Trees

15 February 2024
UCCS ASPREY LECTURE SERIES:

MATT FOSTER '74

FEBRUARY 16, 2024

NE 206
Talk 5:30-6:30pm
Snack and Shmooze 5-5:30pm

VASSAR CS ALUMUS, MATTHEW FOSTER '74, IS THE ADVANCED SENIOR SYSTEMS DESIGNER AT INSOMNIAC GAMES. HE RECENTLY HELPED DESIGN MARVEL'S SPIDER-MAN: WEB SQUIRM, AND HAS SHIPPED A HANDFUL OF TITLES OVER HIS DECADE LONG TENURE IN THE VIDEO GAME INDUSTRY. HE'S WORKED AT LARGE STUDIOS LIKE GEN GAMES IN CREATIVE PROJECTS, AS WELL AS SMALLER INDEPENDENT TEAM TO DESIGN A GAME TO SHARE WITH THE WORLD. HE WILL BE SPEAKING AT A UNIFIED EVENT ON FEBRUARY 16TH WITH CREATORS TO DISCUSS DESIGN PRINCIPLES FOR ROBUST GAMES AS WELL AS A Q&A Session TO ANSWER ANY QUESTIONS ASPIRING DEVELOPERS MIGHT WANT TO ASK THE ANSWER TO.
ColorStack GB Meeting!

Chicago LL 102
February 20th
5:00 - 6:00 PM

See you there!
Where are we?
We’ve seen how lists are defined:

```plaintext
data List:
  | empty
  | link(first :: Any, rest :: List)
end
```

Self-reference
And, given this data definition, we can write functions that recursively process a list:

fun **list-fun**(lst :: List) -> ...:
    doc: "Template for a function that takes a List"
    cases (List) lst:
        | empty => ...
        | link(f, r) =>
            ... f ...
            ... list-fun(r) ...
    end
    where:
    list-fun(...) is ...
end
Every data definition has a corresponding template.

The more complex the data definition is – lots of variants, recursion, etc. – the more helpful it is to use the template!
Given a (recursive) data definition, you write a template by:

1. Creating a function header
2. Using `cases` to break the data input into its variants
3. In each case, listing each of the fields in the answer
4. Calling the function itself on any recursive fields
Warm-up practice
fun list-len(lst :: List) -> Number:
    doc: "Compute the length of a list"
    cases (List) lst:
        | empty => 0
        | link(f, r) => 1 + list-len____
    end
end
fun list-len(lst :: List) -> Number:
    doc: "Compute the length of a list"
    cases (List) lst:
        | empty => 0
        | link(f, r) => 1 + list-len(r)
fun list-product(lst :: List<Number>) -> Number:
   doc: "Compute the product of all the numbers in lst"
   cases (List) lst:
      | empty => 1
      | link(f, r) => ____ * list-product(r)
   end
end
fun list-product(lst :: List<Number>) -> Number:
    doc: "Compute the product of all the numbers in lst"
    cases (List) lst:
        | empty => 1
        | link(f, r) => f * list-product(r)
    end
end
fun is-member(item, lst :: List) -> Boolean:

doc: "Return true if item is a member of lst"

cases (List) lst:
    | empty => ______
    | link(f, r) =>
    | (f == ______) or is-member(______, ______)
end
end
fun is-member(item, lst :: List) -> Boolean:
  doc: "Return true if item is a member of lst"
  cases (List) lst:
    | empty => false
    | link(f, r) =>
      (f == item) or is-member(item, r)
  end
end
Rumor mills
EMMA:
A NOVEL.
IN THREE VOLUMES.

BY THE
AUTHOR OF "PRIDE AND PREJUDICE."
&c. &c.

VOL. I.

LONDON:
PRINTED FOR JOHN MURRAY.
1816.
The news [of *Emma and Mr. Knightley’s engagement*] was universally a surprize wherever it spread; and Mr. Weston had his five minutes share of it…

“It is to be a secret, I conclude,” said he. “*These matters are always a secret, till it is found out that every body knows them.* Only let me be told when I may speak out.—I wonder whether Jane has any suspicion.”

Jane Austen, *Emma*, 1815
He went to Highbury the next morning, and satisfied himself on that point. He told her the news… and Miss Bates being present, it passed, of course, to Mrs. Cole, Mrs. Perry, and Mrs. Elton, immediately afterwards. It was no more than the principals were prepared for; they had calculated from the time of its being known at Randalls, how soon it be over Highbury; and were thinking of themselves, as the evening wonder in many a family circle…

Jane Austen, *Emma*, 1815
Tracking rumors

Suppose we want to track gossip in this rumor mill.
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Tracking rumors

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I said yes!

Emma
Tracking rumors

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Suppose we want to track gossip in this rumor mill.

I said yes!

Emma

Mr Woodhouse
Suppose we want to track gossip in this rumor mill.

I said yes!

Emma

Mr Woodhouse

Mrs Weston
Tracking rumors

Suppose we want to track gossip in this rumor mill.

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Emma

Mr Woodhouse

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Mr Weston
Suppose we want to track gossip in this rumor mill.

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Suppose we want to track gossip in this rumor mill.

I said yes!
Tracking rumors

Suppose we want to track gossip in this rumor mill.

Emma

Mr Woodhouse

Mrs Weston

Mr Weston

Jane

Miss Bates

Mrs Cole

Mrs Perry

Mrs Elton

I said yes!
Suppose we want to track gossip in this rumor mill.

Simplifying assumption: Each person tells at most two others.
Suppose we want to track gossip in this rumor mill.

**Tracking rumors**

Simplifying assumption:
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Suppose we want to track gossip in this rumor mill.

Simplifying assumption: Each person tells at most two others.
The Jane Austen example is a bit frivolous, but otherwise this is an important problem.

A lot of research right now is focused on building models of how information – and misinformation! – spreads through social networks, both in person and online.
Representing rumor mills

Is a rumor mill simply a list of people?
Representing rumor mills

Is a rumor mill simply a list of people?
No, because there are relationships among the people.
Representing rumor mills

We could represent these relations with a table, e.g.,

<table>
<thead>
<tr>
<th>name :: String</th>
<th>next1 :: String</th>
<th>next2 :: String</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Emma&quot;</td>
<td>&quot;Mr Woodhouse&quot;</td>
<td>&quot;Mrs Weston&quot;</td>
</tr>
<tr>
<td>&quot;Mr Woodhouse&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
Representing rumor mills

Using a table doesn’t give us any straightforward way to process the rumor mill.

Could we use something like a list but representing the relations?
Representing rumor mills

data Person:
  | person(name :: String, next1 :: Person, next2 :: Person)
end

How about this?
Representing rumor mills

data Person:
  | person(name :: String, next1 :: Person, next2 :: Person)
end

Some people don’t gossip to anyone else – the red arrows above.
Representing rumor mills

data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end

How about this?
Example rumor mills

data RumorMill:
    | no-one
    | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end

no-one
Example rumor mills

```rust
data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end

gossip("Mrs Cole", no-one, no-one)
```
Example rumor mills

data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end

gossip("Miss Bates",
gossip("Mrs Cole", no-one, no-one)
gossip("Mrs Elton", no-one, no-one))
gossip("Emma",
gossip("Mr Woodhouse", no-one, no-one),
gossip("Mrs Weston",
gossip("Mr Weston",
gossip("Jane", no-one, no-one),
gossip("Miss Bates",
gossip("Mrs Cole", no-one, no-one),
gossip("Mrs Elton", no-one, no-one))),
no-one))
Example using names for parts:

\[
\begin{align*}
MRS\text{-}COLE\text{-}MILL & =\ \text{gossip}("Mrs\ Cole",\ \text{no-one},\ \text{no-one}) \\
MRS\text{-}ELTON\text{-}MILL & =\ \text{gossip}("Mrs\ Elton",\ \text{no-one},\ \text{no-one}) \\
MISS\text{-}BATES\text{-}MILL & =\ \text{gossip}("Miss\ Bates",\ MRS\text{-}COLE\text{-}MILL,\ MRS\text{-}ELTON\text{-}MILL) \\
JANE\text{-}MILL & =\ \text{gossip}("Jane",\ \text{no-one},\ \text{no-one}) \\
MR\text{-}WESTON\text{-}MILL & =\ \text{gossip}("Mr\ Weston",\ JANE\text{-}MILL,\ MISS\text{-}BATES\text{-}MILL) \\
MRS\text{-}WESTON\text{-}MILL & =\ \text{gossip}("Mrs\ Weston",\ MR\text{-}WESTON\text{-}MILL,\ \text{no-one}) \\
MR\text{-}WOODHOUSE\text{-}MILL & =\ \text{gossip}("Mr\ Woodhouse",\ \text{no-one},\ \text{no-one}) \\
EMMA\text{-}MILL & =\ \text{gossip}("Emma",\ MR\text{-}WOODHOUSE\text{-}MILL,\ MRS\text{-}WESTON\text{-}MILL)
\end{align*}
\]
A *RumorMill* is a type of structure called a *tree*.

Each element in the tree is called a *node*.

The first node in the tree is called the *root*.

A node with no children is called a *leaf*.

Like a list, a tree is recursive: Every subtree is a tree.
I said yes!
Draw it vertically and you can see it’s a tree!
Computer scientists are weird.
data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
Programming with rumors

data RumorMill:
    | no-one
    | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
Programming with rumors

data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end

Self-reference × 2
Programming with rumors

data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end

For each element, there’s not just one “next” element; there are two!
Programming with rumors

data RumorMill:
    | no-one
    | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end

#|
fun rumor-mill-fun(rm :: RumorMill) -> ...:
  doc: "Template for a function with a RumorMill as input"
  cases (RumorMill) rm:
    | no-one => ...
    | gossip(name, next1, next2) =>
      ... name
      ... rumor-mill-fun(next1)
      ... rumor-mill-fun(next2)
  end
end
|#
Programming with rumors

data RumorMill:
  | no-one
  | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end

#|
fun rumor-mill-fun(rm :: RumorMill) -> ...:
doc: "Template for a function with a RumorMill as input"
cases (RumorMill) rm:
  | no-one => ...
  | gossip(name, next1, next2) =>
      ... name
      ... rumor-mill-fun(next1)
      ... rumor-mill-fun(next2)
end
end
|#

Self-reference × 2

Natural recursion × 2
Starter file:

tinyurl.com/101-2024-02-15-starter
Rumor program examples

Design the function \texttt{is-informed} that takes a person’s name and a rumor mill and determines whether the person is part of the rumor mill.
Rumor program examples

Design the function `gossip-length` that takes a rumor mill and determines the length of the longest sequence of people transmitting the rumor.
Rumor program examples

Design the function \texttt{add-gossip} that takes a rumor mill and two names – one new and one old – and adds the new person to the rumor mill, receiving rumors from the old person. (You can assume the old person does not already have two next persons!)
Solutions:

tinyurl.com/101-2024-02-15
Acknowledgments

This lecture incorporates material from:

Jane Austen, *Emma*
Tom Ellman, Vassar College
Marc Smith, Vassar College