# CMPU 101 § $53 \cdot$ Computer Science I 

## Trees

15 February 2024


NLECS ASPAEV LEETUHE SEATISS:


## Colorstack GB Meeting

Chicago LL 102
February 20th
5:00-6:00 PM

See you there!

Where are we?

We've seen how lists are defined:


And, given this data definition, we can write functions that recursively process a list:

```
fun list-fun(lst : List) -> ... :
    doc: "Template for a function that takes a List"
    cases (List) lst:
    | empty => ... Recursive call
    | link(f, r) =>
            ... f ...
            ... list-fun(r) ...
    end
where:
    list-fun(...) is ...
end
```

Every data definition has a corresponding template.
The more complex the data definition is - lots of variants, recursion, etc. - the more helpful it is to use the template!

Given a (recursive) data definition, you write a template by:

1 Creating a function header
2 Using cases to break the data input into its variants
3 In each case, listing each of the fields in the answer
4 Calling the function itself on any recursive fields

Warm-up practice

```
fun list-len(lst :: List) -> Number:
    doc: "Compute the length of a list"
    cases (List) lst:
        | empty => 0
        | link(f, r) => 1 + list-len(____)
    end
end
```

```
fun list-len(lst :: List) -> Number:
    doc: "Compute the length of a list"
    cases (List) lst:
        | empty => 0
        | link(f, r) => 1 + list-len(r)
    end
end
```

```
fun list-product(lst :: List<Number>) -> Number:
    doc: "Compute the product of all the numbers in lst"
    cases (List) lst:
        | empty => 1
        | link(f, r) => ___ * list-product(r)
    end
end
```

```
fun list-product(lst :: List<Number>) -> Number:
    doc: "Compute the product of all the numbers in lst"
    cases (List) lst:
        | empty => 1
        | link(f, r) => f * list-product(r)
    end
end
```

fun is-member (item, lst :: List) -> Boolean:
doc: "Return true if item is a member of lst"
cases (List) lst:
| empty =>
| link(f, r) =>
( $\mathrm{f}==$ _ ) or is-member (____ $\qquad$
end
end
fun is-member (item, lst :: List) -> Boolean:
doc: "Return true if item is a member of lst"
cases (List) lst:
| empty => false
| link(f, r) =>
(f == item) or is-member(item, r)

## end

end

Rumor mills

## EMMA:

A NOVEL.
in three volumes
$\qquad$
${ }_{\text {by the }}$
AUTHOR OF " PRIDE AND PREJUDICE,"
sc. 昏c.
$\qquad$

VOL. I.
$\qquad$

LONDON:
PRINTED FOR JOHN MURRAY. $\overline{1816 .}$

The news [of Emma and Mr. Knightley's engagement] was universally a surprize wherever it spread; and Mr. Weston had his five minutes share of it...
"It is to be a secret, I conclude," said he. "These every body knows them. Only let me be told when I may speak out.—I wonder whether Jane has any suspicion."

He went to Highbury the next morning, and satisfied himself on that point. He told her the news... and Miss Bates being present, it passed, of course, to Mrs. Cole, Mrs. Perry, and Mrs. Elton, immediately afterwards. It was no more than the principals were prepared for; they had calculated from the time of its being known at Randalls, how soon it be over Highbury; and were thinking of themselves, as the evening wonder in many a family circle...

## Tracking rumors

Suppose we want to track gossip in this rumor mill.

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Suppose we want to track gossip in this rumor mill.


The Jane Austen example is a bit frivolous, but otherwise this is an important problem.

A lot of research right now is focused on building models of how information - and misinformation! spreads through social networks, both in person and online.

## Representing rumor mills



Is a rumor mill simply a list of people?

## Representing rumor mills



Is a rumor mill simply a list of people?
No, because there are relationships among the people.

## Representing rumor mills



We could represent these relations with a table, e.g.,

| name :: String | next1 :: String | next2 :: String |
| :--- | :--- | :--- |
| "Emma" | "Mr Woodhouse" | "Mrs Weston" |
| "Mr Woodhouse" |  |  |
| $\ldots$ | $\ldots$ | $\ldots$ |

## Representing rumor mills



Using a table doesn't give us any straightforward way to process the rumor mill.

Could we use something like a list but representing the relations?

## Representing rumor mills



## data Person:

| person(name :: String, next1 :: Person, next2 :: Person)

```
How about this?
```


## Representing rumor mills



## data Person:

| person(name :: String, next1 : Person, next2 : Person

## Representing rumor mills



## data RumorMill:

| no-one
| gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)

## Example rumor mills

```
data RumorMill:
    | no-one
    | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
```

no-one

## Example rumor mills

data RumorMill:
| no-one
| gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill) end

```
gossip("Mrs Cole", no-one, no-one)
```



## Example rumor mills

data RumorMill:
| no-one
| gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill) end

```
gossip("Miss Bates",
    gossip("Mrs Cole", no-one, no-one)
    gossip("Mrs Elton", no-one, no-one))
```



```
gossip("Emma",
    gossip("Mr Woodhouse", no-one, no-one),
    gossip("Mrs Weston",
        gossip("Mr Weston",
            gossip("Jane", no-one, no-one),
            gossip("Miss Bates",
            gossip("Mrs Cole", no-one, no-one),
            gossip("Mrs Elton", no-one, no-one))),
    no-one))
```



## Example using names for parts:

```
MRS-COLE-MILL = gossip("Mrs Cole", no-one, no-one)
MRS-ELTON-MILL = gossip("Mrs Elton", no-one, no-one)
MISS-BATES-MILL = gossip("Miss Bates", MRS-COLE-MILL, MRS-ELTON-MILL)
JANE-MILL = gossip("Jane", no-one, no-one)
MR-WESTON-MILL = gossip("Mr Weston", JANE-MILL, MISS-BATES-MILL)
MRS-WESTON-MILL = gossip("Mrs Weston", MR-WESTON-MILL, no-one)
MR-WOODHOUSE-MILL = gossip("Mr Woodhouse", no-one, no-one)
EMMA-MILL = gossip("Emma", MR-WOODHOUSE-MILL, MRS-WESTON-MILL)
```


## A RumorMill is a type of structure called a tree.

Each element in the tree is called a node.
The first node in the tree is called the root.
A node with no children is called a leaf.
Like a list, a tree is recursive: Every subtree is a tree.



Draw it vertically and you can see it's a tree!
Root


Computer scientists are weird.


## Programming with rumors

```
data RumorMill:
    | no-one
    | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
```


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## Programming with rumors

data RumorMill:
| no-one
| gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill) end

For each element, there's not just one "next" element; there are two!

```
Programming with rumors
data RumorMill:
    | no-one
    | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
#|
fun rumor-mill-fun(rm :: RumorMill) -> ...:
    doc: "Template for a function with a RumorMill as input"
    cases (RumorMill) rm:
        | no-one =>
        / gossip(name, next1, next2) =>
            ... name
            ... rumor-mill-fun(next1)
            ... rumor-mill-fun(next2)
    end
end
|#
```


## Programming with rumors

```
data RumorMill:
    | no-one
    | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end
#|
fun rumor-mill-fun(rm %RumorMill) -> ..."
    doc:" "Template for a function with a RumorMill as input"
    cases (RumorMill) rm.
        | no-one => ... Natural recursion }\times
        / gossip(name, next1, next2) =>
            ... name
                ...rumor-mill-fun(nexf1)
            ... rumor-mill-fun(next2)
    end
end
|#
```

Starter file:
tinyurl.com/101-2024-02-15-starter

## Rumor program examples

Design the function is-informed that takes a
person's name and a rumor mill and determines
whether the person is part of the rumor mill.

## Rumor program examples

Design the function gossip-length that takes a
rumor mill and determines the length of the longest
sequence of people transmitting the rumor.

## Rumor program examples

Design the function add-gossip that takes a rumor mill and two names - one new and one old - and adds the new person to the rumor mill, receiving rumors from the old person. (You can assume the old person does not already have two next persons!)

Solutions:
tinyurl.com/101-2024-02-15

## Acknowledgments

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Jane Austen, Emma

Tom Ellman, Vassar College<br>Marc Smith, Vassar College



