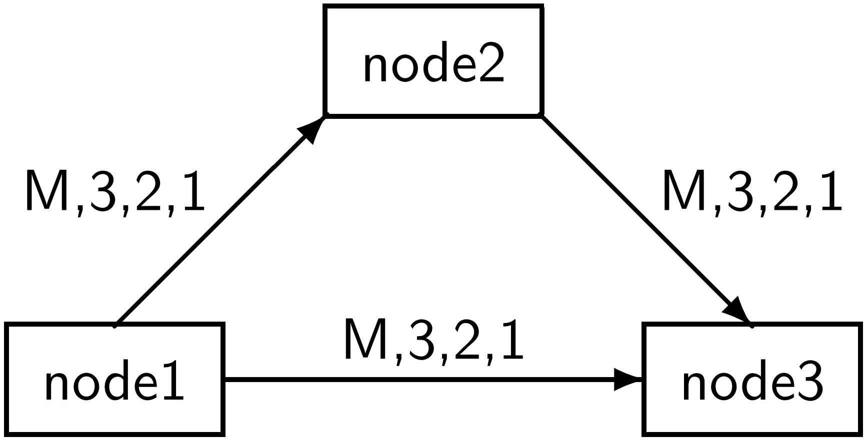
receive marker

p6: receive(marker, source)
p7: messageAtMarker[source] $\leftarrow$ lastReceived[source]
p8: if stateAtRecord $= [-1,\ldots,-1]$ // Not yet recorded
p9:   stateAtRecord $\leftarrow$ lastSent
p10:   messageAtRecord $\leftarrow$ lastReceived
p11:   for all outgoing edges E
p12:   send(marker, E, myID)

record state

p13: await markers received on all incoming edges
p14: recordState

Messages and Markers for a Scenario



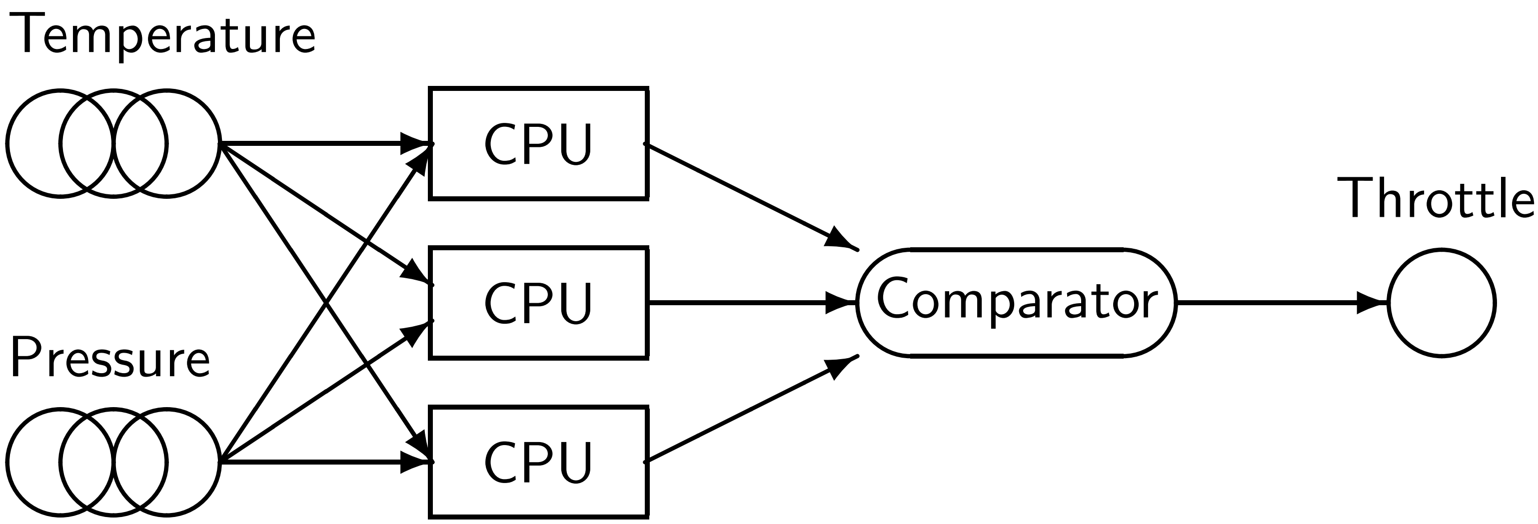
Scenario for CL Algorithm (1)

Action	node1					node2				
	ls	lr	st	rc	mk	ls	lr	st	rc	mk
	[3,3]					[3]	[3]			
1M⇒2	[3,3]		[3,3]			[3]	[3]			
1M⇒3	[3,3]		[3,3]			[3]	[3]			
2⇐1M	[3,3]		[3,3]			[3]	[3]			
2M⇒3	[3,3]		[3,3]			[3]	[3]	[3]	[3]	[3]

Scenario for CL Algorithm (2)

Action	node3				
	ls	lr	st	rc	mk
3⇐2					
3⇐2		[0,1]			
3⇐2		[0,2]			
3⇐2M		[0,3]			
3⇐1		[0,3]		[0,3]	[0,3]
3⇐1		[1,3]		[0,3]	[0,3]
3⇐1		[2,3]		[0,3]	[0,3]
3⇐1M		[3,3]		[0,3]	[0,3]
		[3,3]		[0,3]	[3,3]

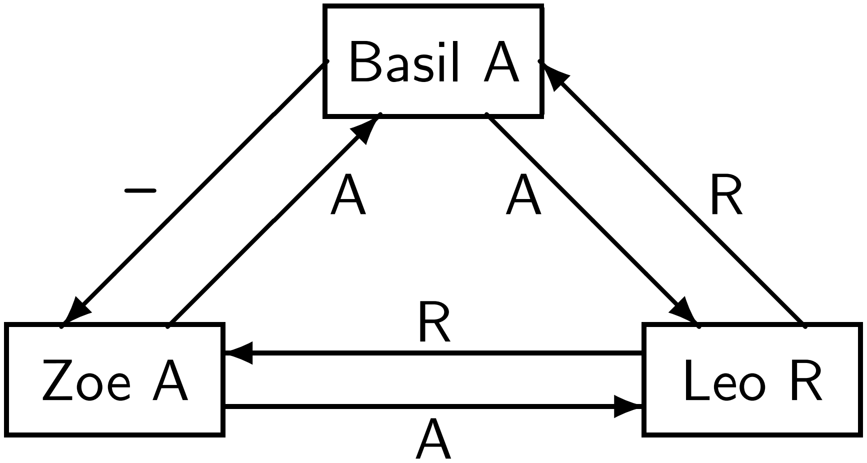
Architecture for a Reliable System



Algorithm 12.1: Consensus - one-round algorithm

```
planType finalPlan
planType array[generals] plan

p1: plan[myID] ← chooseAttackOrRetreat
p2: for all other generals G
p3:     send(G, myID, plan[myID])
p4: for all other generals G
p5:     receive(G, plan[G])
p6: finalPlan ← majority(plan)
```

Messages Sent in a One-Round Algorithm



Data Structures in a One-Round Algorithm

Leo	
general	plan
Basil	A
Leo	R
Zoe	A
majority	A

Zoe	
general	plans
Basil	–
Leo	R
Zoe	A
majority	R

Algorithm 12.2: Consensus - Byzantine Generals algorithm

```
planType finalPlan
planType array[generals] plan, majorityPlan
planType array[generals, generals] reportedPlan

p1:  plan[myID] ← chooseAttackOrRetreat
p2:  for all other generals G                    // First round
p3:     send(G, myID, plan[myID])
p4:  for all other generals G
p5:     receive(G, plan[G])
p6:  for all other generals G                    // Second round
p7:     for all other  generals G’ except G
p8:        send(G’, myID, G, plan[G])
p9:  for all other generals G
p10:    for all other generals G’ except G
p11:       receive(G, G’, reportedPlan[G, G’])
p12: for all other generals G                    // First vote
p13:    majorityPlan[G] ← majority(plan[G] ∪ reportedPlan[*, G])
p14: majorityPlan[myID] ← plan[myID]             // Second vote
p15: finalPlan ← majority(majorityPlan)
```

Crash Failure - First Scenario (Leo)

Leo				
general	plan	reported by		majority
		Basil	Zoe	
Basil	A		–	A
Leo	R			R
Zoe	A	–		A
majority				A

Crash Failure - First Scenario (Zoe)

Zoe				
general	plan	reported by		majority
		Basil	Leo	
Basil	–		**A**	A
Leo	R	–		R
Zoe	A			A
majority				A

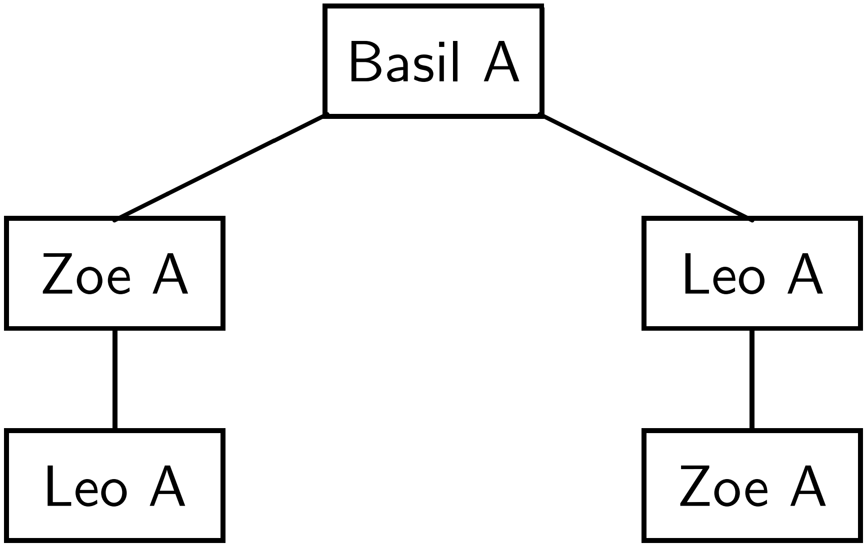
Crash Failure - Second Scenario (Leo)

Leo				
general	plan	reported by		majority
		Basil	Zoe	
Basil	A		A	A
Leo	R			R
Zoe	A	A		A
majority				A

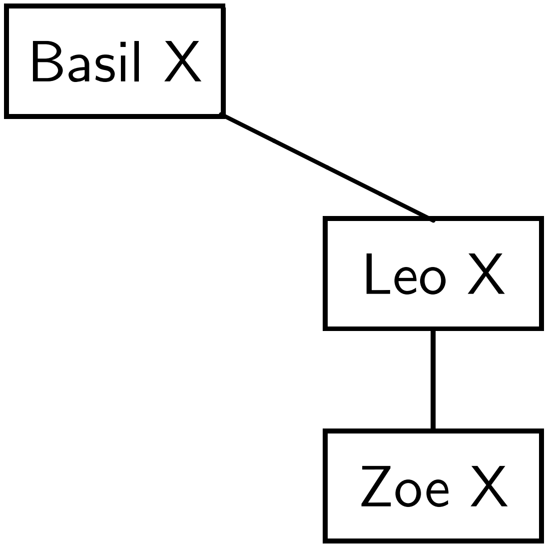
Crash Failure - Second Scenario (Zoe)

Zoe				
general	plan	reported by		majority
		Basil	Leo	
Basil	A		A	A
Leo	R	–		R
Zoe	A			A
majority				A

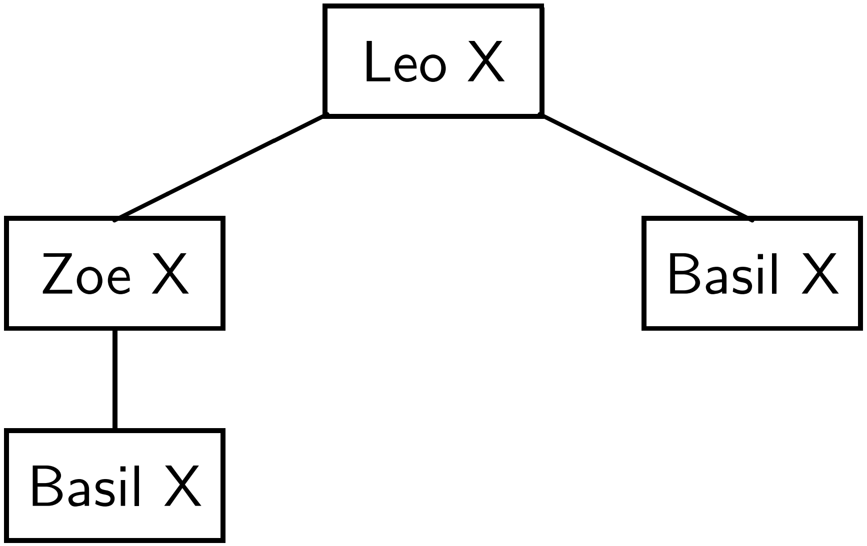
Knowledge Tree about Basil - First Scenario



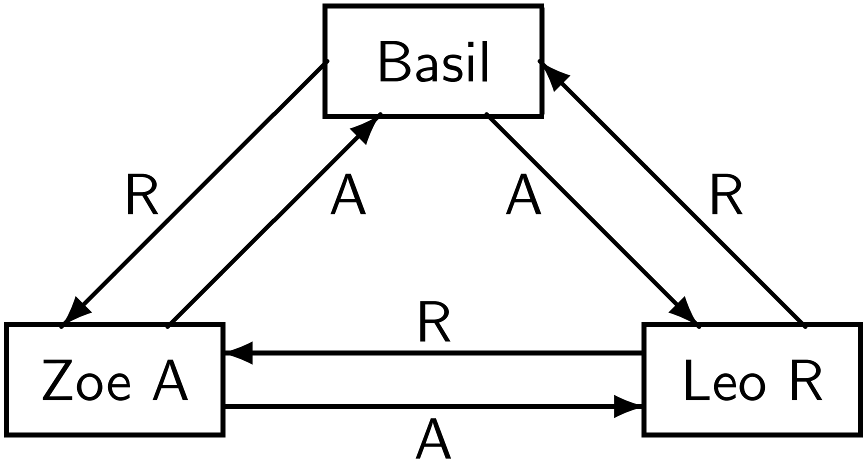
Knowledge Tree about Basil - Second Scenario



Knowledge Tree about Leo



Byzantine Failure with Three Generals



Data Stuctures for Leo and Zoe After First Round

Leo	
general	plans
Basil	A
Leo	R
Zoe	A
majority	A

Zoe	
general	plans
Basil	R
Leo	R
Zoe	A
majority	R

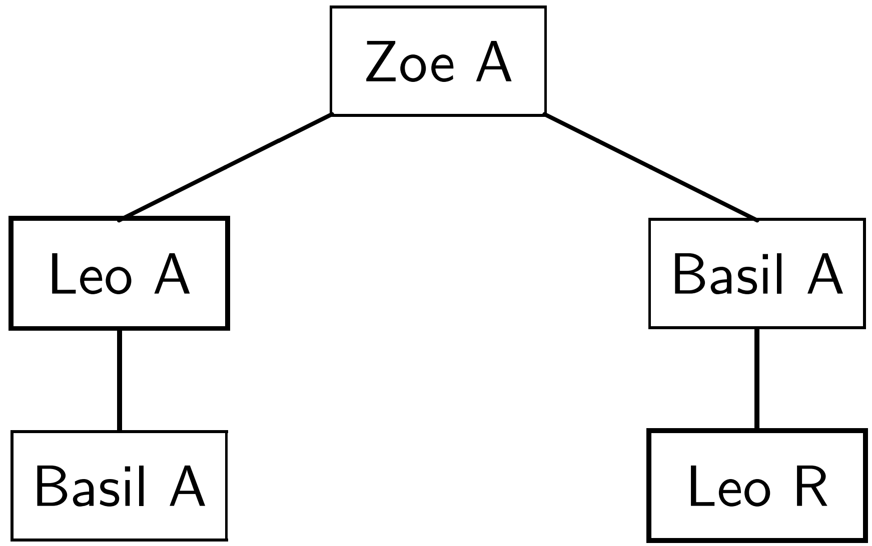
Data Stuctures for Leo After Second Round

Leo				
general	plans	reported by		majority
		Basil	Zoe	
Basil	A		A	A
Leo	R			R
Zoe	A	**R**		R
majority				R

Data Stuctures for Zoe After Second Round

Zoe				
general	plans	reported by		majority
		Basil	Leo	
Basil	A		A	A
Leo	R	R		R
Zoe	A			A
majority				A

Knowledge Tree About Zoe



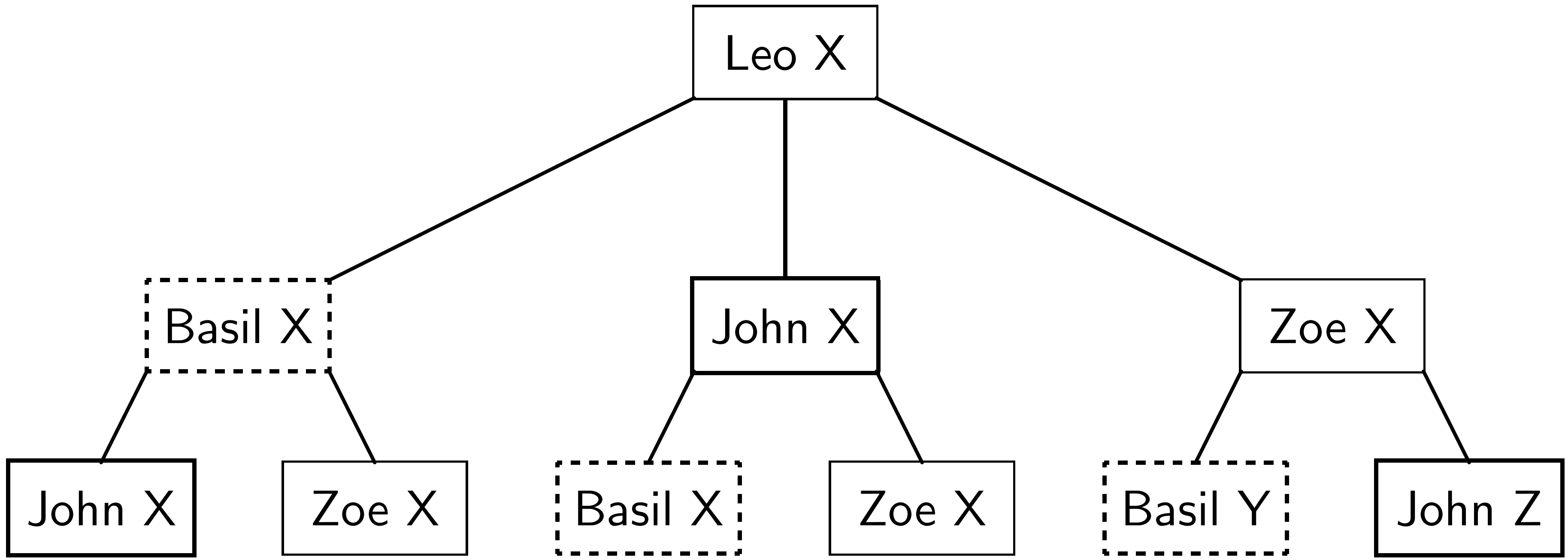
Four Generals: Data Structure of Basil (1)

Basil					
general	plan	reported by			majority
		John	Leo	Zoe	
Basil	A				A
John	**A**		**A**	?	A
Leo	R	R		?	R
Zoe	?	?	?		?
majority					?

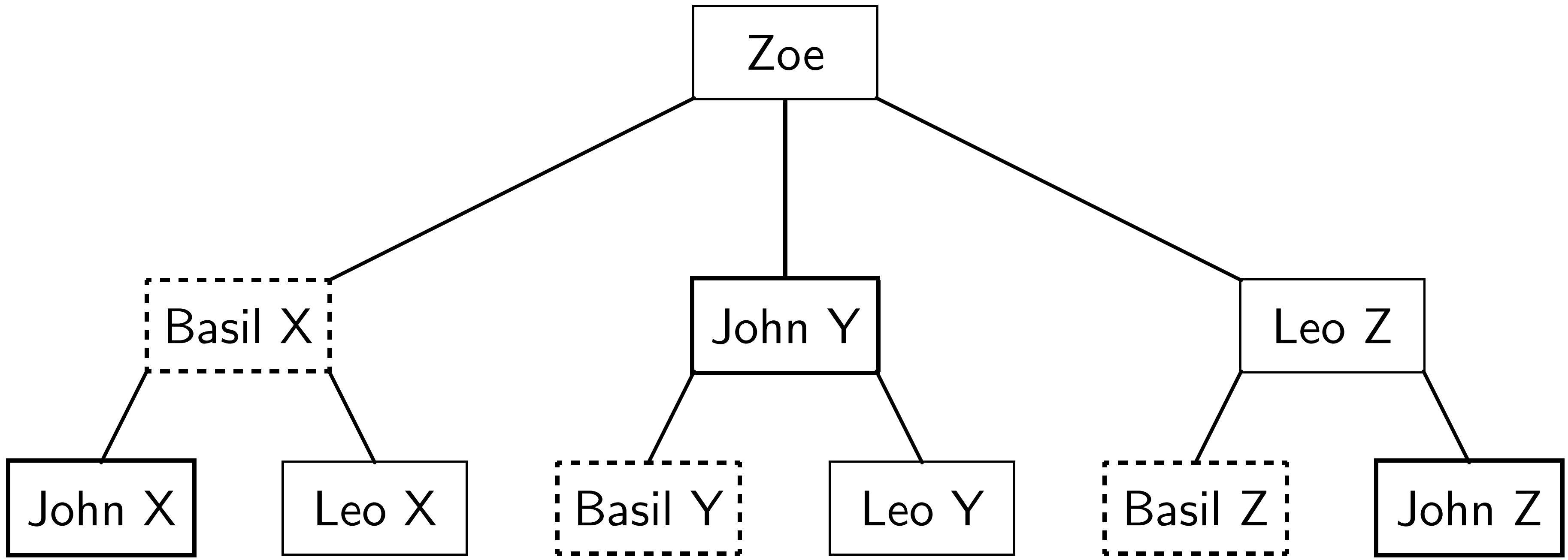
Four Generals: Data Structure of Basil (2)

Basil					
general	plans	reported by			majority
		John	Leo	Zoe	
Basil	A				A
John	A		A	?	A
Leo	R	R		?	R
Zoe	**R**	**A**	**R**		R
					R

Knowledge Tree About Loyal General Leo



Knowledge Tree About Traitor Zoe



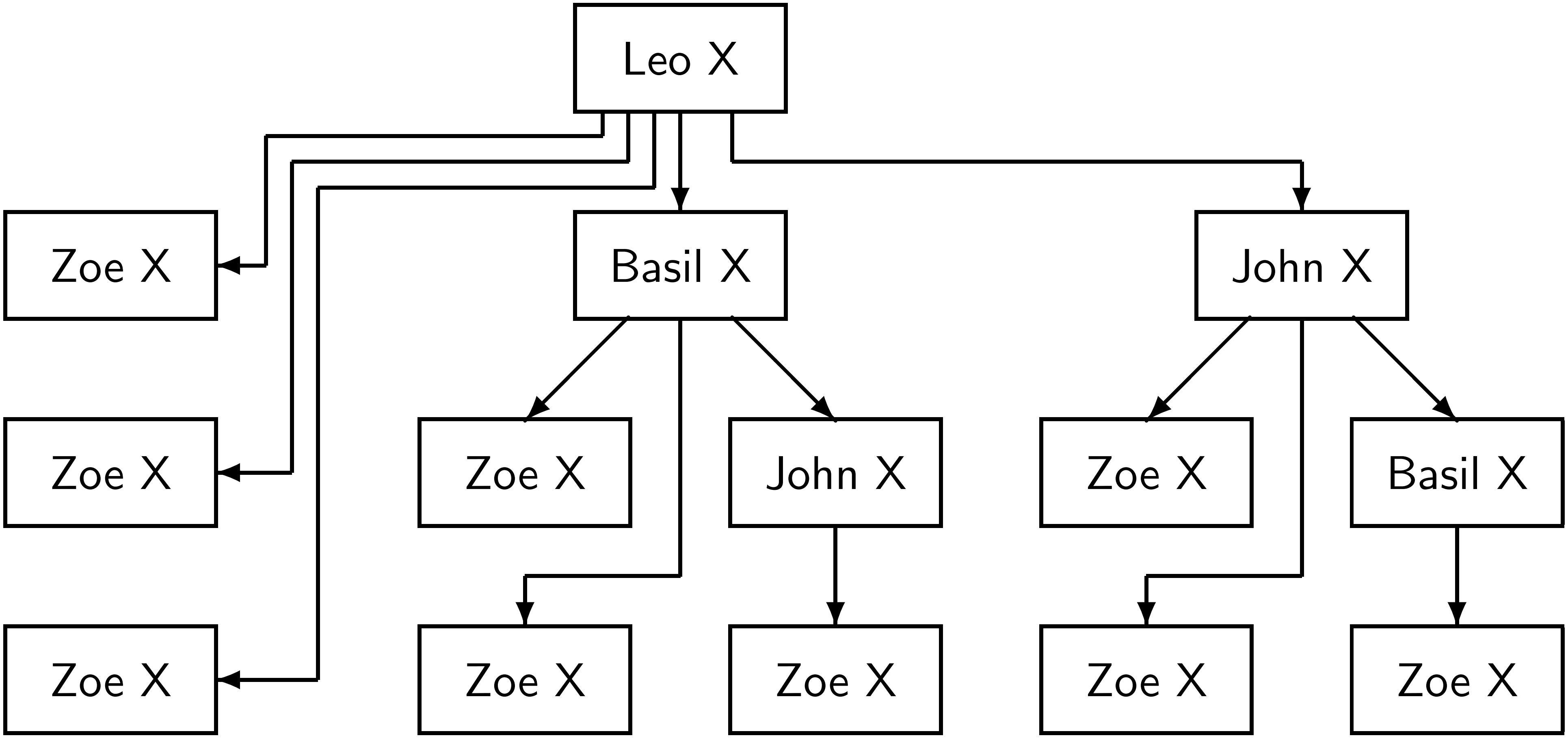
Complexity of the Byzantine Generals Algorithm

traitors	generals	messages
1	4	36
2	7	392
3	10	1790
4	13	5408

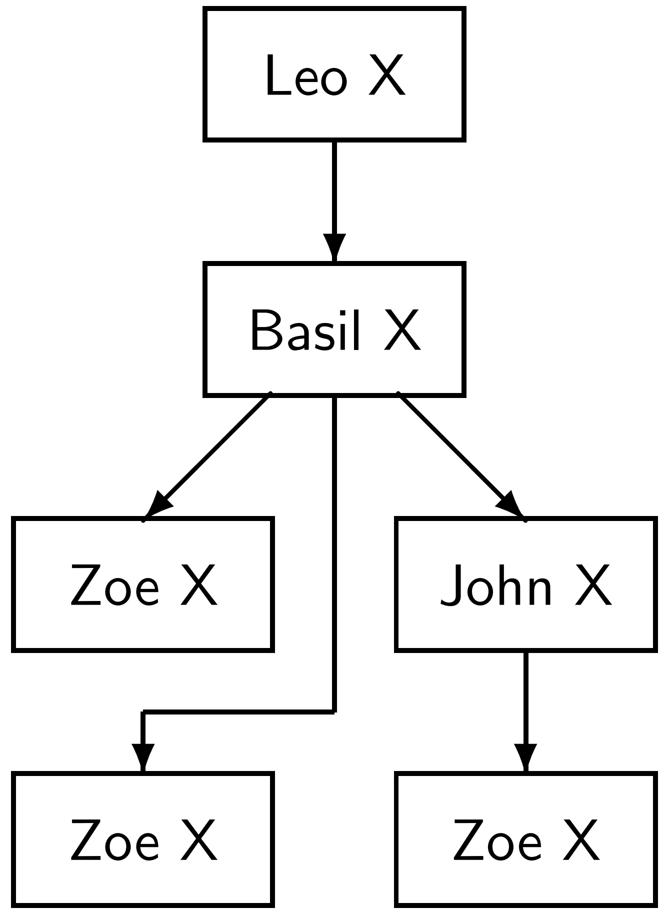
Algorithm 12.3: Consensus - flooding algorithm
planType finalPlan set of planType plan ← { chooseAttackOrRetreat } set of planType receivedPlan

```
p1: do t + 1 times
p2:     for all other generals G
p3:         send(G, plan)
p4:     for all other generals G
p5:         receive(G, receivedPlan)
p6:         plan ← plan ∪ receivedPlan
p7: finalPlan ← majority(plan)
```

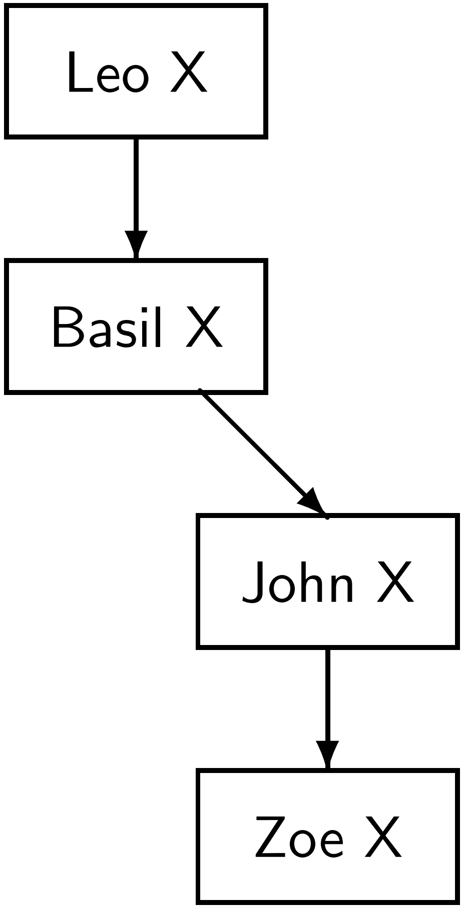
Flooding Algorithm with No Crash: Knowledge Tree About Leo



Flooding Algorithm with Crash: Knowledge Tree About Leo (1)



Flooding Algorithm with Crash: Knowledge Tree About Leo (2)



Algorithm 12.4: Consensus - King algorithm

```
                planType finalPlan, myMajority, kingPlan
                planType array[generals] plan
                integer votesMajority
p1:  plan[myID] ← chooseAttackOrRetreat

p2:  do two times
p3:     for all other generals G                // First and third rounds
p4:        send(G, myID, plan[myID])
p5:     for all other generals G
p6:        receive(G, plan[G])
p7:     myMajority ← majority(plan)
p8:     votesMajority ← number of votes for myMajority
```

Algorithm 12.4: Consensus - King algorithm (continued)

```
p9:     if my turn to be king              // Second and fourth rounds
p10:        for all other generals G
p11:            send(G, myID, myMajority)
p12:        plan[myID] ← myMajority
        else
p13:        receive(kingID, kingPlan)
p14:        if votesMajority ¿ 3
p15:            plan[myID] ← myMajority
            else
p16:            plan[myID] ← kingPlan

p17: finalPlan ← plan[myID]               // Final decision
```

Scenario for King Algorithm:
First King Loyal General Zoe (1)

Basil							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
A	A	R	R	R	R	3	

John							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
A	A	R	A	R	A	3	

Leo							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
A	A	R	A	R	A	3	

Zoe							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
A	A	R	R	R	R	3	

Scenario for King Algorithm: First King Loyal General Zoe (2)

Basil							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R							R

John							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
	R						R

Leo							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
		R					R

Zoe							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
				R			

Scenario for King Algorithm:
First King Loyal General Zoe (3)

Basil							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	R	R	?	R	R	4–5	

John							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	R	R	?	R	R	4–5	

Leo							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	R	R	?	R	R	4–5	

Zoe							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	R	R	?	R	R	4–5	

Scenario for King Algorithm: First King Traitor Mike (1)

Basil							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R							R

John							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
	A						A

Leo							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
		A					A

Zoe							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
				R			R

Scenario for King Algorithm:
First King Traitor Mike (2)

Basil							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	A	A	?	R	?	3	

John							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	A	A	?	R	?	3	

Leo							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	A	A	?	R	?	3	

Zoe							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	A	A	?	R	?	3	

Scenario for King Algorithm: First King Traitor Mike (3)

Basil							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
A							A

John							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
	A						A

Leo							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
		A					A

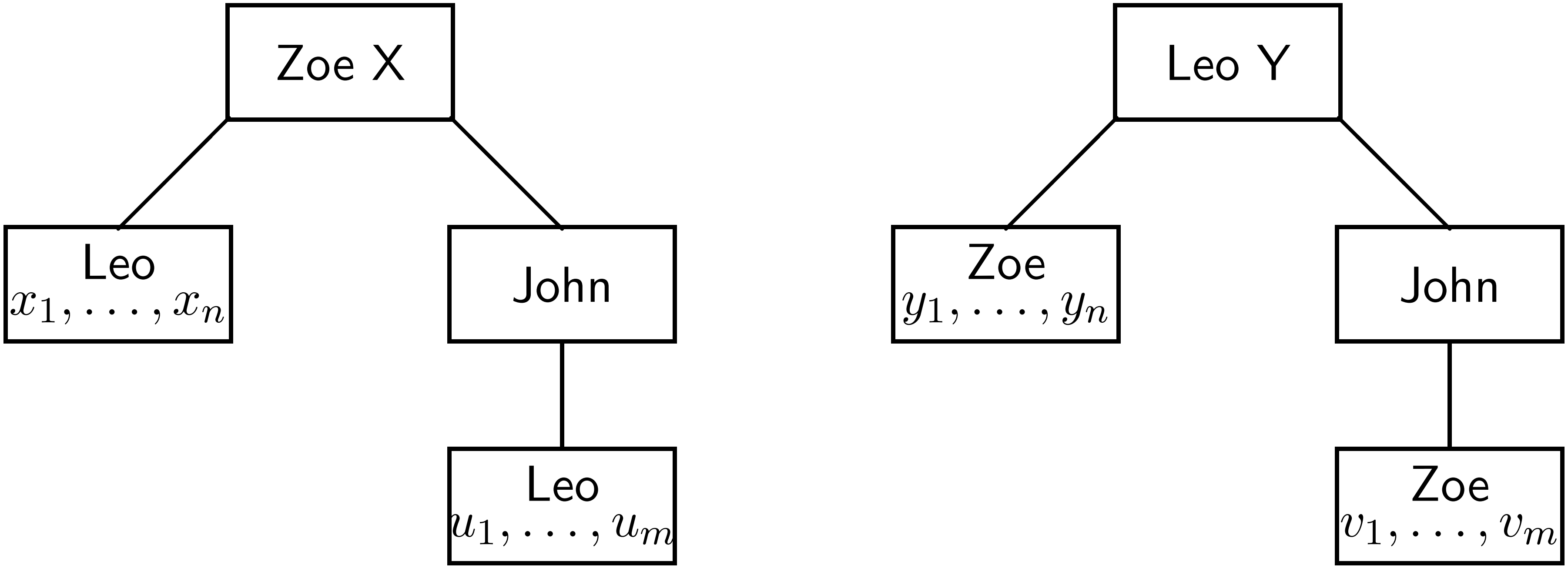
Zoe							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
				A			

Complexity of Byzantine Generals and King Algorithms

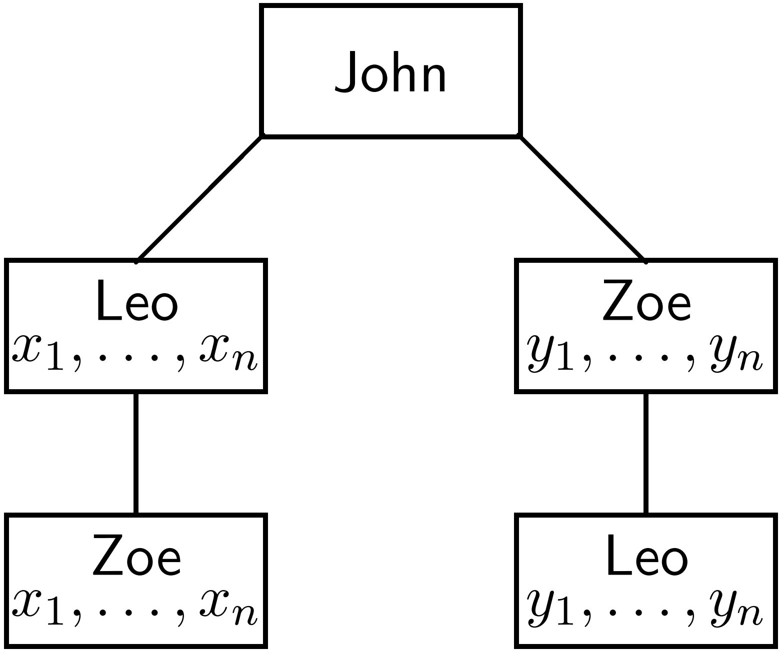
traitors	generals	messages
1	4	36
2	7	392
3	10	1790
4	13	5408

traitors	generals	messages
1	5	48
2	9	240
3	13	672
4	17	1440

Impossibility with Three Generals (1)



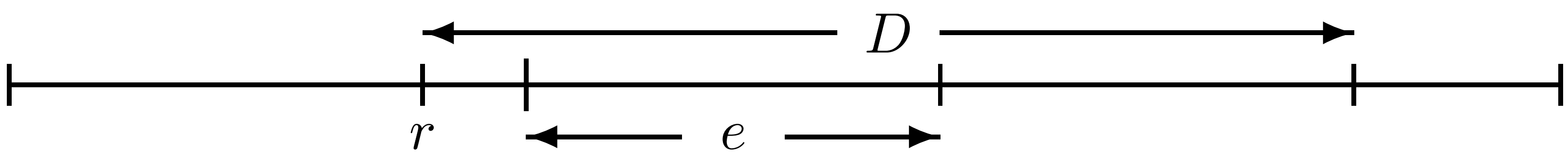
Impossibility with Three Generals (2)



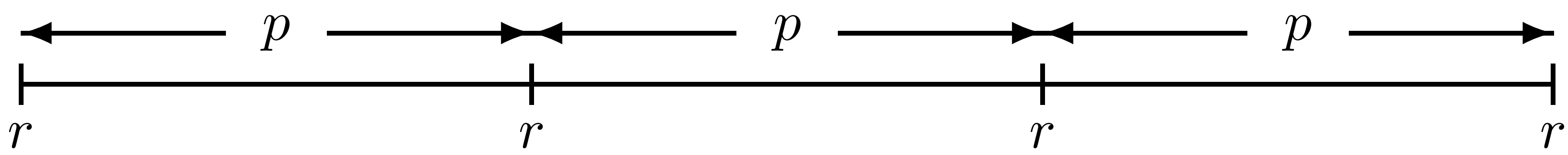
Exercise for Byzantine Generals Algorithm

Zoe					
general	plan	reported by			majority
		Basil	John	Leo	
Basil	R		A	R	?
John	A	R		A	?
Leo	R	R	R		?
Zoe	A				A
					?

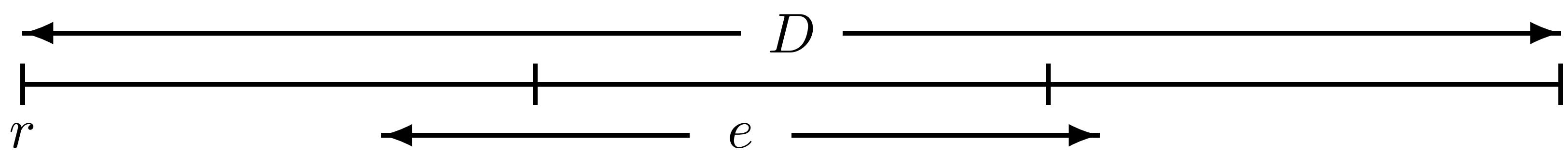
Release Time, Execution Time and Relative Deadline



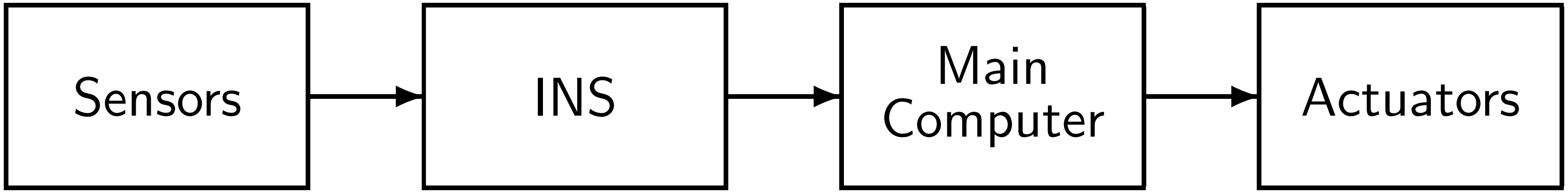
Periodic Task



Deadline is a Multiple of the Period

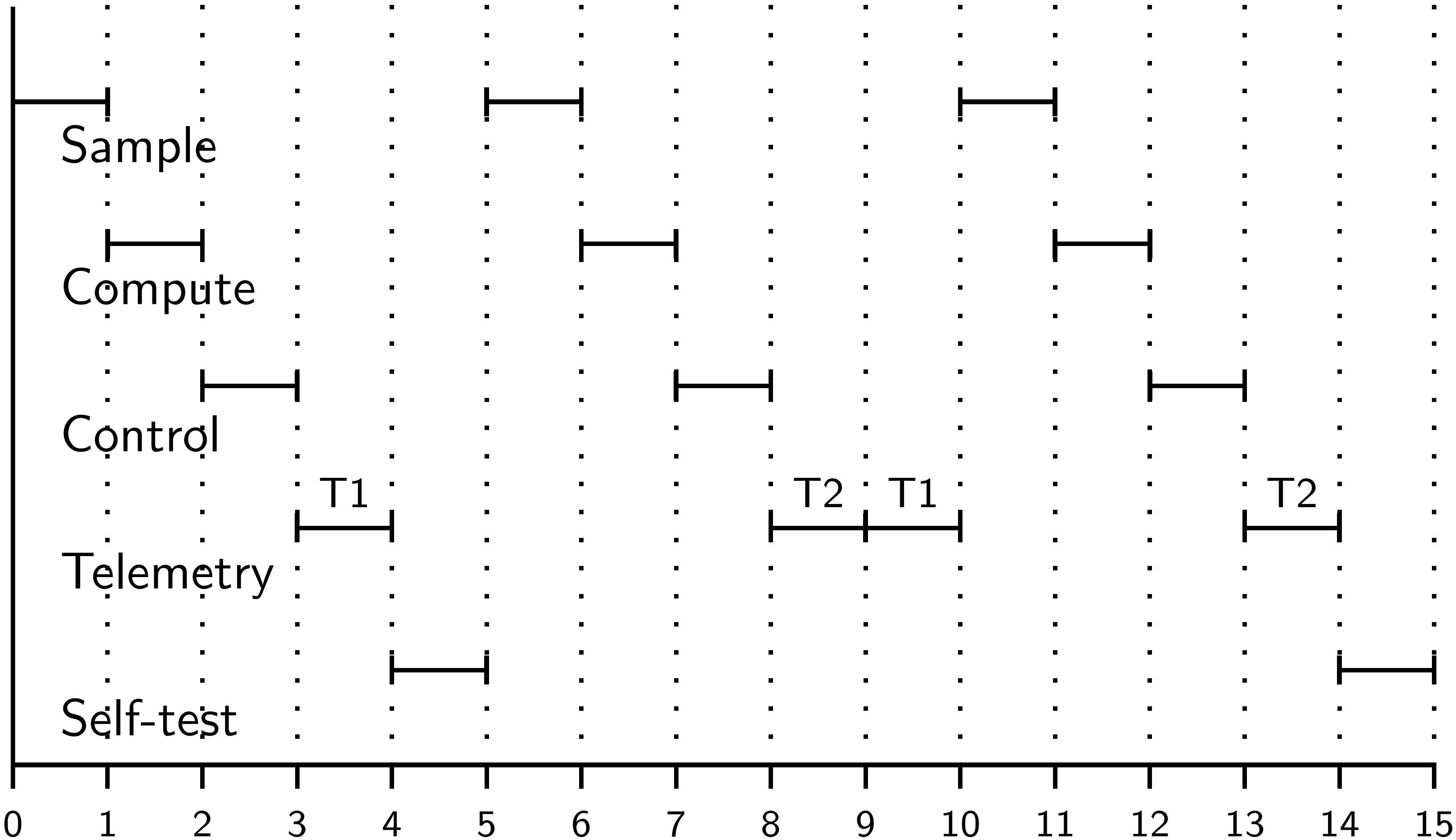


Architecture of Ariane Control System



Synchronization Window in the Space Shuttle

Synchronous System



Synchronous System Scheduling Table

0	1	2	3	4
Sample	Compute	Control	Telemetry 1	Self-test

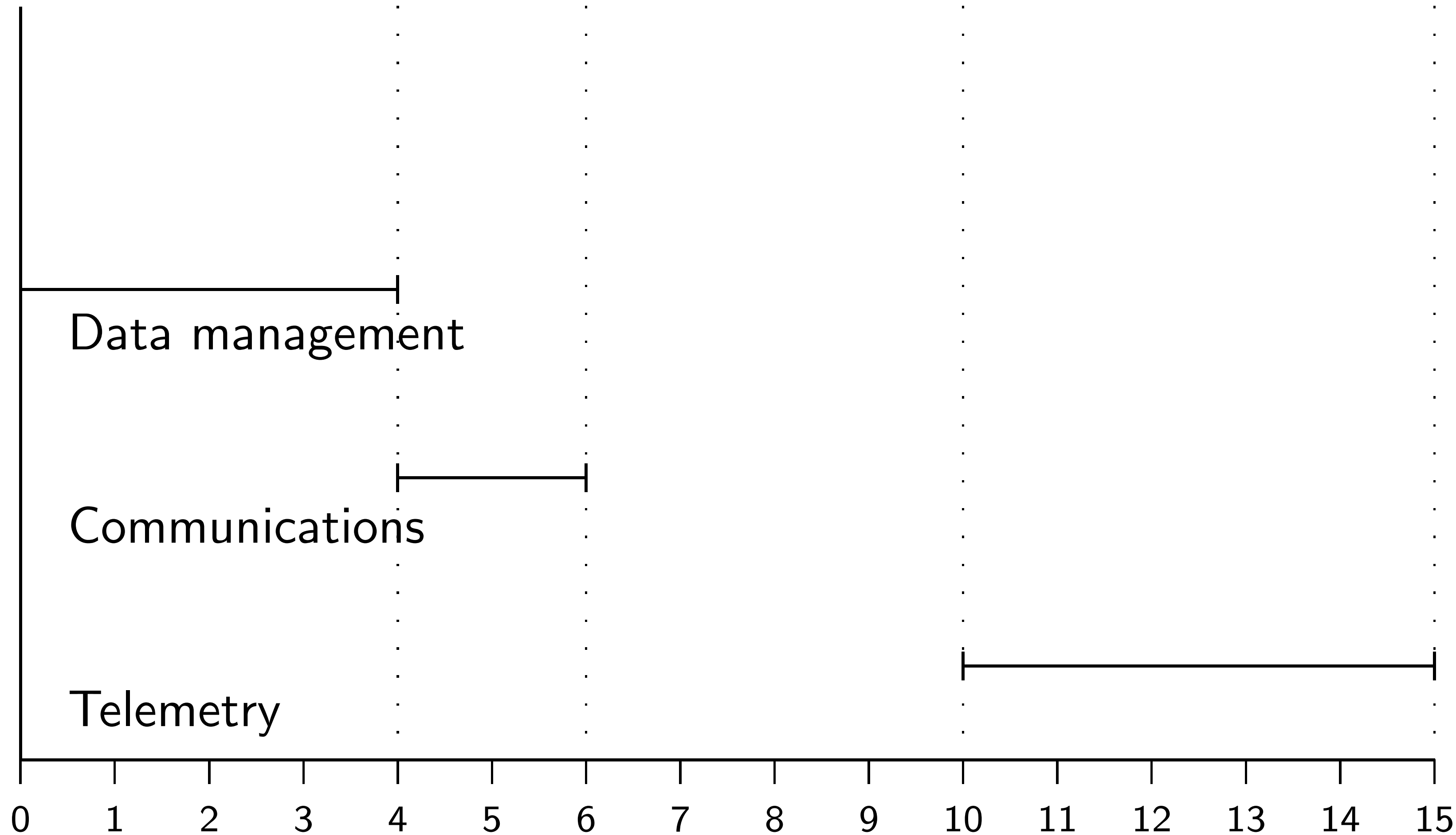
5	6	7	8	9
Sample	Compute	Control	Telemetry 2	Telemetry 1

10	11	12	13	14
Sample	Compute	Control	Telemetry 2	Self-test

Algorithm 13.1: Synchronous scheduler
taskAddressType array[0..numberFrames-1] tasks ←   [task address,. . . ,task address]   integer currentFrame ← 0
p1: loop   p2: await beginning of frame   p3: invoke tasks[currentFrame]   p4: increment currentFrame modulo numberFrames

Algorithm 13.2: Producer-consumer (synchronous system)		
queue of dataType buffer1, buffer2		
sample	**compute**	**control**
dataType d	dataType d1, d2	dataType d
p1: d ← sample	q1: d1 ← take(buffer1)	r1: d ← take(buffer2)
p2: append(d, buffer1)	q2: d2 ← compute(d1)	r2: control(d)
p3:	q3: append(d2, buffer2)	r3:

Asynchronous System



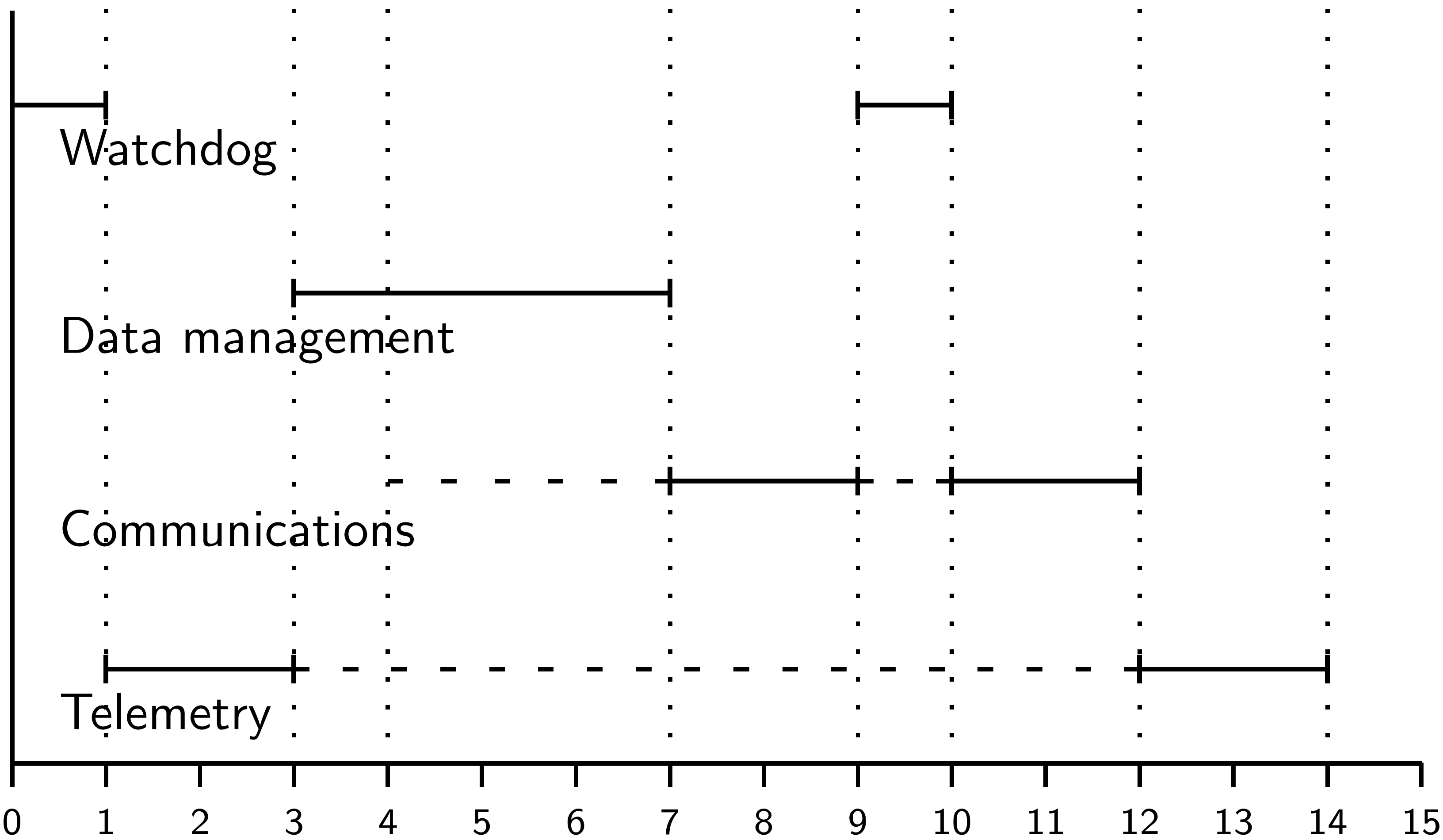
Algorithm 13.3: Asynchronous scheduler
queue of taskAddressType readyQueue ← . . .
taskAddressType currentTask

```
     loop forever
p1:    await readyQueue not empty
p2:      currentTask ← take head of readyQueue
p3:      invoke currentTask
```

Algorithm 13.4: Preemptive scheduler

```
        queue of taskAddressType readyQueue ← ...
        taskAddressType currentTask

      loop forever
p1:     await a scheduling event
p2:     if currentTask.priority ¡ highest priority of a task on readyQueue
p3:        save partial computation of currentTask and place on readyQueue
p4:        currentTask ← take task of highest priority from readyQueue
p5:        invoke currentTask
p6:     else if currentTask's timeslice is past and
              currentTask.priority = priority of some task on readyQueue
p7:        save partial computation of currentTask and place on readyQueue
p8:        currentTask ← take a task of the same priority from readyQueue
p9:        invoke currentTask
p10:    else resume currentTask
```

Preemptive Scheduling

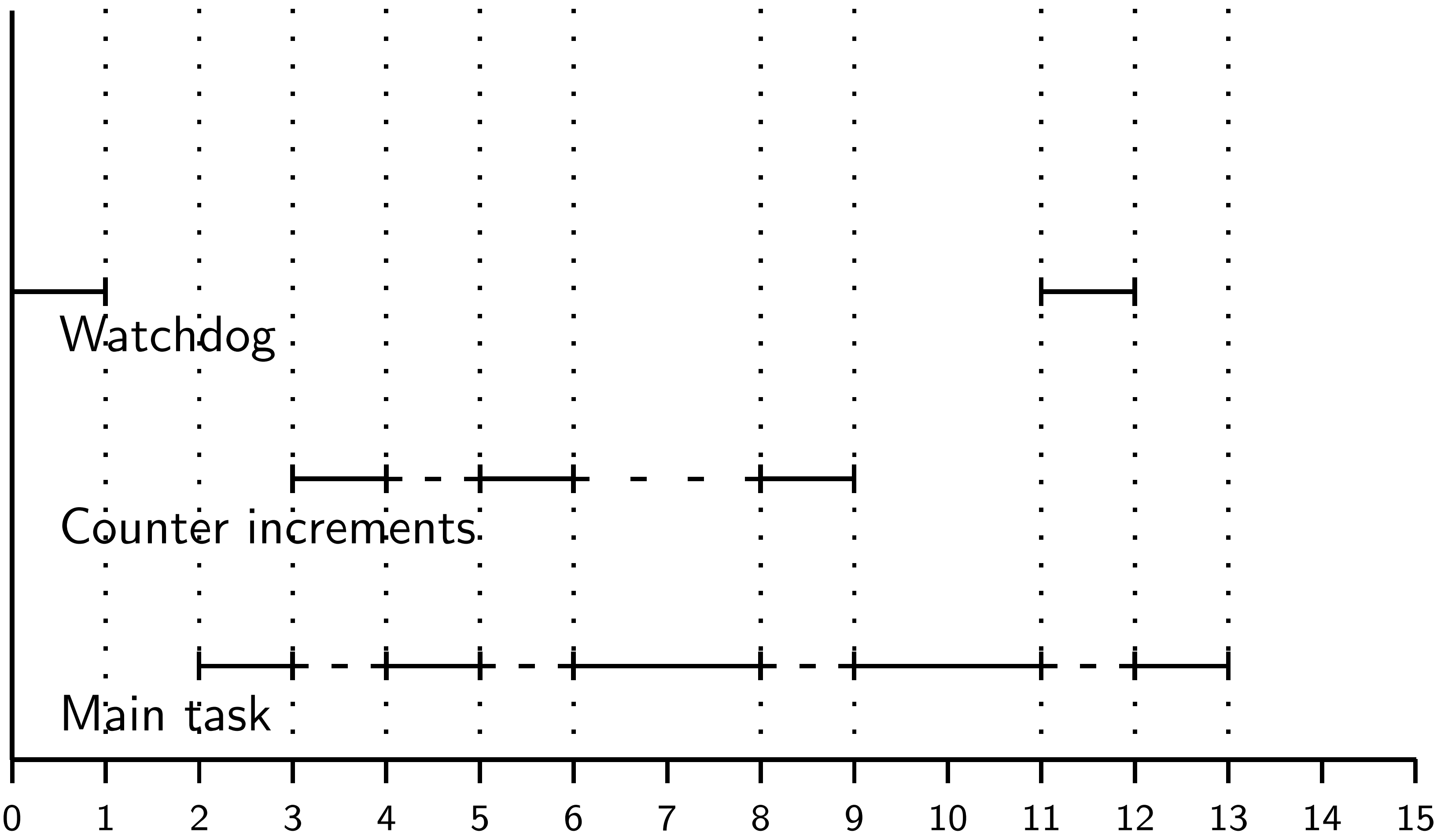


Algorithm 13.5: Watchdog supervision of response time	
boolean ran ← false	
data management	**watchdog**
loop forever	loop forever
p1: do data management	q1: await ninth frame
p2: ran ← true	q2: if ran is false
p3: rejoin readyQueue	q3: notify response-time overflow
p4:	q4: ran ← false
p5:	q5: rejoin readyQueue

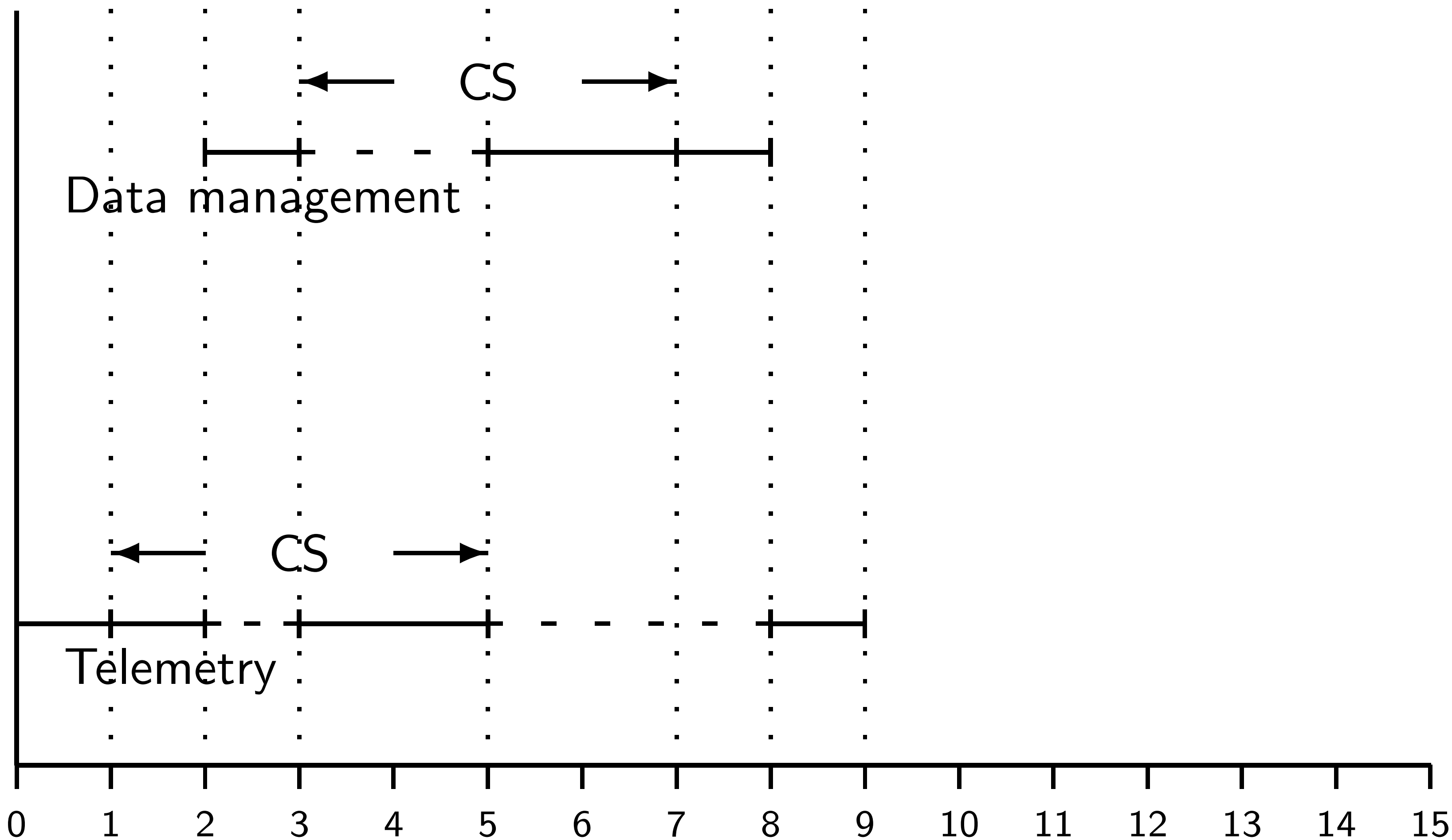
Algorithm 13.6: Real-time buffering - throw away new data	
queue of dataType buffer ← empty queue	
sample	**compute**
dataType d	dataType d
loop forever	loop forever
p1: d ← sample	q1: await buffer not empty
p2: if buffer is full do nothing	q2: d ← take(buffer)
p3: else append(d,buffer)	q3: compute(d)

Algorithm 13.7: Real-time buffering - overwrite old data	
queue of dataType buffer ← empty queue	
sample	**compute**
dataType d loop forever p1: d ← sample p2: append(d, buffer) p3:	dataType d loop forever q1: await buffer not empty q2: d ← take(buffer) q3: compute(d)

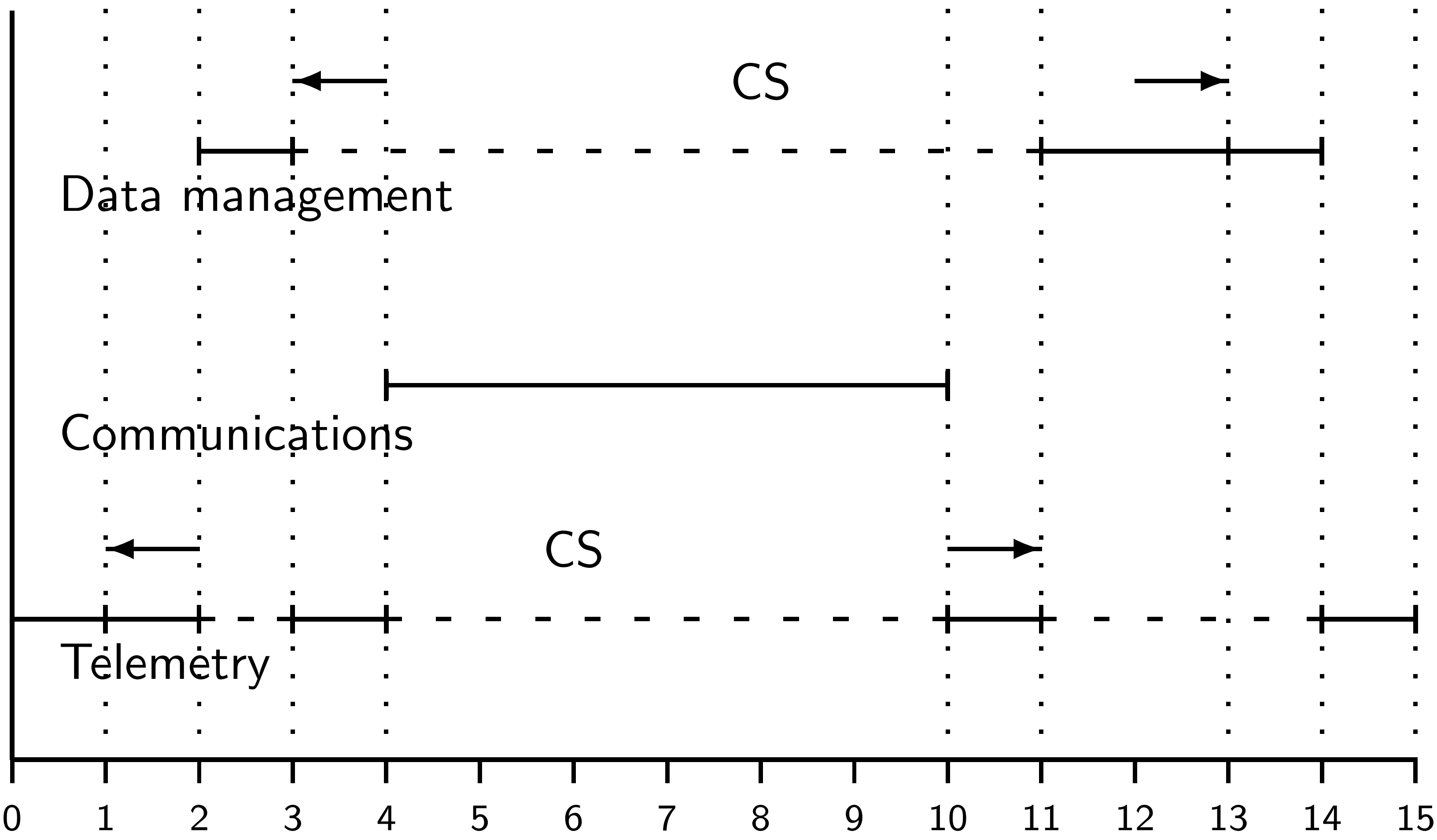
Interrupt Overflow on Apollo 11



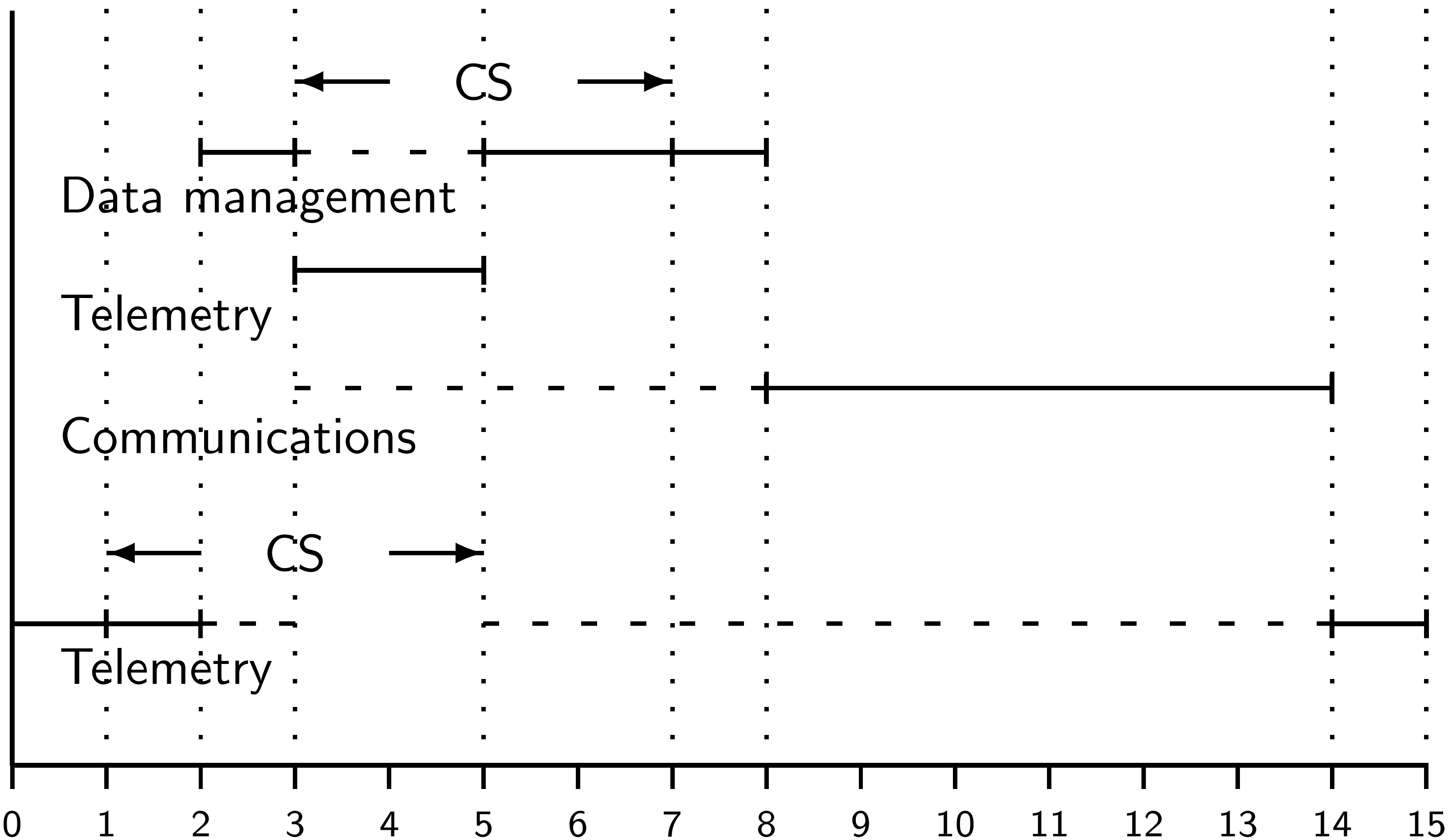
Priority Inversion (1)



Priority Inversion (2)



Priority Inheritance



Priority Inversion in Promela (1)

```
mtype = { idle, blocked, nonCS, CS, long };

mtype data = idle, comm = idle, telem = idle;

#define ready(p) (p != idle && p != blocked)

active proctype Data() {
    do
    ::  data = nonCS;
        enterCS(data);
        exitCS(data);
        data = idle;
    od
}
```

Priority Inversion in Promela (2)

```
active proctype Comm() provided (!ready(data)) {
    do
    ::  comm = long;
        comm = idle;
    od
}

active proctype Telem() provided (!ready(data) && !ready(comm)) {
    do
    ::  telem = nonCS;
        enterCS(telem);
        exitCS(telem);
        telem = idle;
    od
}
```

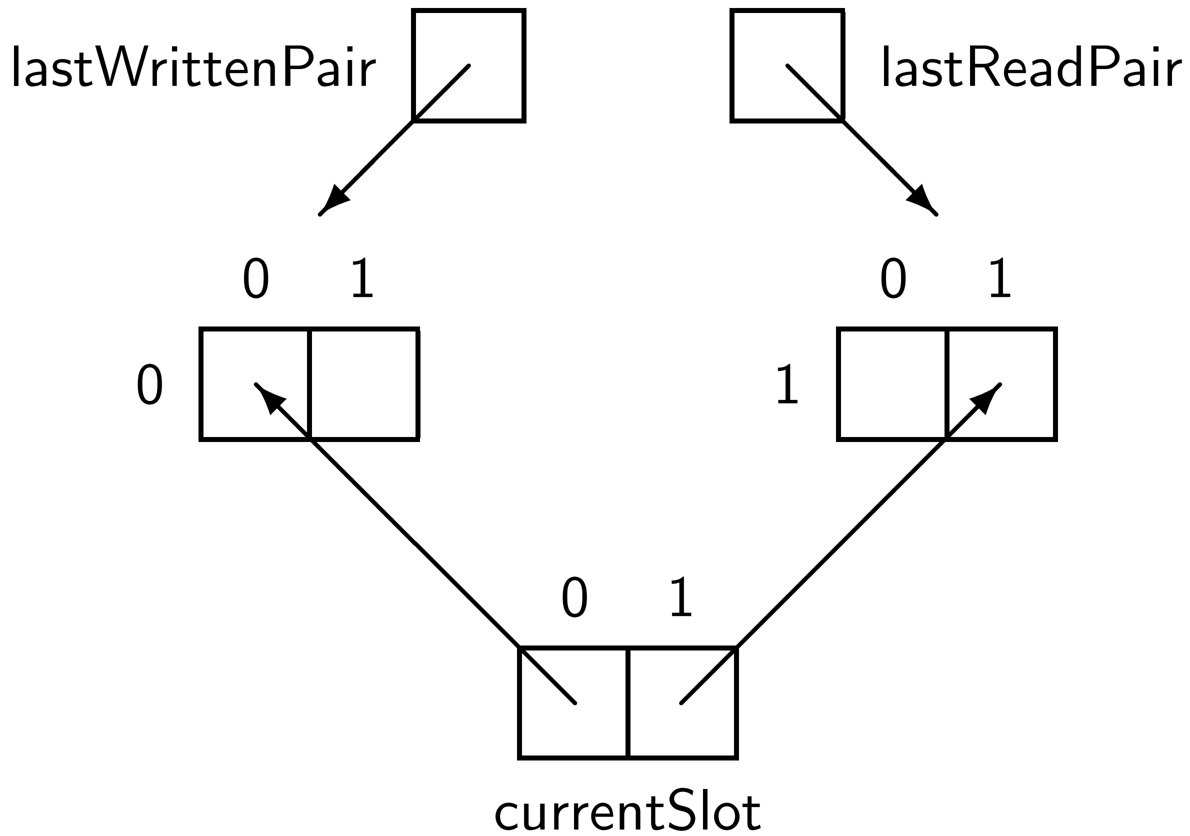
Priority Inversion in Promela (3)

```
bit sem = 1;

inline enterCS(state) {
    atomic {
        if
        :: sem == 0 ->
            state = blocked;
            sem != 0;
        :: else ->
        fi;
        sem = 0;
        state = CS;
    }
}

inline exitCS(state) {
    atomic {
        sem = 1;
        state = idle
    }
}
```

Priority Inheritance in Promela

```
#define inherit(p) (p == CS)

active proctype Data() {
  do
  :: data = nonCS;
     assert( ! (telem == CS && comm == long) );
     enterCS(data); exitCS(data);
     data = idle;
  od
}

active proctype Comm()
  provided (!ready(data) && !inherit(telem))
    { ... }

active proctype Telem()
  provided (!ready(data) && !ready(comm) || inherit(telem))
    { ... }
```

Data Structures in Simpson's Algorithm



Algorithm 13.8: Simpson's four-slot algorithm	
	dataType array[0..1,0..1] data ← default initial values
	bit array[0..1] currentSlot ← { 0, 0 }
	bit lastWrittenPair ← 1, lastReadPair ← 1
	writer
	bit writePair, writeSlot
	dataType item
	loop forever
p1:	item ← produce
p2:	writePair ← $1-$ lastReadPair
p3:	writeSlot ← $1-$ currentSlot[writePair]
p4:	data[writePair, writeSlot] ← item
p5:	currentSlot[writePair] ← writeSlot
p6:	lastWrittenPair ← writePair

Algorithm 13.8: Simpson's four-slot algorithm (continued)

reader

```
      bit readPair, readSlot
      dataType item
      loop forever
p7:      readPair ← lastWrittenPair
p8:      lastReadPair ← readPair
p9:      readSlot ← currentSlot[readPair]
p10:     item ← data[readPair, readSlot]
p11:     consume(item)
```

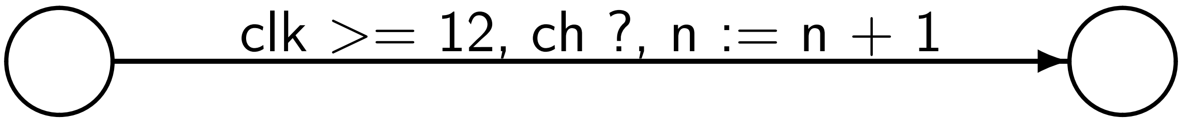
Algorithm 13.9: Event signaling	
binary semaphore s $\leftarrow$ 0	
p	**q**
p1: if decision is to wait for event p2: wait(s)	q1: do something to cause event q2: signal(s)

Suspension Objects in Ada

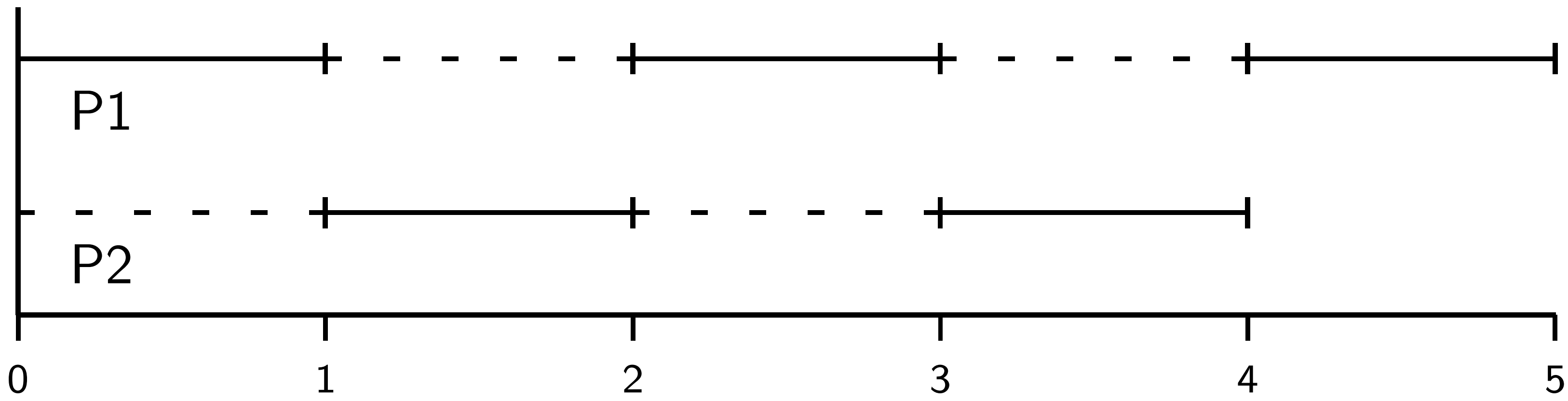
```
package Ada.Synchronous_Task_Control is
  type Suspension_Object is limited private;
  procedure Set_True(S : in out Suspension_Object);
  procedure Set_False(S : in out Suspension_Object);
  function Current_State(S : Suspension_Object)
    return Boolean;
  procedure Suspend_Until_True(
    S : in out Suspension_Object);
private
  -- not specified by the language
end Ada.Synchronous_Task_Control;
```

Algorithm 13.10: Suspension object - event signaling	
Suspension_Object SO ← (false by default)	
p	**q**
p1: if decision is to wait for event p2: Suspend_Until_True(SO)	q1: do something to cause event q2: Set_True(SO)

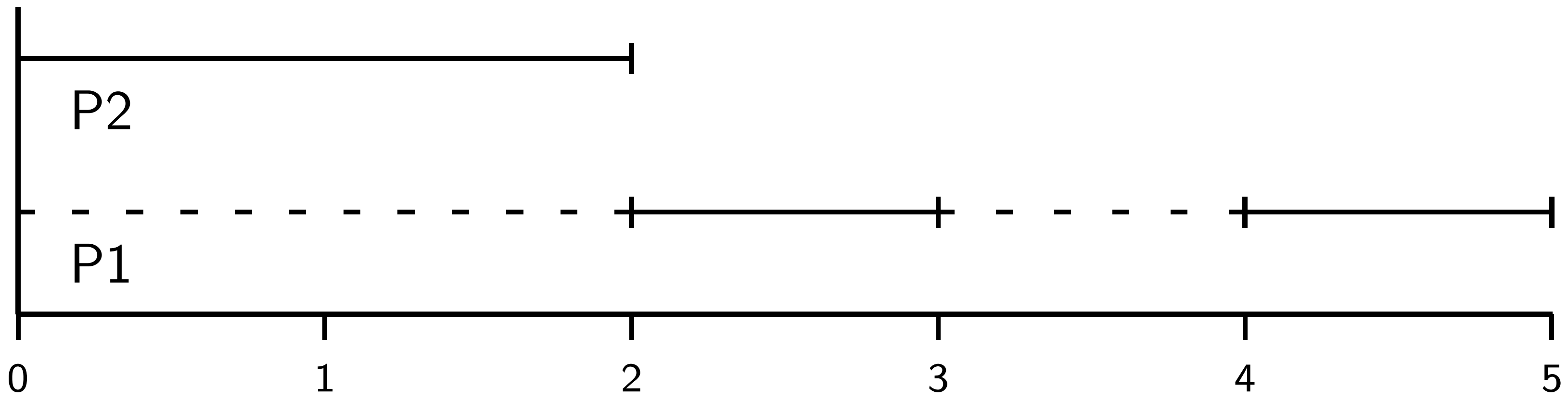
Transition in UPPAAL



Feasible Priority Assignment



Infeasible Priority Assignment



Algorithm 13.11: Periodic task	
constant integer period ← . . .	
	integer next ← currentTime
	loop forever
p1:	delay next − currentTime
p2:	compute
p3:	next ← next + period

Semantics of Propositional Operators

A	$v(A_1)$	$v(A_2)$	$v(A)$
$\neg A_1$	T		F
$\neg A_1$	F		T
$A_1 \vee A_2$	F	F	F
$A_1 \vee A_2$	otherwise		T
$A_1 \wedge A_2$	T	T	T
$A_1 \wedge A_2$	otherwise		F
$A_1 \rightarrow A_2$	T	F	F
$A_1 \rightarrow A_2$	otherwise		T
$A_1 \leftrightarrow A_2$	$v(A_1) = v(A_2)$		T
$A_1 \leftrightarrow A_2$	$v(A_1) \neq v(A_2)$		F

Wason Selection Task

$p3$	$p5$	$flag = 1$	$flag = 0$

Algorithm 2.1: Verification example

```
                    integer x1, integer x2
                    integer y1 ← 0, integer y2 ← 0, integer y3
p1:  read(x1,x2)
p2:  y3 ← x1
p3:  while y3 ≠ 0
p4:     if y2+1 = x2
p5:        y1 ← y1 + 1
p6:        y2 ← 0
p7:     else
p8:        y2 ← y2 + 1
p9:     y3 ← y3 − 1
p10: write(y1,y2)
```

Spark Program for Integer Division

```
--# main_program;
procedure Divide(X1,X2: in Integer ;  Q,R : out Integer )
--# derives Q, R from X1,X2;
--# pre (X1 >= 0) and (X2 > 0);
--# post (X1 = Q * X2 + R) and (X2 > R) and (R >= 0);
is
     N: Integer ;
begin
    Q := 0; R := 0; N := X1;
    while N /= 0
    --# assert (X1 = Q*X2+R+N) and (X2 > R) and (R >= 0);
    loop
        if R+1 = X2 then
            Q := Q + 1; R := 0;
        else
            R := R + 1;
        end if;
        N := N - 1;
    end loop;
end Divide;
```

Integer Division

```
procedure Divide(X1,X2: in Integer ;  Q,R : out Integer ) is
     N: Integer ;
begin
    -- pre (X1 >= 0) and (X2 > 0);
    Q := 0; R := 0; N := X1;
    while N /= 0
    -- assert (X1 = Q*X2+R+N) and (X2 > R) and (R >= 0);
    loop
        if  R+1 = X2 then Q := Q + 1; R := 0;
        else  R := R + 1;
        end if ;
        N := N - 1;
    end loop;
    -- post (X1 = Q * X2 + R) and (X2 > R) and (R >= 0);
end Divide;
```

Verification Conditions for Integer Division

Precondition to assertion:

$$(X1 \geq 0) \wedge (X2 > 0) \rightarrow$$
$$(X1 = Q \cdot X2 + R + N) \wedge (X2 > R) \wedge (R \geq 0).$$

Assertion to postcondition:

$$(X1 = Q \cdot X2 + R + N) \wedge (X2 > R) \wedge (R \geq 0) \wedge (N = 0) \rightarrow$$
$$(X1 = Q \cdot X2 + R) \wedge (X2 > R) \wedge (R \geq 0).$$

Assertion to assertion by then branch:

$$(X1 = Q \cdot X2 + R + N) \wedge (X2 > R) \wedge (R \geq 0) \wedge (R + 1 = X2) \rightarrow$$
$$(X1 = Q' \cdot X2 + R' + N') \wedge (X2 > R') \wedge (R' \geq 0).$$

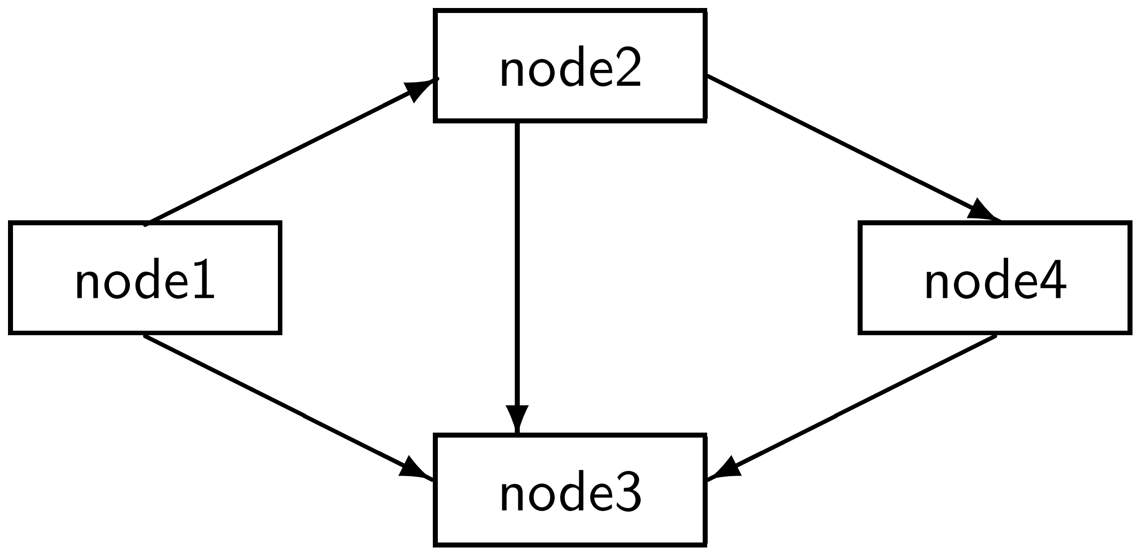
Assertion to assertion by else branch:

$$(X1 = Q \cdot X2 + R + N) \wedge (X2 > R) \wedge (R \geq 0) \wedge (R + 1 \neq X2) \rightarrow$$
$$(X1 = Q' \cdot X2 + R' + N') \wedge (X2 > R') \wedge (R' \geq 0).$$

The Sleeping Barber

n	producer	consumer	Buffer	notEmpty
1	**append(d, Buffer)**	wait(notEmpty)	[]	0
2	**signal(notEmpty)**	wait(notEmpty)	[1]	0
3	append(d, Buffer)	**wait(notEmpty)**	[1]	1
4	append(d, Buffer)	**d $\leftarrow$ take(Buffer)**	[1]	0
5	**append(d, Buffer)**	wait(notEmpty)	[]	0

Synchronizing Precedence



Algorithm 3.1: Barrier synchronization
global variables for synchronization

```
     loop forever
p1:     wait to be released
p2:     computation
p3:     wait for all processes to finish their computation
```

The Stable Marriage Problem

Man	List of women			
1	2	4	1	3
2	3	1	4	2
3	2	3	1	4
4	4	1	3	2

Woman	List of men			
1	2	1	4	3
2	4	3	1	2
3	1	4	3	2
4	2	1	4	3

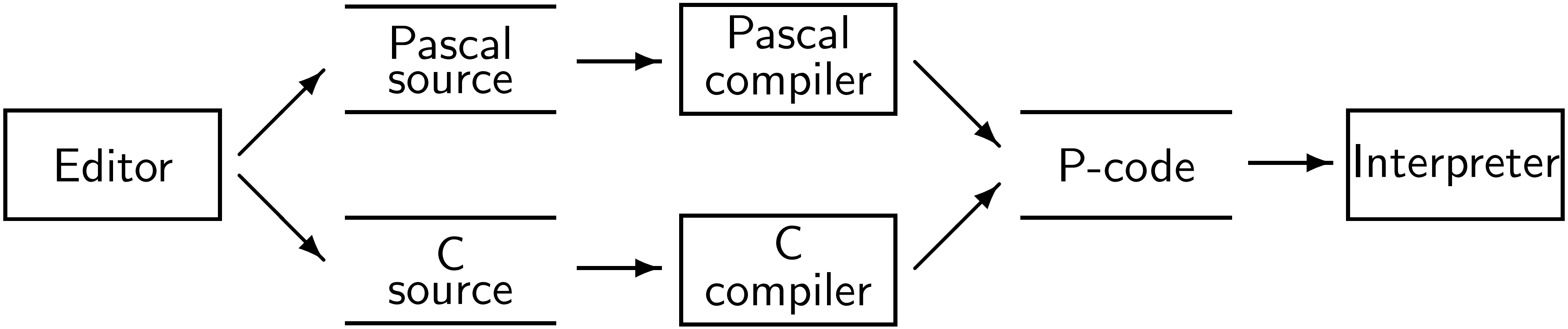
Algorithm 3.2: Gale-Shapley algorithm for stable marriage

```
integer list freeMen ← {1,. . . ,n}
integer list freeWomen ← {1,. . . ,n}
integer pair-list matched ← ∅
integer array[1..n, 1..n] menPrefs ← . . .
integer array[1..n, 1..n] womenPrefs ← . . .
integer array[1..n] next ← 1

p1: while freeMen ≠ ∅, choose some m from freeMen
p2:     w ← menPrefs[m, next[m]]
p3:     next[m] ← next[m] + 1
p4:     if w in freeWomen
p5:         add (m,w) to matched, and remove w from freeWomen
p6:     else if w prefers m to m' // where (m',w) in matched
p7:         replace (m',w) in matched by (m,w), and remove m' from freeMen
p8:     else // w rejects m, and nothing is changed
```

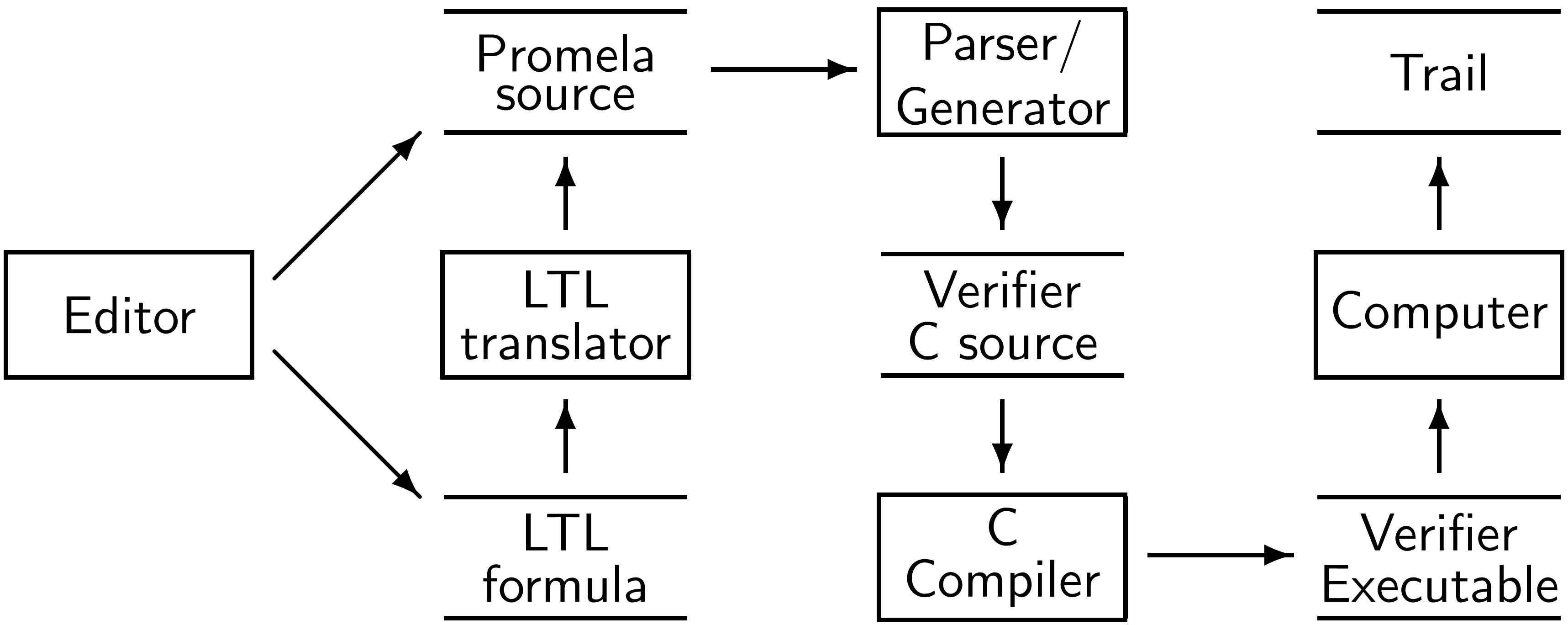
The n-Queens Problem

1	Q							
2							Q	
3					Q			
4								Q
5		Q						
6				Q				
7						Q		
8			Q					
	1	2	3	4	5	6	7	8

The Architecture of BACI



The Architecture of Spin



Cycles in a State Diagram