

CMPU 101 § 53 · Computer Science I

## Trees

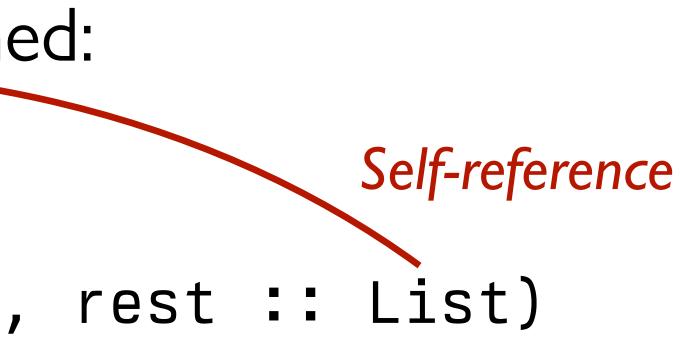
15 February 2024





# Where are we?

#### We've seen how lists are defined: data List: empty link(first :: Any, rest :: List) end



And, given this data definition, we can write functions that recursively process a list: fun list-fun(lst : List) -> ...:
 doc: "Template for a function that takes a List" cases (List) lst: empty => ... Recursive call | link(f, r) =>
... f ... ... list-fun(r) ... end where: list-fun(...) is ... end

Every data definition has a corresponding template. The more complex the data definition is – lots of variants, recursion, etc. – the more helpful it is to use the template!

## Given a (recursive) data definition, you write a template by:

- 1 Creating a function header
- 2 Using cases to break the data input into its variants
- 3 In each case, listing each of the fields in the answer
- 4 Calling the function itself on any recursive fields

# Warm-up practice

fun list-len(lst :: List) -> Number:
 doc: "Compute the length of a list"
 cases (List) lst:
 l empty => 0
 l link(f, r) => 1 + list-len(\_\_\_)
 end
end

fun list-len(lst :: List) -> Number:
 doc: "Compute the length of a list"
 cases (List) lst:
 l empty => 0
 l link(f, r) => 1 + list-len(r)
 end
end

```
fun list-product(lst :: List<Number>) -> Number:
  cases (List) lst:
    | empty => 1
    | link(f, r) => ____ * list-product(r)
  end
end
```

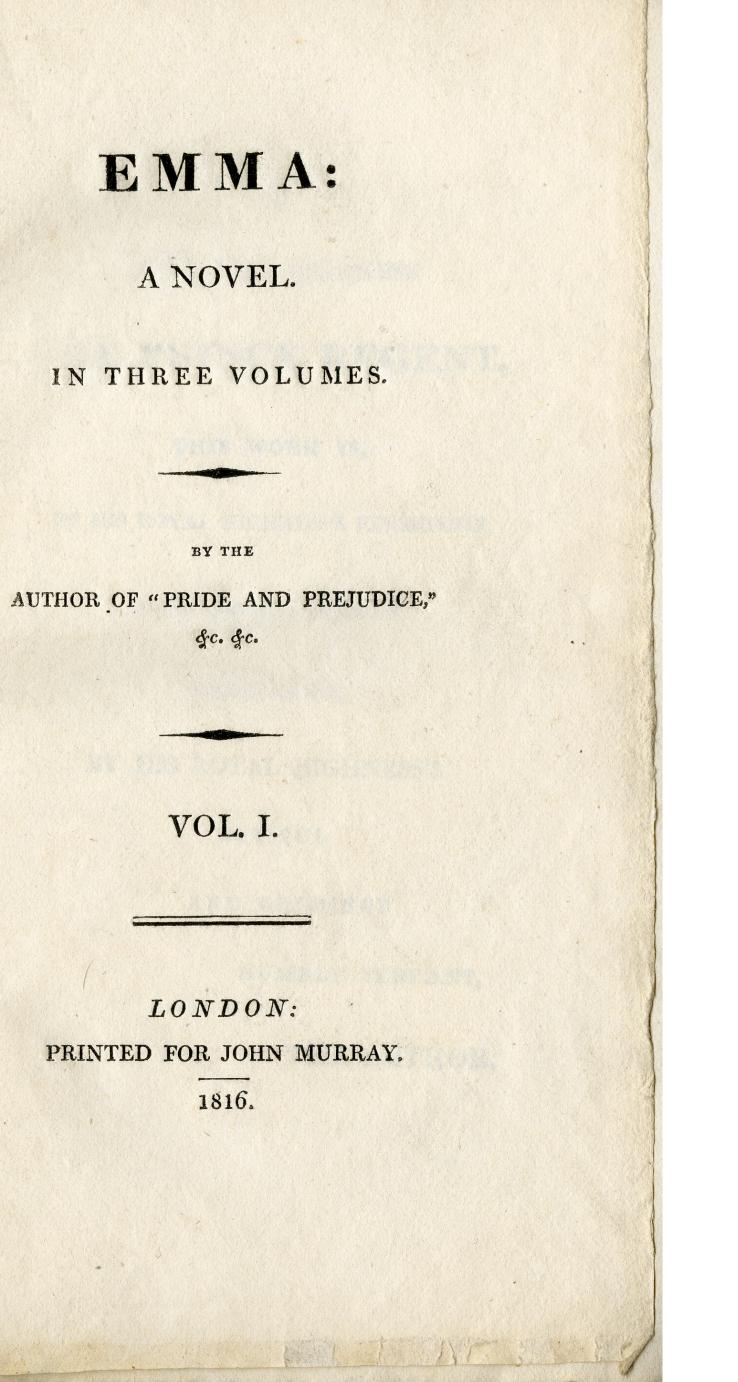
doc: "Compute the product of all the numbers in lst"

fun list-product(lst :: List<Number>) -> Number: doc: "Compute the product of all the numbers in lst" cases (List) lst: [ empty => 1 [ link(f, r) => f \* list-product(r) end end

fun is-member(item, lst :: List) -> Boolean: doc: "Return true if item is a member of lst" cases (List) lst: | empty => \_\_\_\_\_ | link(f, r) => (f == \_\_\_\_\_) or is-member(\_\_\_\_\_, \_\_\_\_ end end

fun is-member(item, lst :: List) -> Boolean: doc: "Return true if item is a member of lst" cases (List) lst: | empty => false | link(f, r) => (f == item) or is-member(item, r) end end

# Rumor mills



The news [of Emma and Mr. Knightley's engagement] was universally a surprize wherever it spread; and Mr. Weston had his five minutes share of it...

"It is to be a secret, I conclude," said he. "These matters are always a secret, till it is found out that every body knows them. Only let me be told when I may speak out.—I wonder whether Jane has any suspicion."

Jane Austen, Emma, 1815



He went to Highbury the next morning, and satisfied himself on that point. He told her the news... and Miss Bates being present, it passed, of course, to Mrs. Cole, Mrs. Perry, and Mrs. Elton, immediately afterwards. It was no more than the principals were prepared for; they had calculated from the time of its being known at Randalls, how soon it be over Highbury; and were thinking of themselves, as the evening wonder in many a family circle...

Jane Austen, Emma, 1815



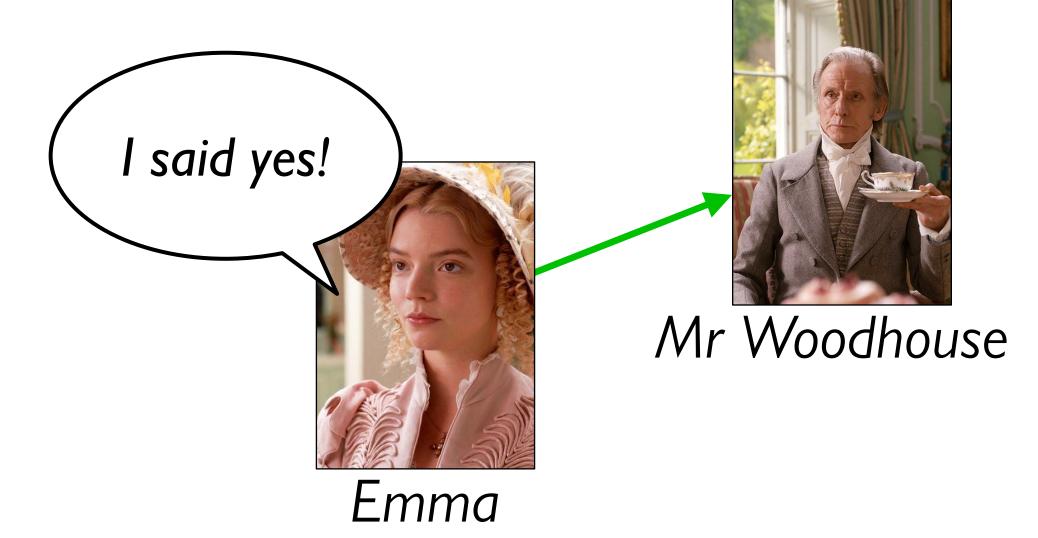




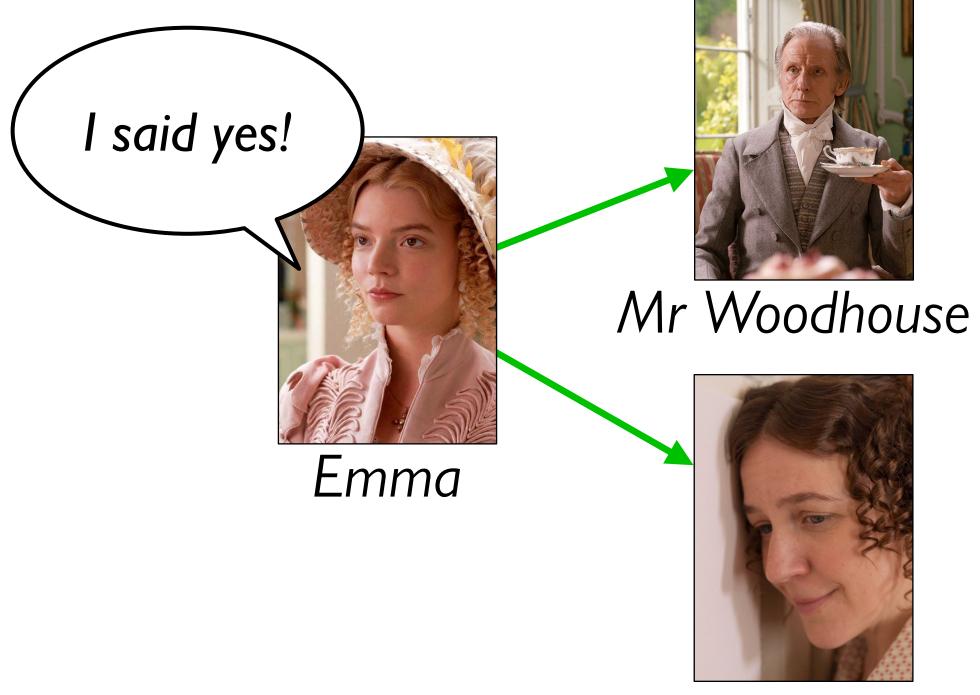






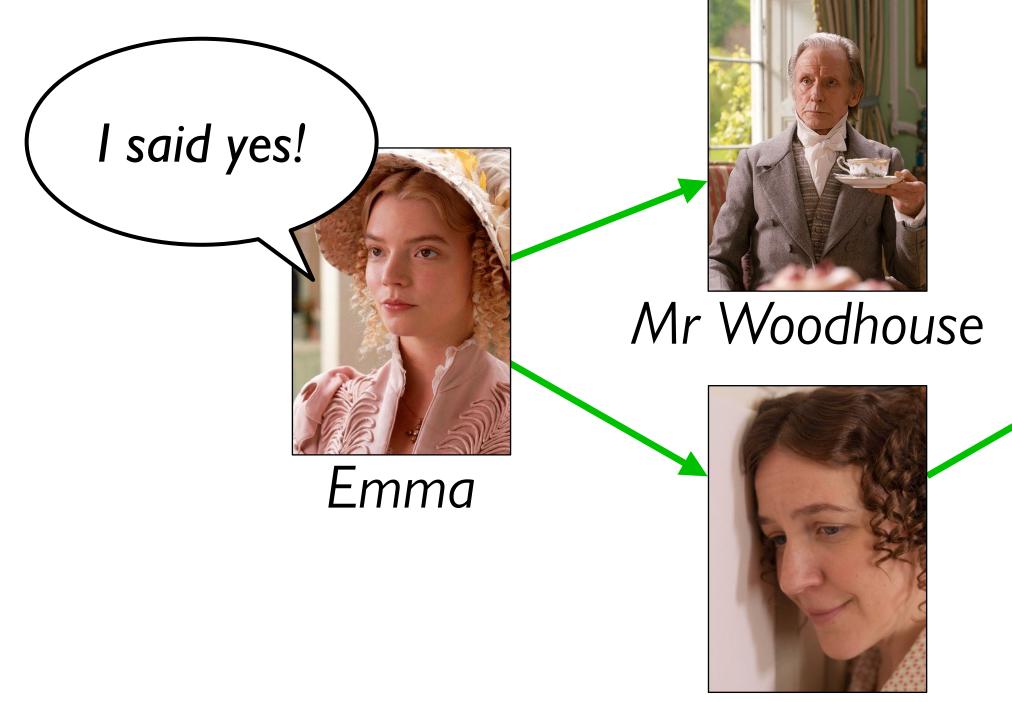


### Suppose we want to track gossip in this rumor mill.



Mrs Weston

#### Suppose we want to track gossip in this rumor mill.

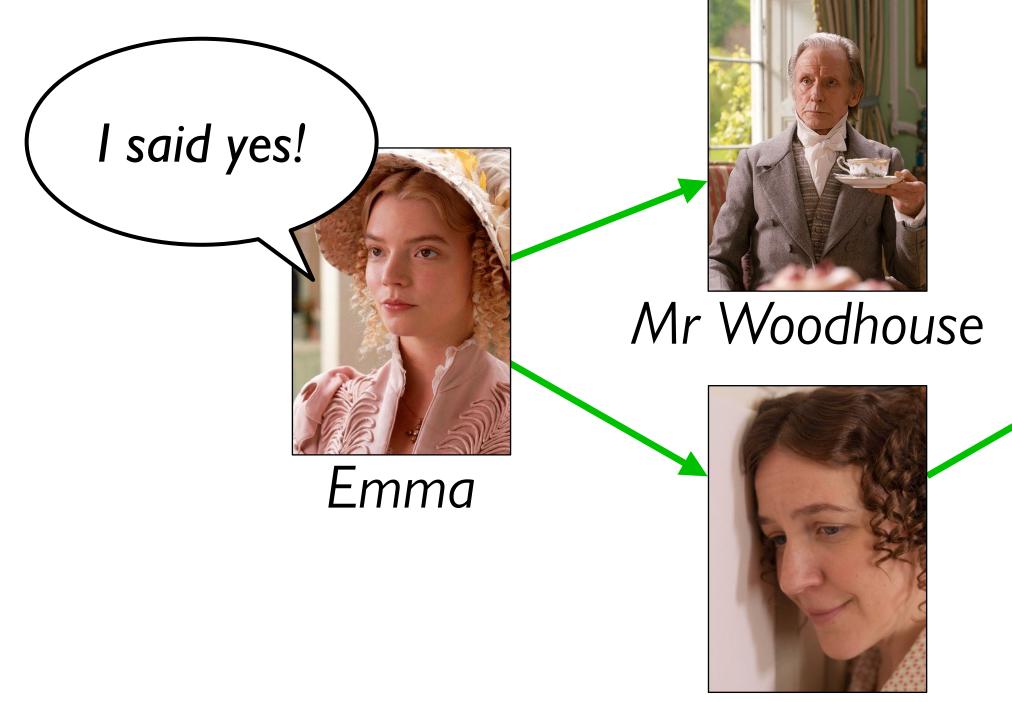


Mrs Weston

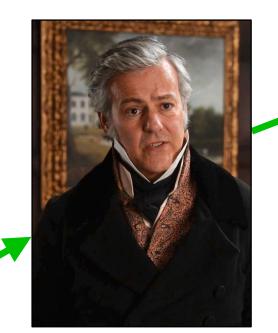


Mr Weston

## Suppose we want to track gossip in this rumor mill.



Mrs Weston

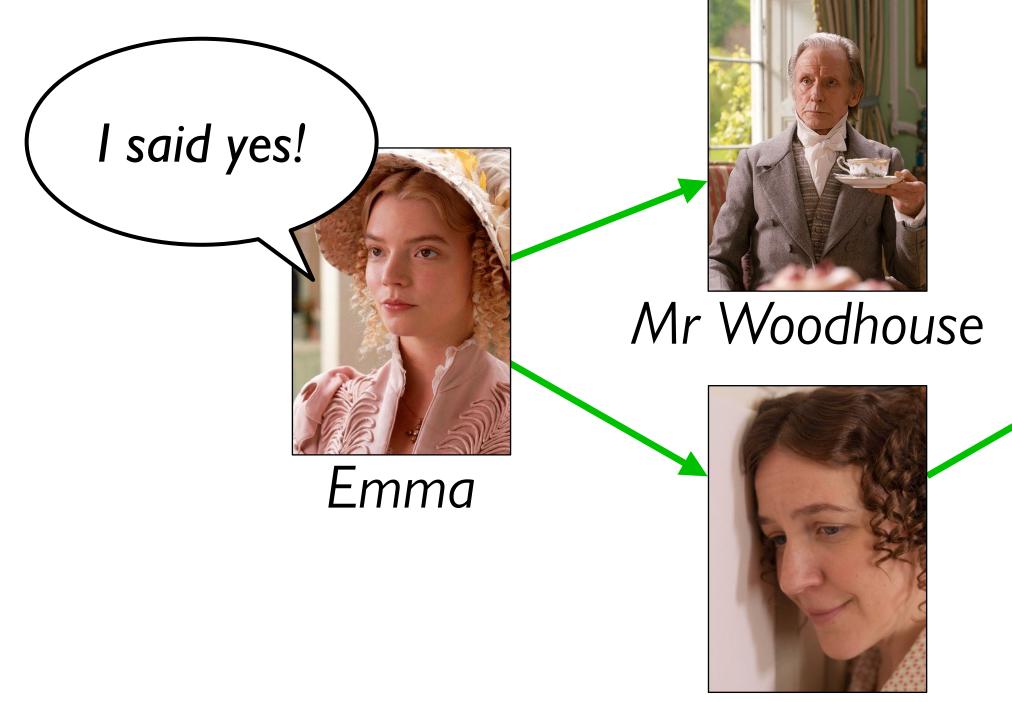




Jane

Mr Weston

## Suppose we want to track gossip in this rumor mill.



Mrs Weston



Mr Weston

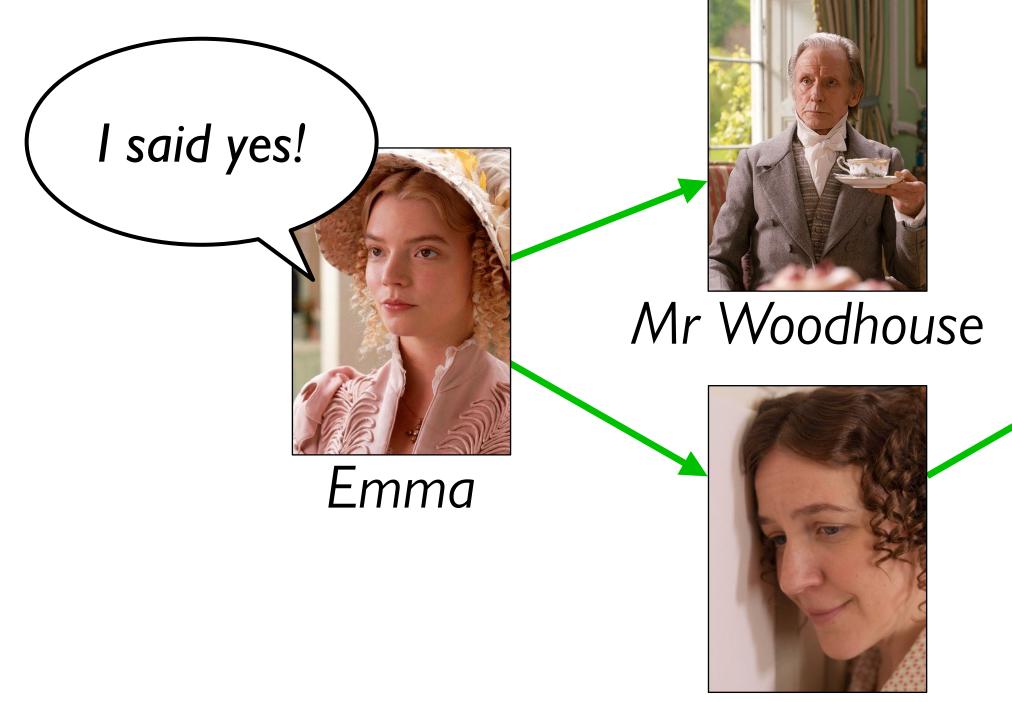


Jane

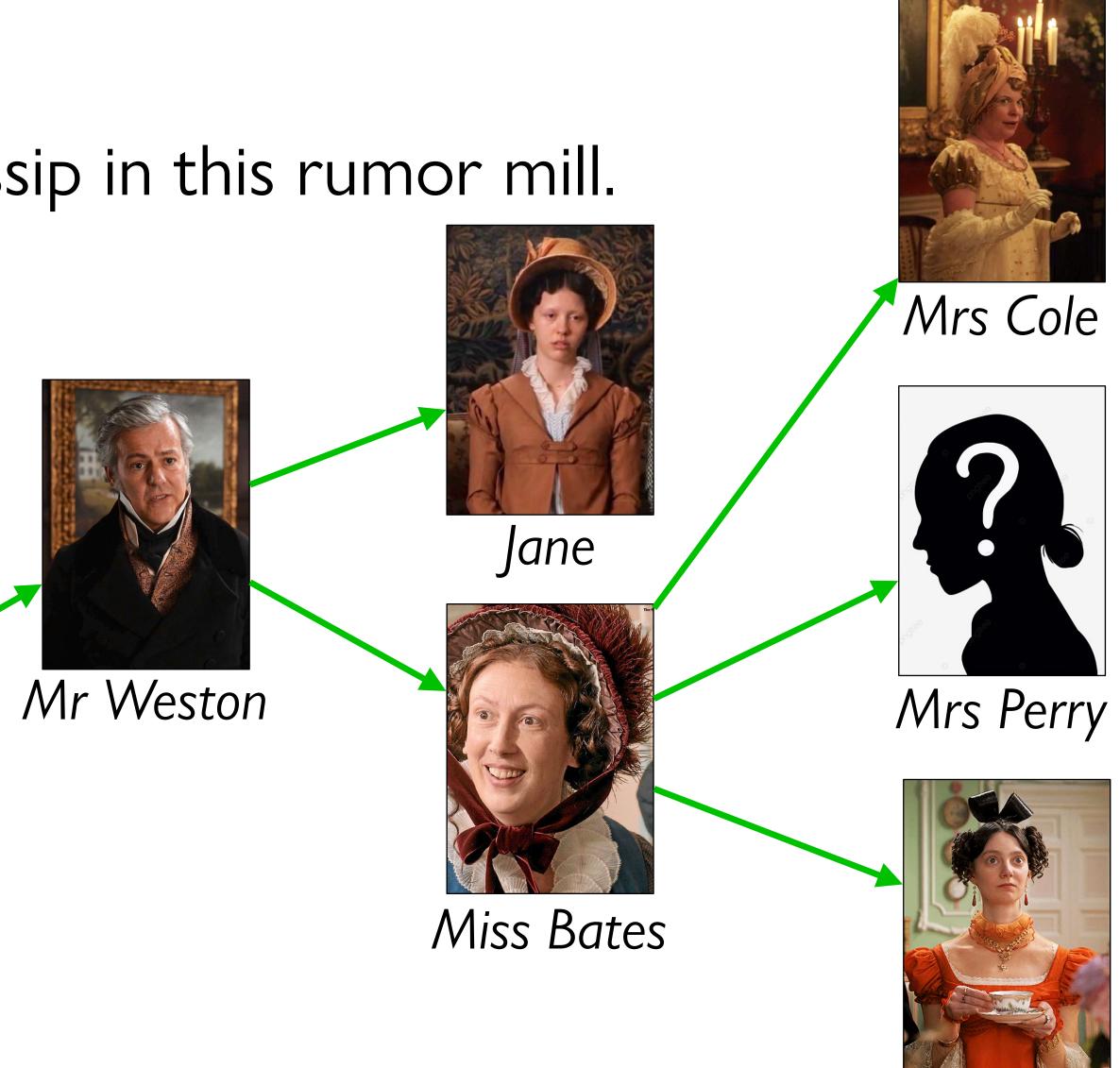


Miss Bates

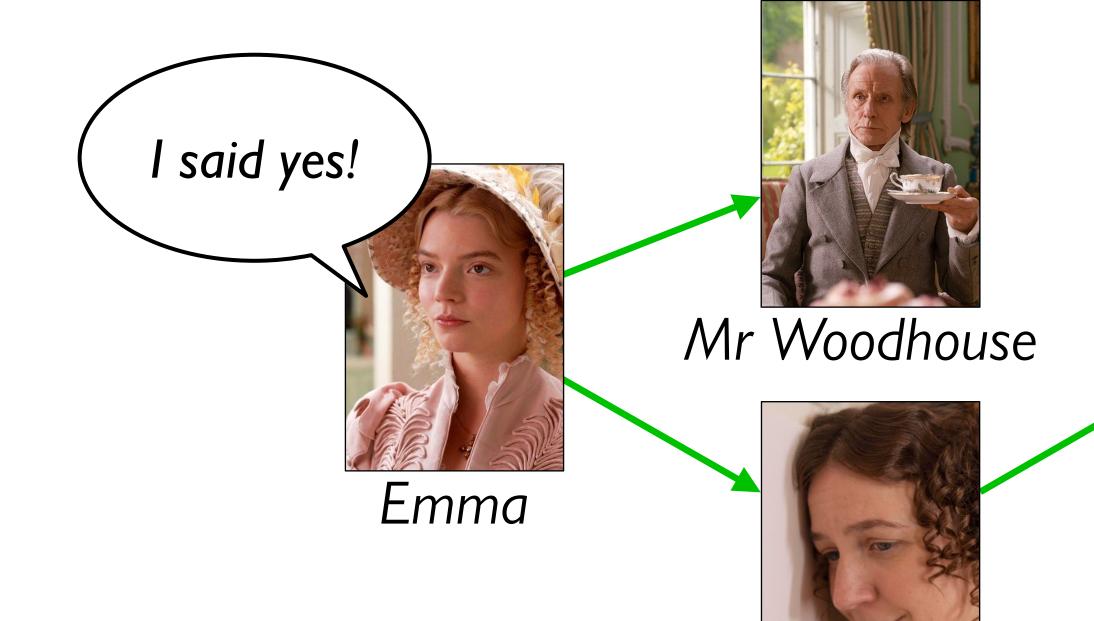
#### Suppose we want to track gossip in this rumor mill.



Mrs Weston

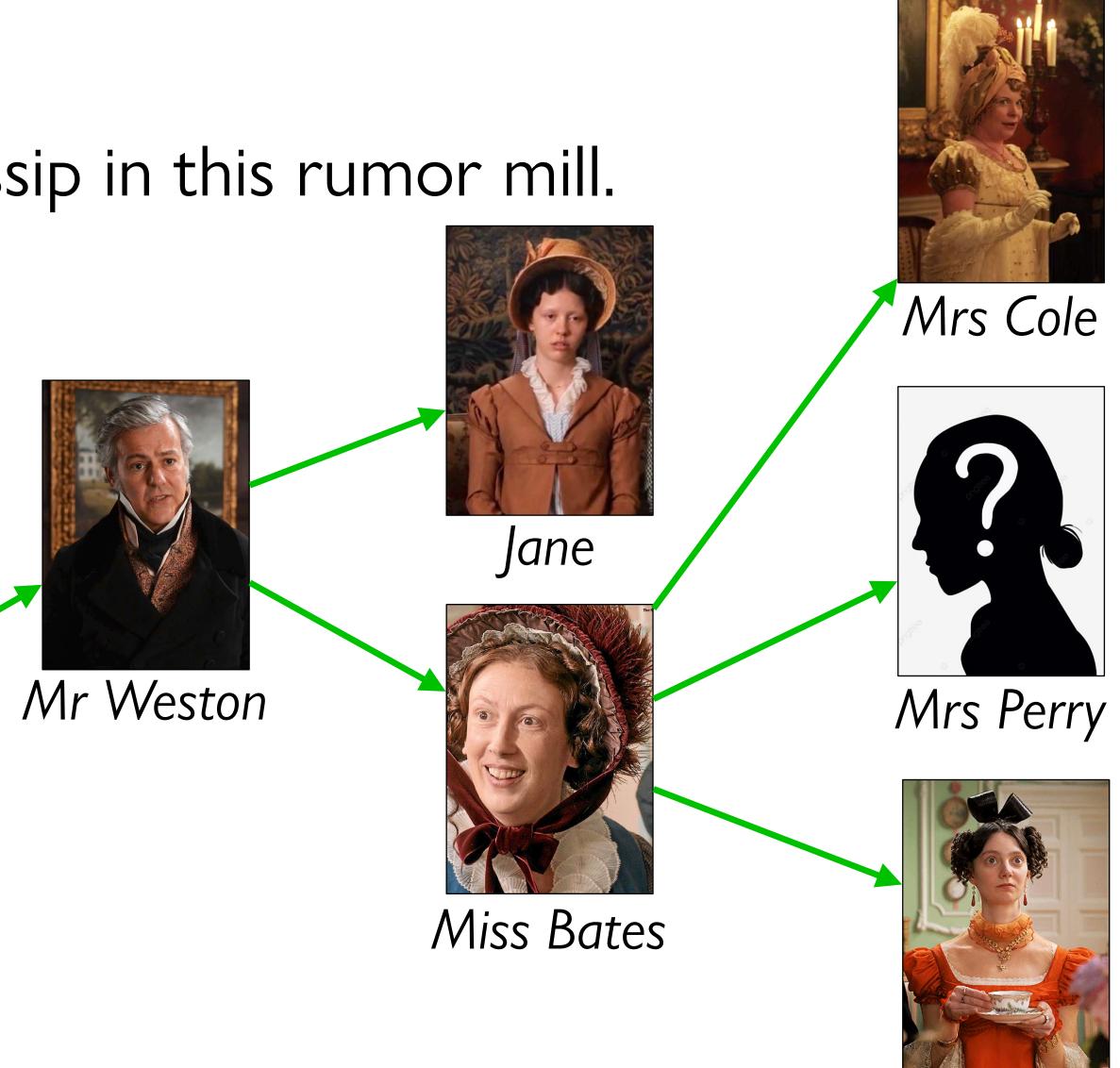


#### Suppose we want to track gossip in this rumor mill.

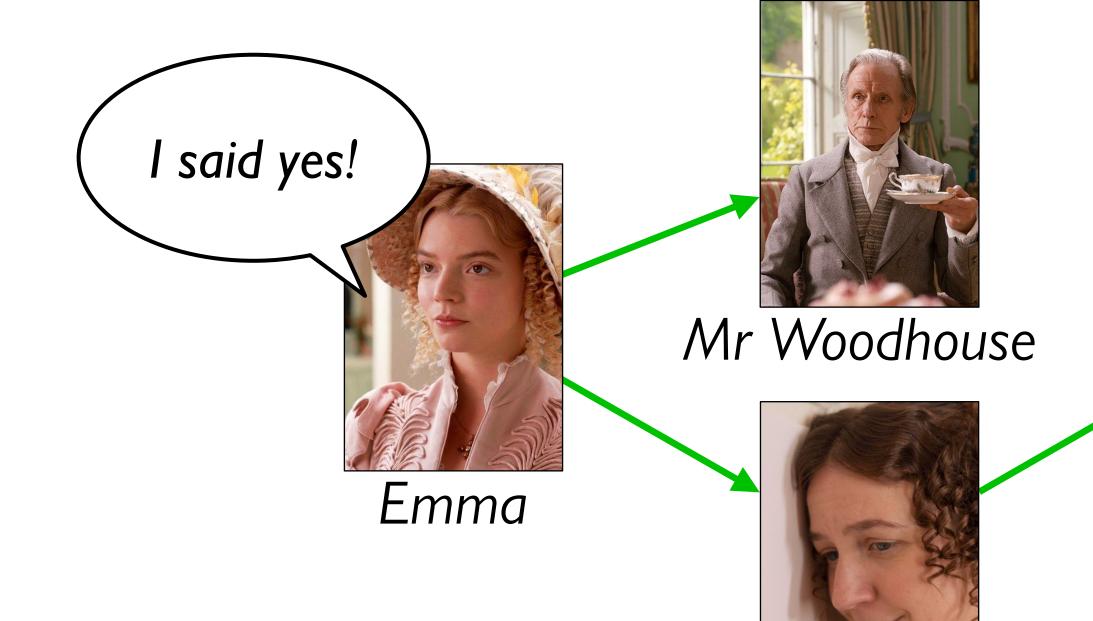


Simplifying assumption: Each person tells at most two others

Mrs Weston

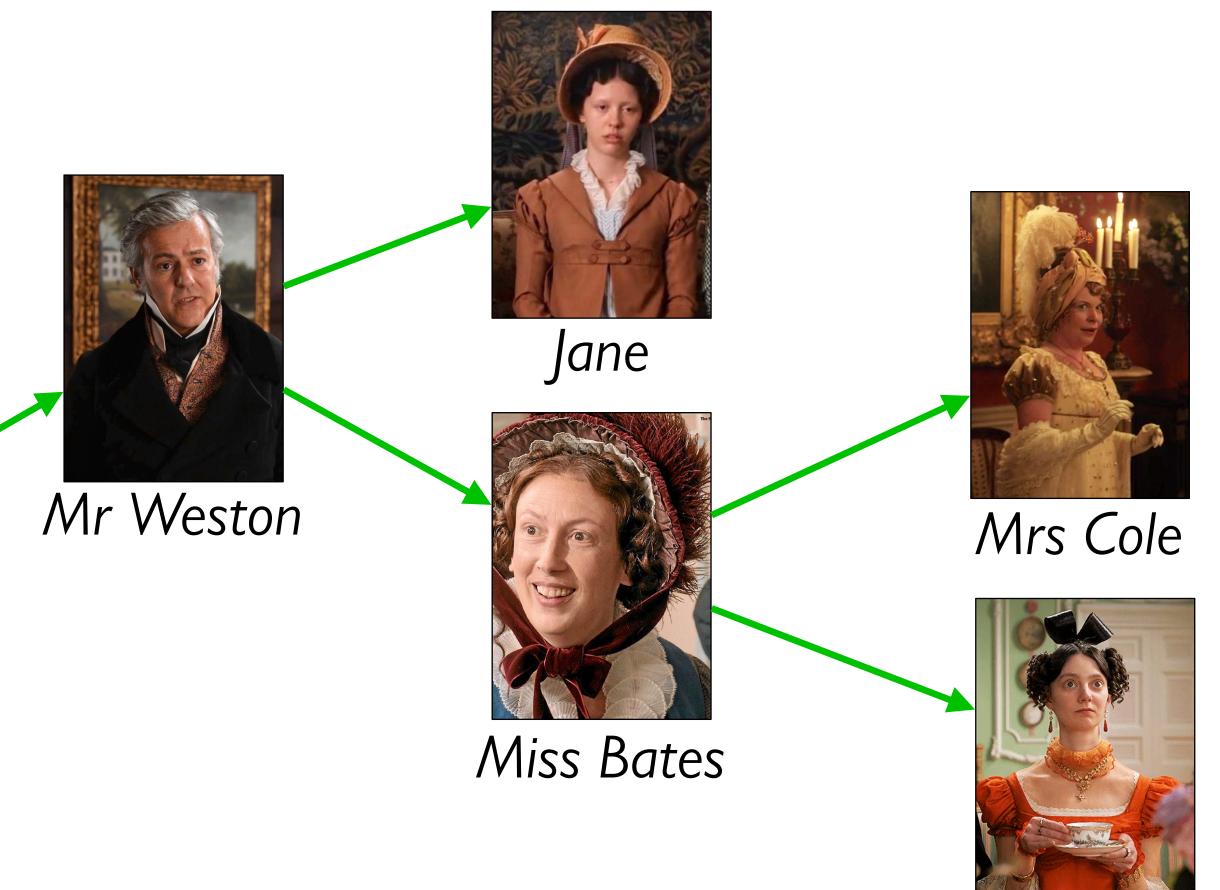


## Suppose we want to track gossip in this rumor mill.

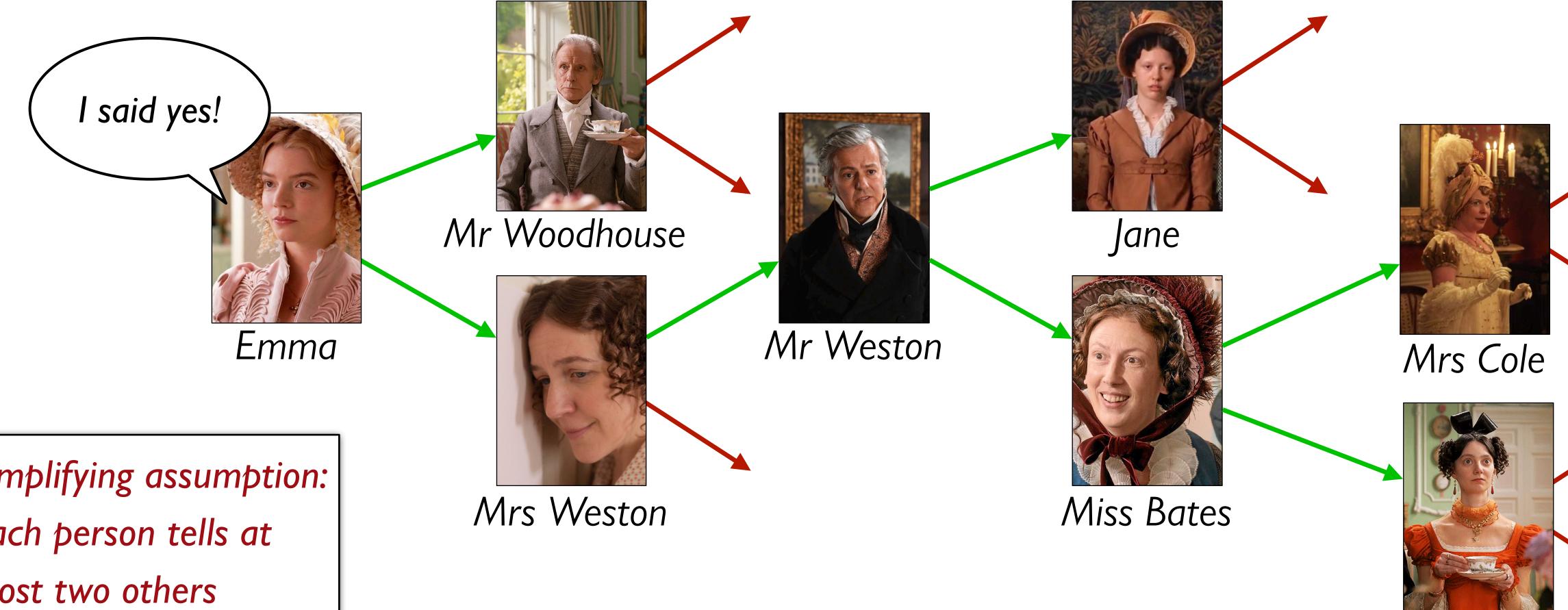


Simplifying assumption: Each person tells at most two others

Mrs Weston



### Suppose we want to track gossip in this rumor mill.



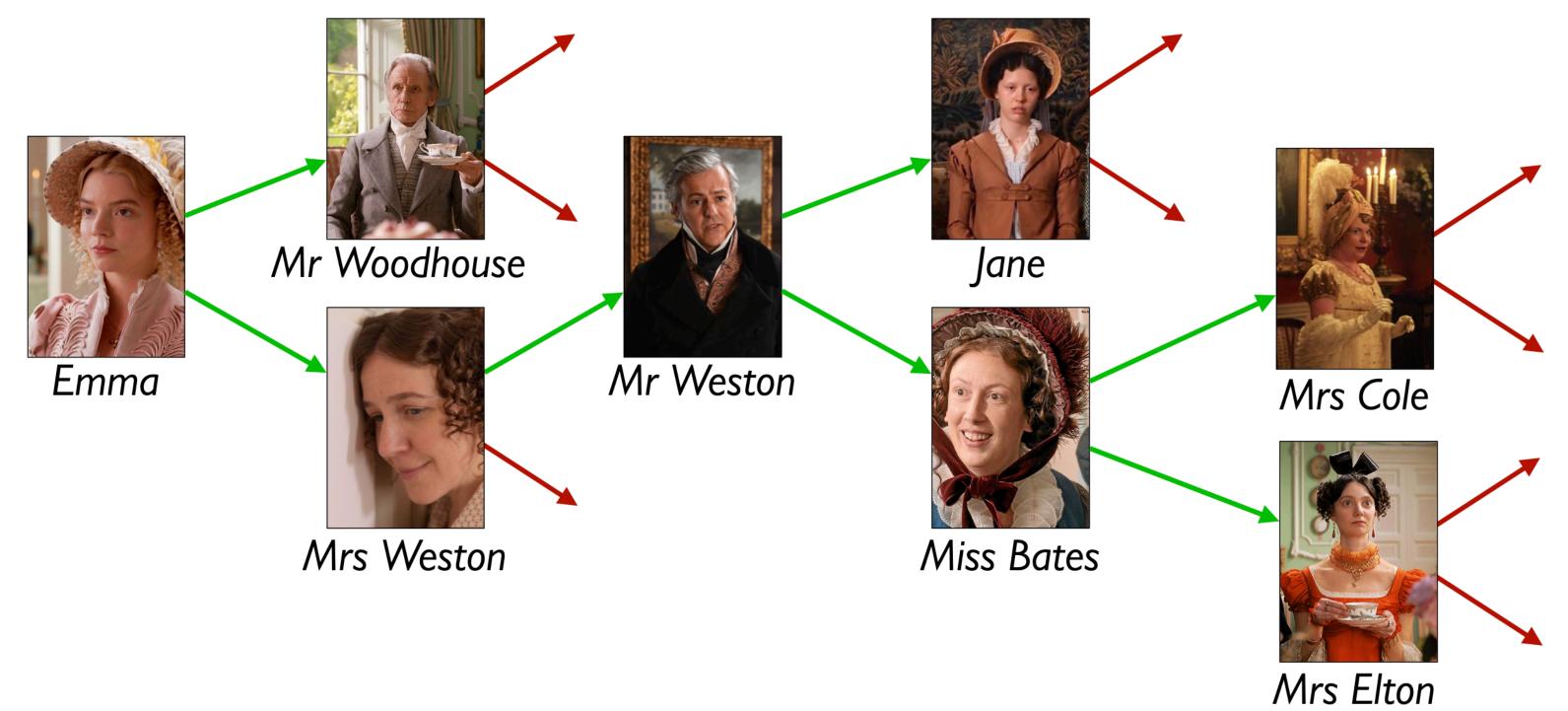
Simplifying assumption: Each person tells at most two others



The Jane Austen example is a bit frivolous, but otherwise this is an important problem.

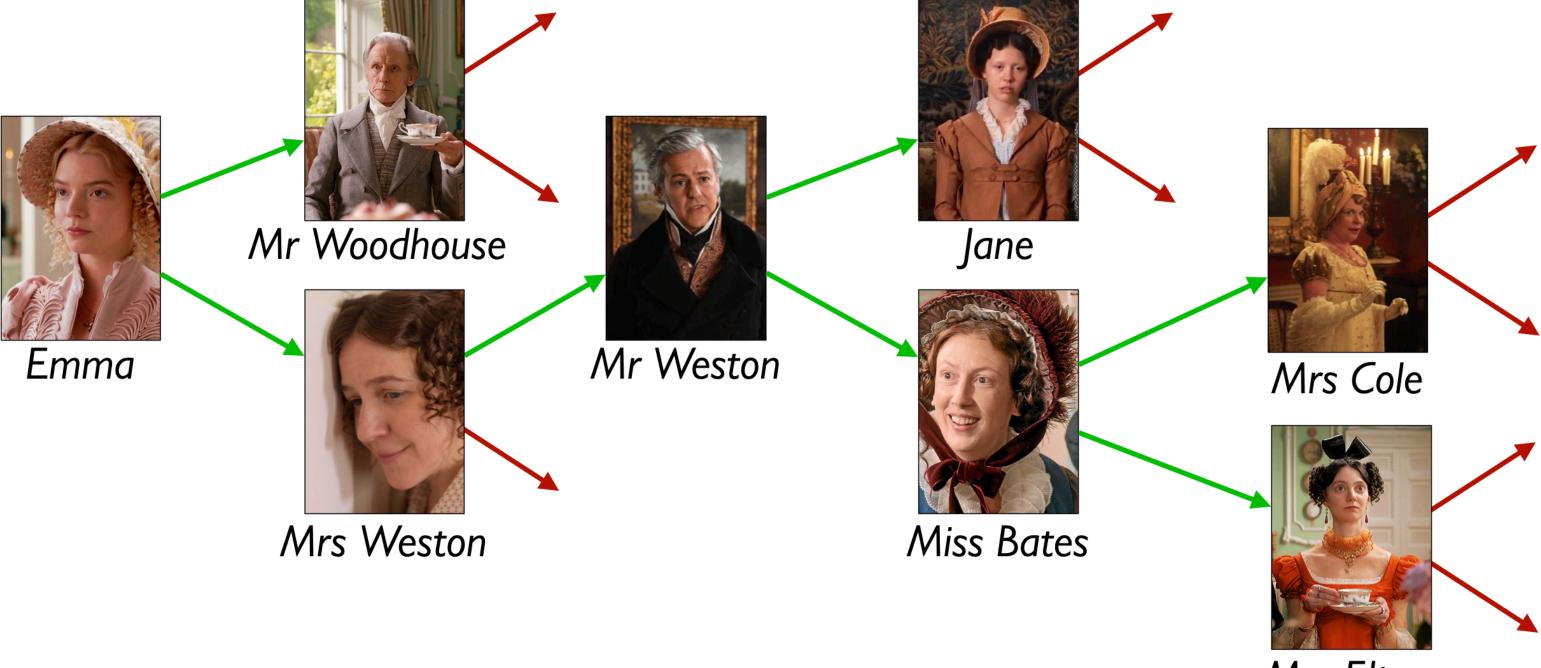
A lot of research right now is focused on building models of how information – and misinformation! – spreads through social networks, both in person and online.

# Representing rumor mills



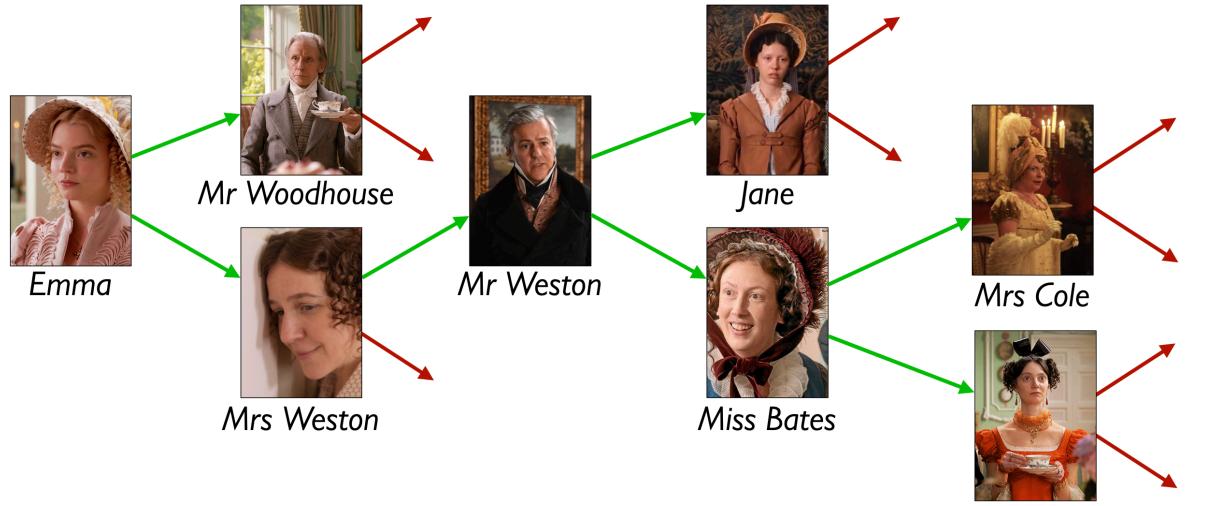
Is a rumor mill simply a list of people?

# Representing rumor mills



# among the people.

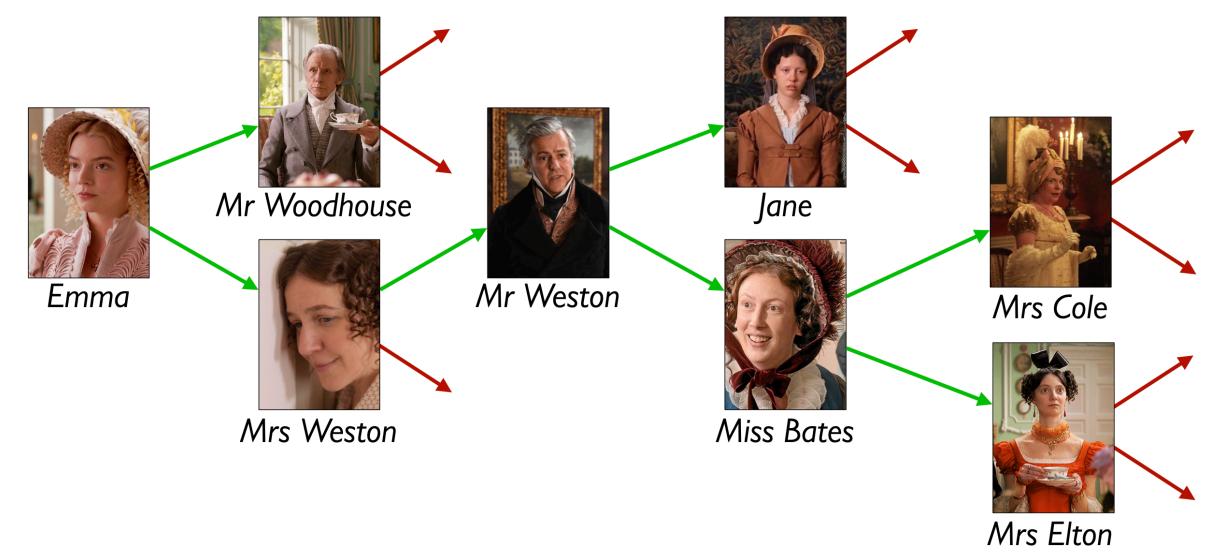
- Is a rumor mill simply a list of people?
- No, because there are relationships



## We could represent these relations with a table, e.g.,

name :: String	next1 :: String	next2 :: String
"Emma"	"Mr Woodhouse"	"Mrs Weston"
"Mr Woodhouse"		

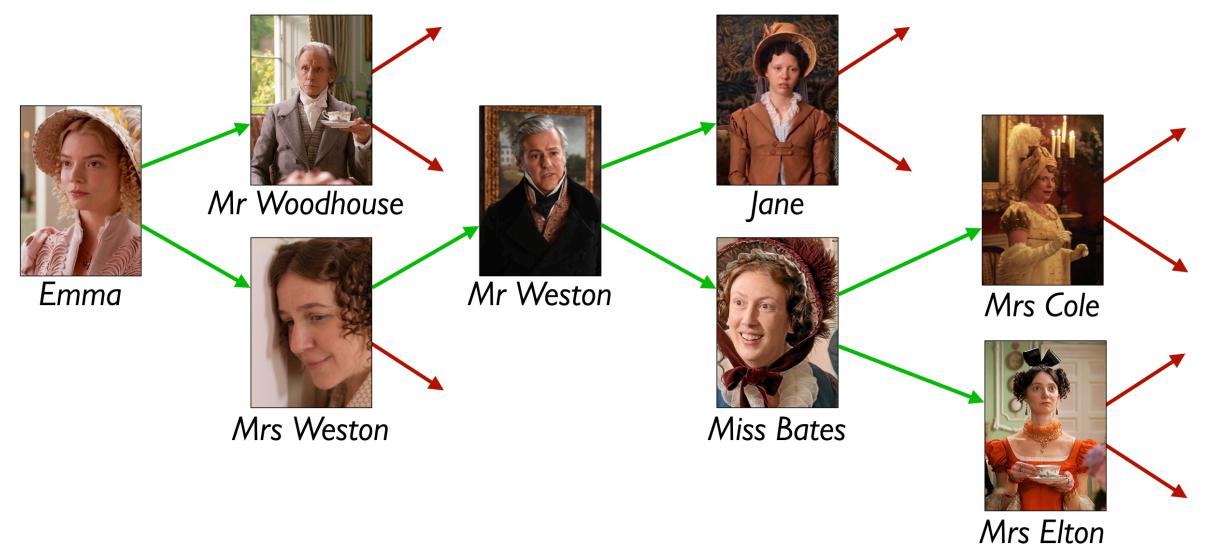
Mrs Elton



way to process the rumor mill.

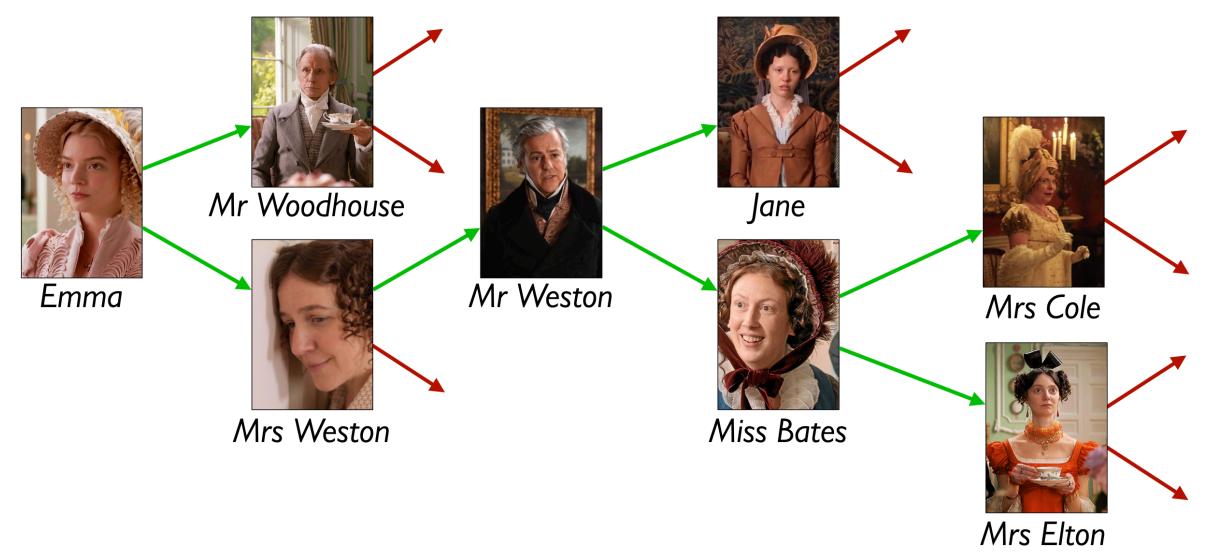
the relations?

- Using a table doesn't give us any straightforward
- Could we use something *like* a list but representing



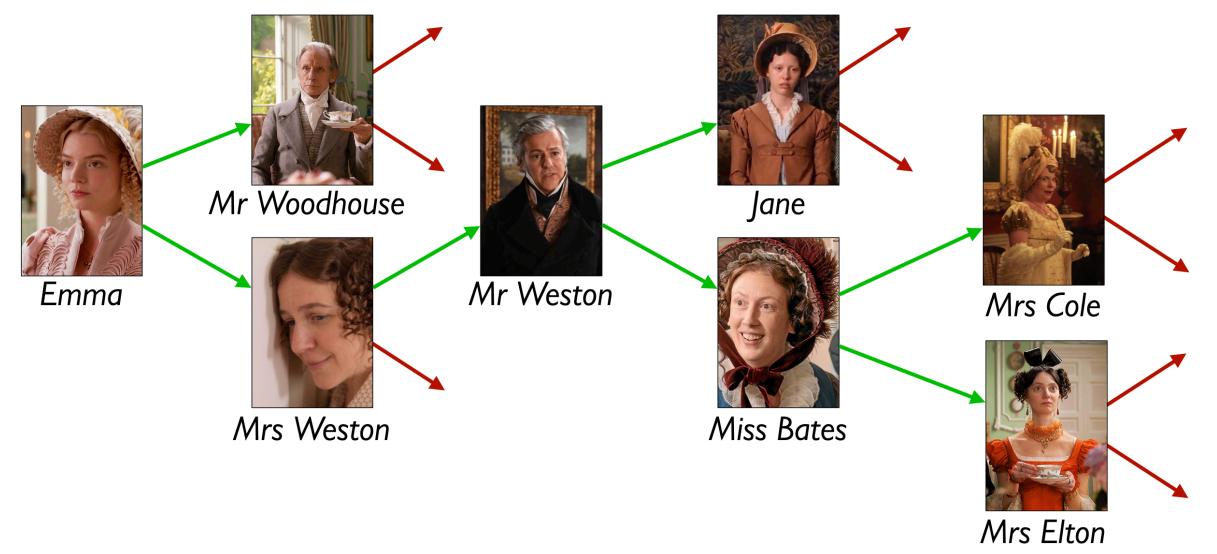
## data Person: | person(name :: String, next1 :: Person, next2 :: Person) end

How about this?



## data **Person**: | person(name :: String, ne end

Some people don't gossip to



### data RumorMill: no-one | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill) end

How about this?

## Example rumor mills

data RumorMill:
 | no-one
 | gossip(name :: String, next
end

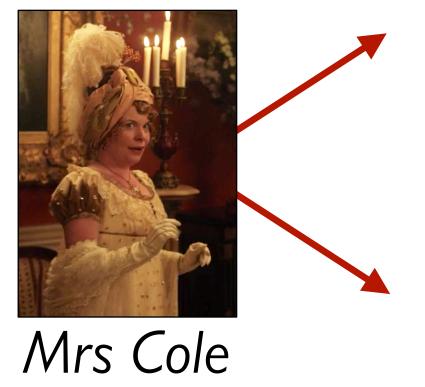
#### | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)

no-one

## Example rumor mills

data **RumorMill:** no-one end

#### gossip("Mrs Cole", no-one, no-one)



#### gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)

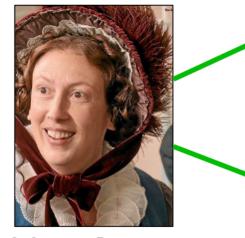
## Example rumor mills

data **RumorMill**:

no-one

end

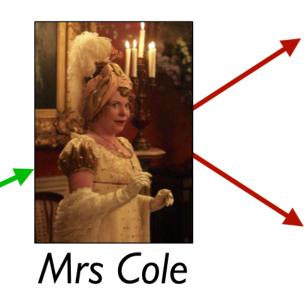
gossip("Miss Bates",

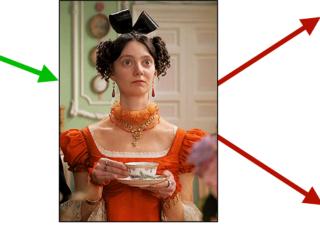


Miss Bates

#### gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)

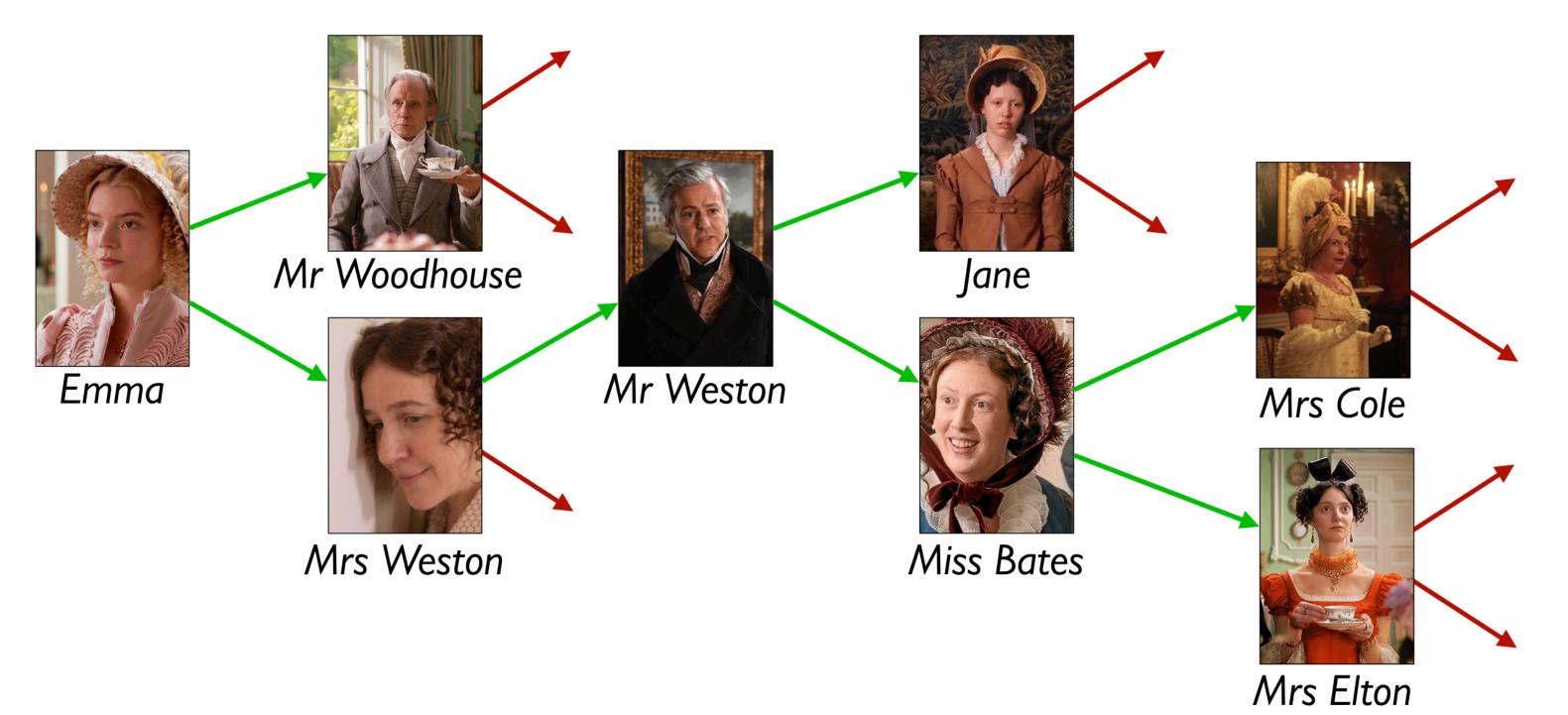
```
gossip("Mrs Cole", no-one, no-one)
gossip("Mrs Elton", no-one, no-one))
```





Mrs Elton

gossip("Emma", gossip("Mr Woodhouse", no-one, no-one), gossip("Mrs Weston", gossip("Mr Weston", gossip("Jane", no-one, no-one), gossip("Miss Bates", gossip("Mrs Cole", no-one, no-one), gossip("Mrs Elton", no-one, no-one))), no-one))



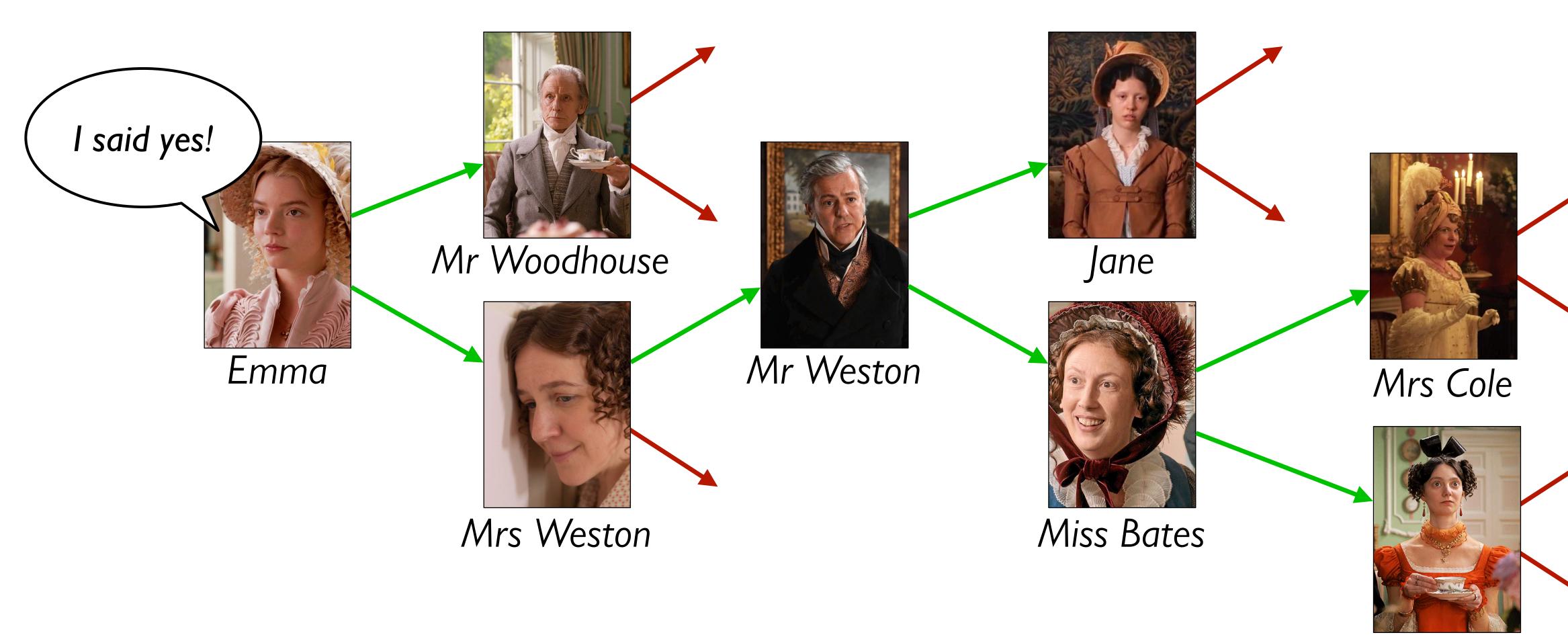
# Example using names for parts: MRS-COLE-MILL = gossip("Mrs Cole", no-one, no-one) MRS-ELTON-MILL = gossip("Mrs Elton", no-one, no-one) JANE-MILL = gossip("Jane", no-one, no-one) MR-WOODHOUSE-MILL = gossip("Mr Woodhouse", no-one, no-one)

```
MISS-BATES-MILL = gossip("Miss Bates", MRS-COLE-MILL, MRS-ELTON-MILL)
MR-WESTON-MILL = gossip("Mr Weston", JANE-MILL, MISS-BATES-MILL)
MRS-WESTON-MILL = gossip("Mrs Weston", MR-WESTON-MILL, no-one)
```

EMMA-MILL = gossip("Emma", MR-WOODHOUSE-MILL, MRS-WESTON-MILL)

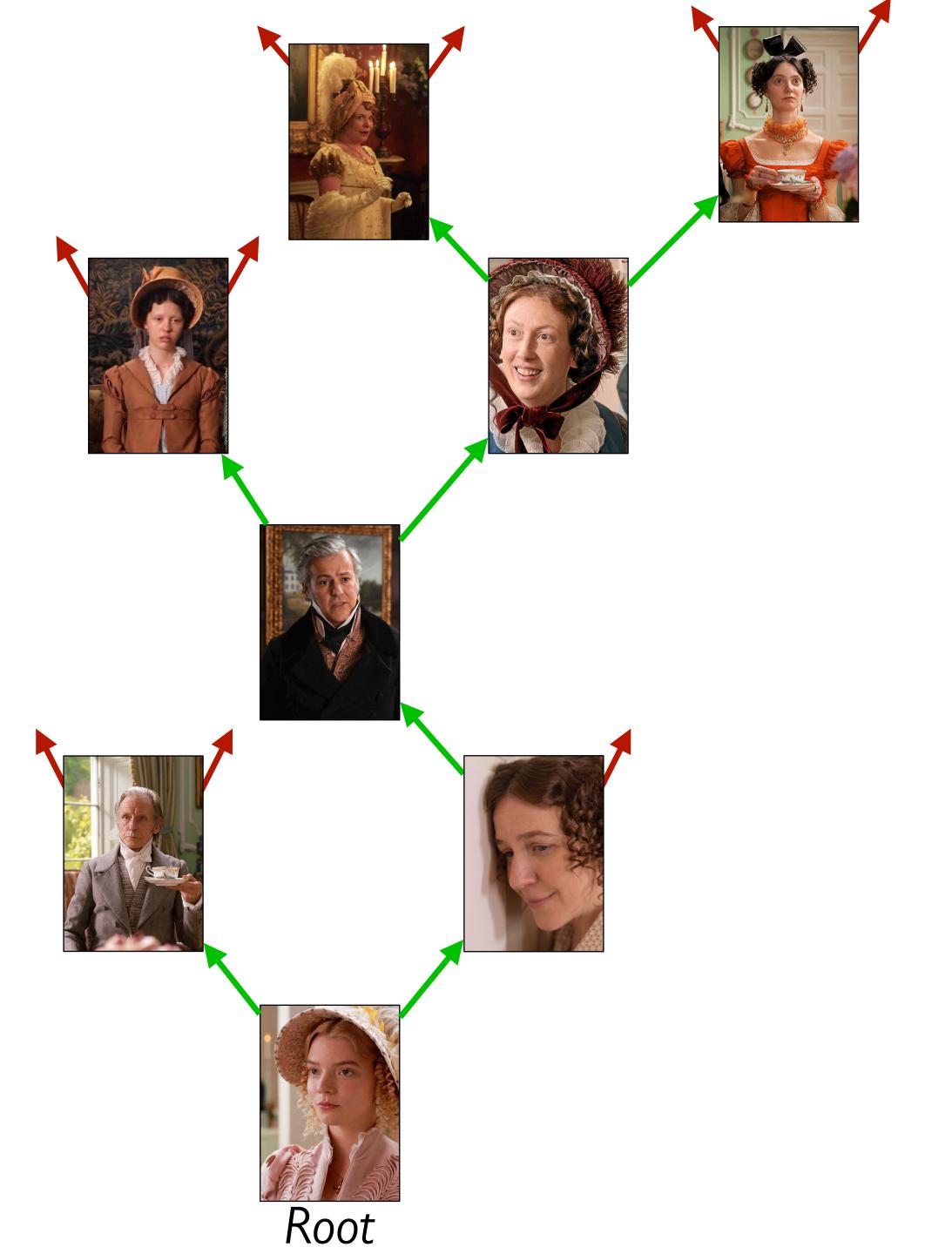
A RumorMill is a type of structure called a tree. Each element in the tree is called a *node*. The first node in the tree is called the *root*. A node with no children is called a *leaf*.

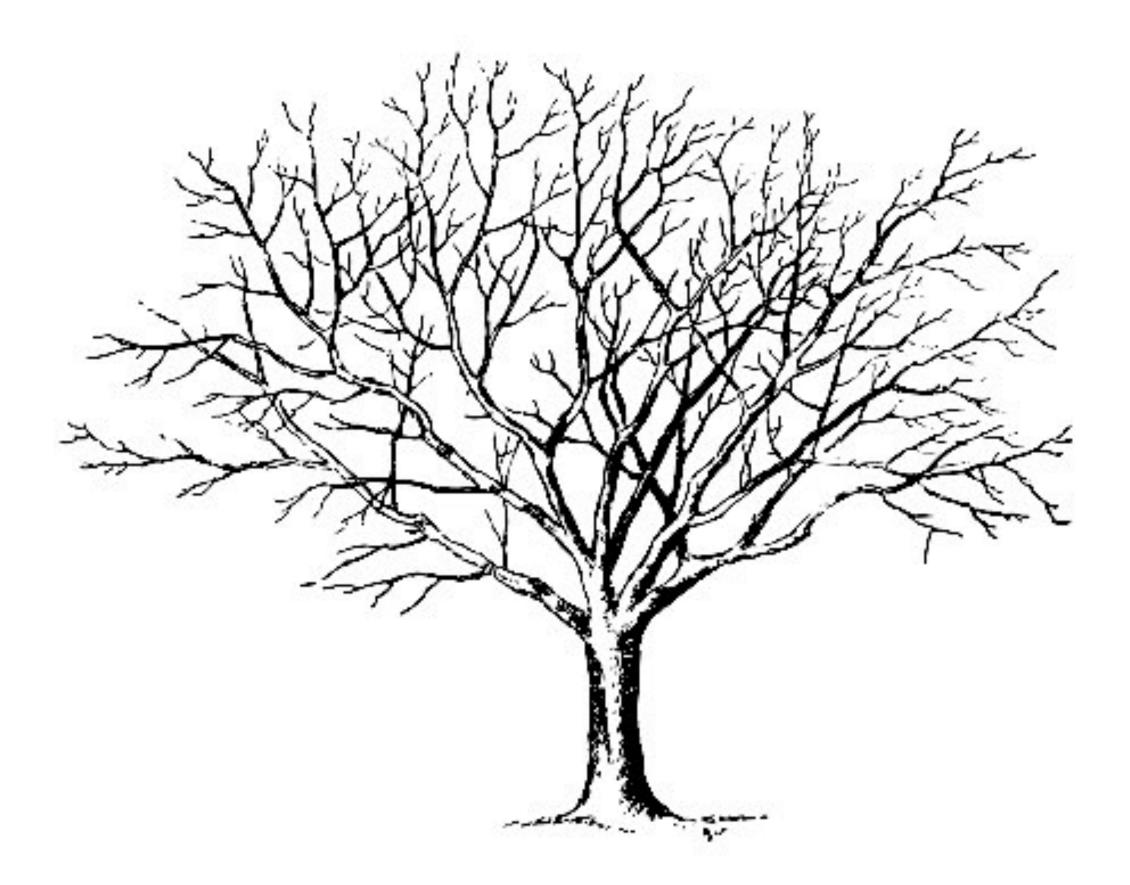
Like a list, a tree is recursive: Every subtree is a tree.



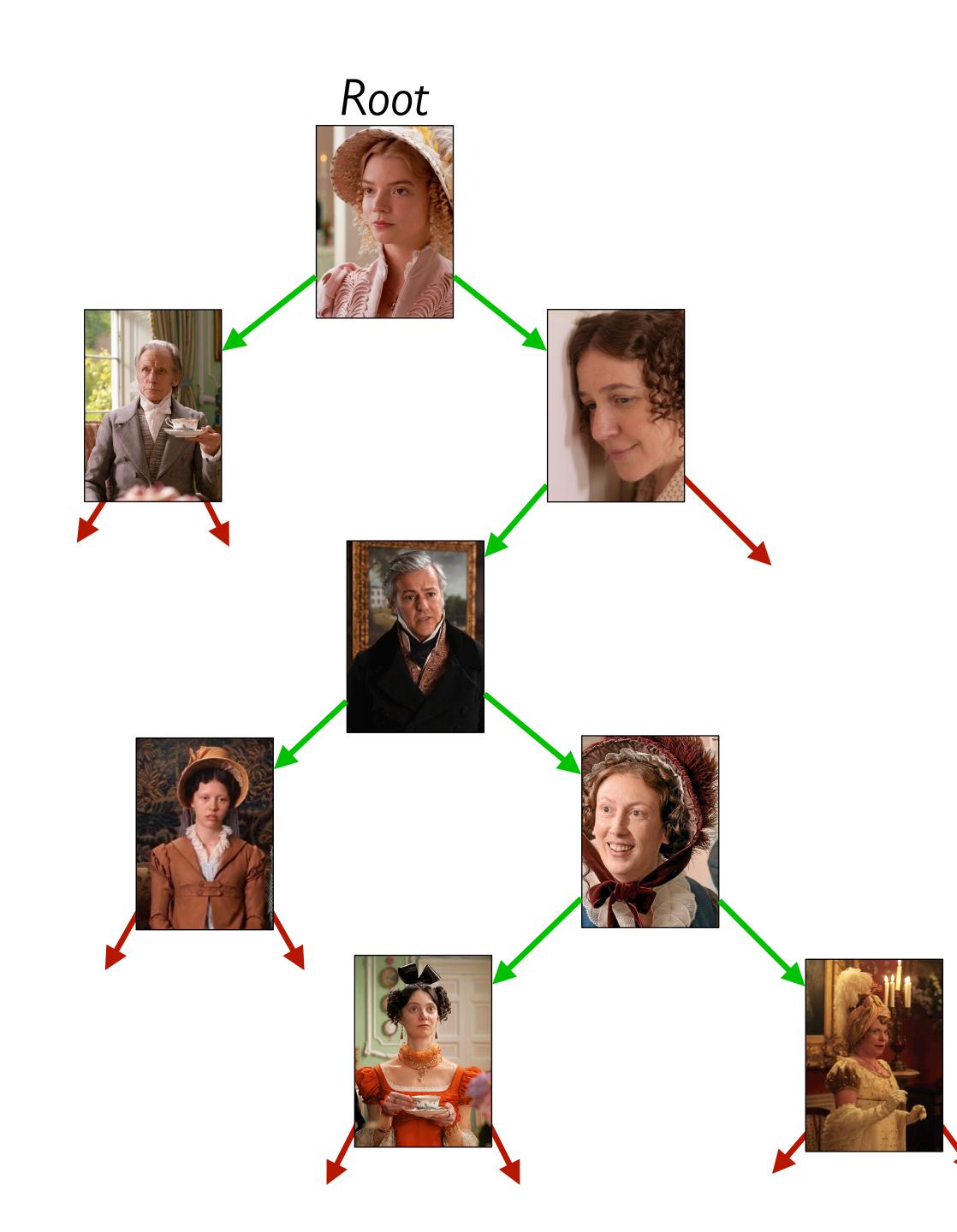
#### Mrs Elton

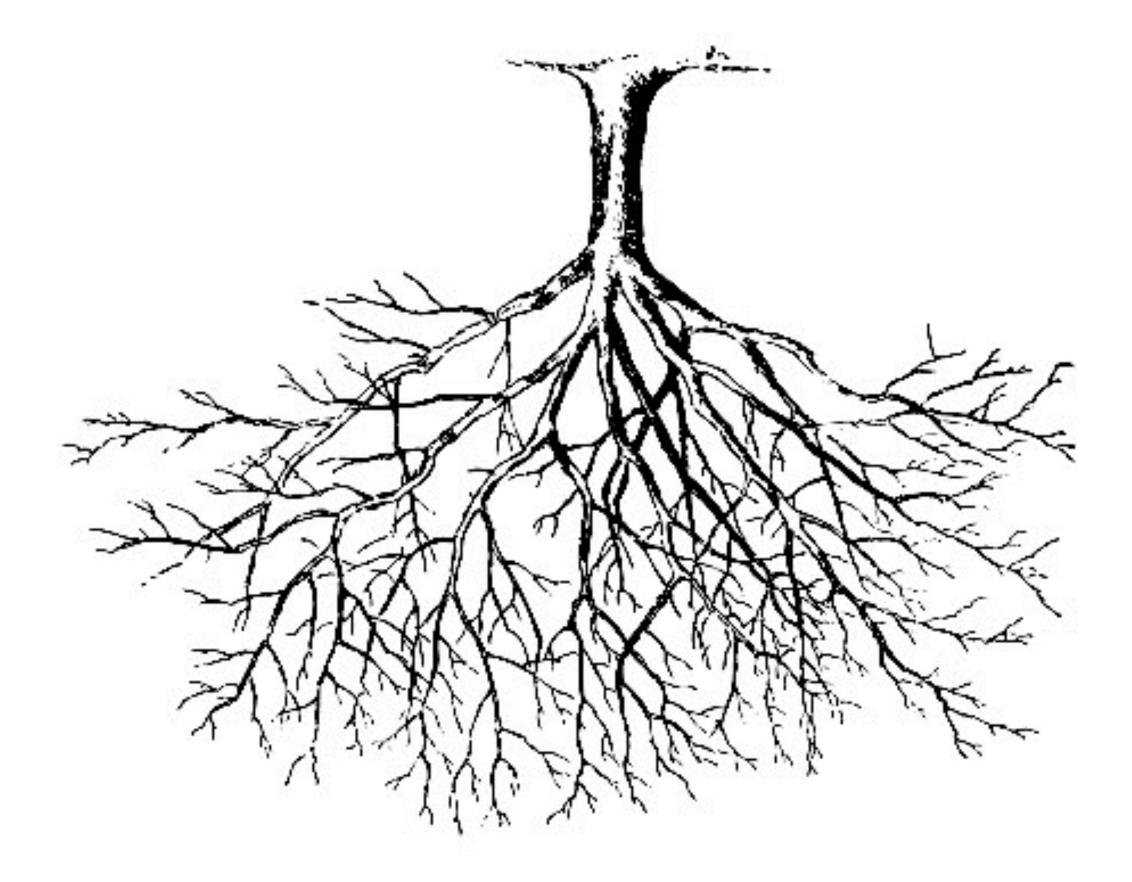






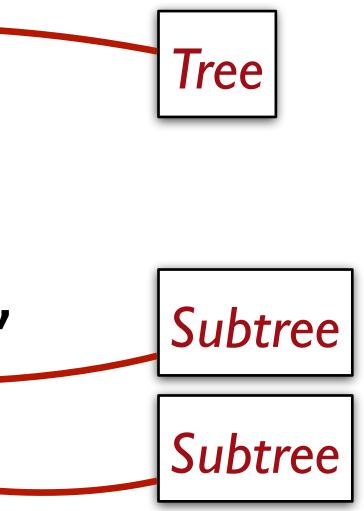
Draw it vertically and you can see it's a tree!





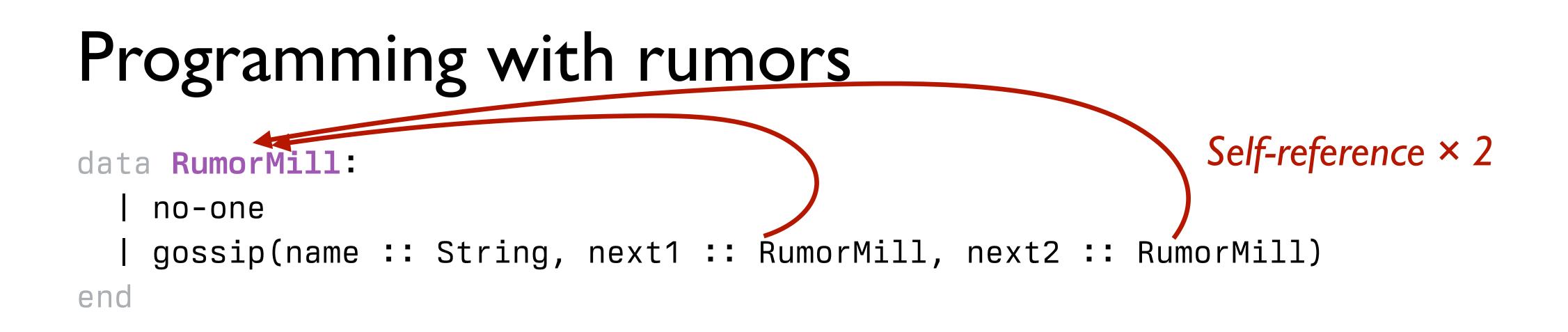
Computer scientists are weird.

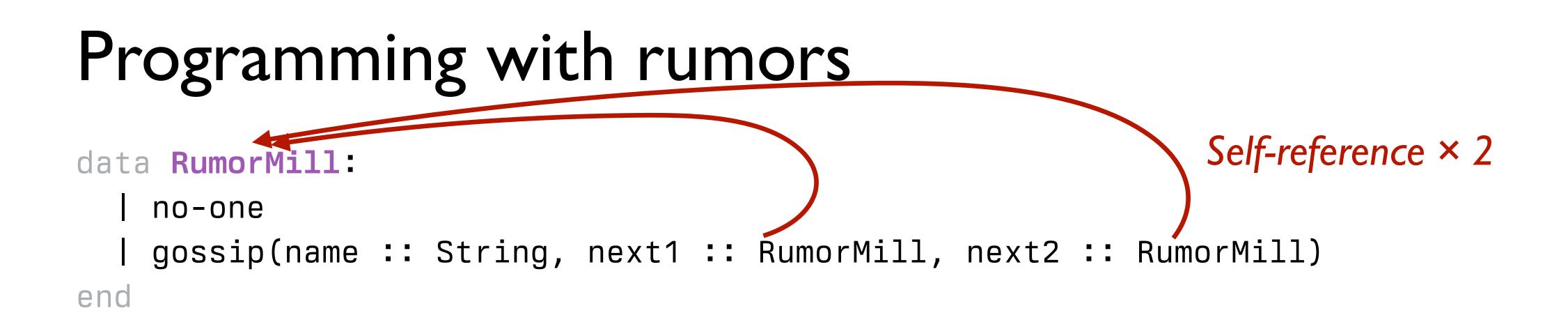
# data RumorMill: | no-one | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill) end



## Programming with rumors

data RumorMill:
 | no-one
 | gossip(name :: String, next1 :: RumorMill, next2 :: RumorMill)
end





For each element, there's not just one "next" element; there are two!

## Programming with rumors

data RumorMill:

no-one

end

#

fun rumor-mill-fun(rm :: RumorMill) -> ...: doc: "Template for a function with a RumorMill as input" cases (RumorMill) rm:

/ no-one => ...

/ gossip(name, next1, next2) =>

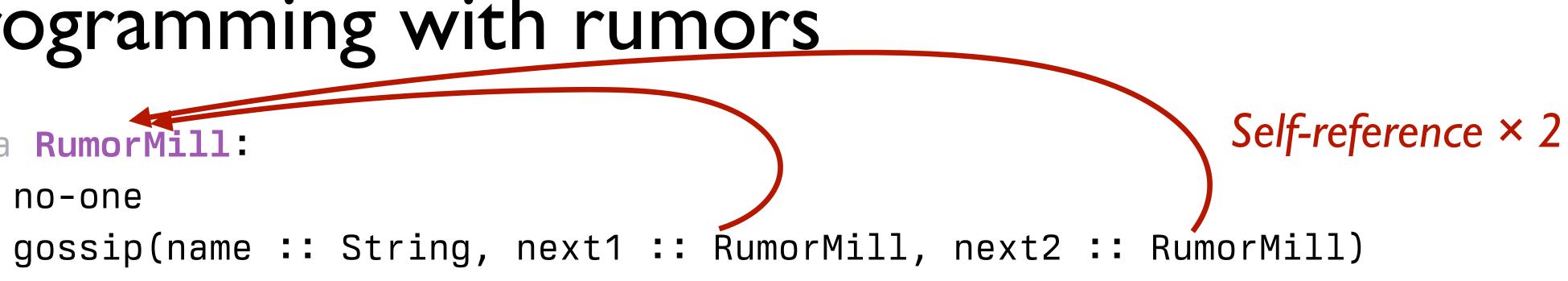
... name

- ... rumor-mill-fun(next1)
- ... rumor-mill-fun(next2)

end

end

#



## Programming with rumors

data RumorMill:

no-one

end

#### #

fun rumor-mill-fun(rm TRumorMill) -> ...: doc: "Template for a function with a RumorMill as input" cases (RumorMill) rm:

/ no-one => ...

| gossip(name, next1, next2) =>

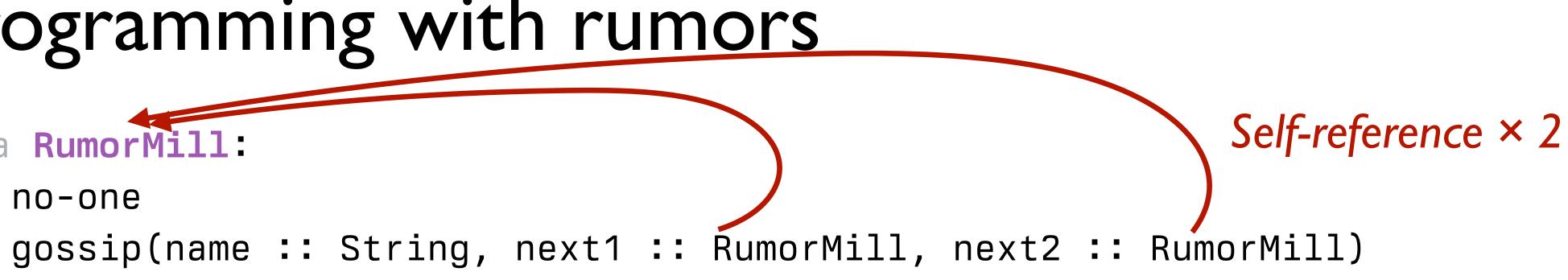
... name

- ... rumor-mill-fun(next1)
- ... rumor-mill-fun(next2)

end

end

#



- Natural recursion × 2

Starter file: tinyurl.com/101-2024-02-15-starter

## Rumor program examples

Design the function **is-informed** that takes a person's name and a rumor mill and determines whether the person is part of the rumor mill.

## Rumor program examples

Design the function **gossip-length** that takes a rumor mill and determines the length of the longest sequence of people transmitting the rumor.

## Rumor program examples

Design the function **add-gossip** that takes a rumor mill and two names – one new and one old – and adds the new person to the rumor mill, receiving rumors from the old person. (You can assume the old person does not already have two next persons!)

Solutions: tinyurl.com/101-2024-02-15

## Acknowledgments

This lecture incorporates material from: Jane Austen, Emma Tom Ellman, Vassar College Marc Smith, Vassar College



