Recursive predicates, continued

Checking whether a symbol is a member of a list of symbols

```
(define member?
  (lambda (item lst)
    ...?...))

> (member? 'a '())
#f
> (member? 'a '(z a z))
#t
> (member? 'a '(a z a))
#t
> (member? 'a '(x y z))
#f
```
Recursive definition of member?

```
(define member?
  (lambda (item lst)
    ;; ITEM is in LST if...
    (and (not (null? lst))
      (or (equal? item (first lst))
        (member? item (rest lst))))))
```

The built-in member function is like this, except it returns its input when it finds a match:

```
> (member 'b '(a b c))
(b c)
```

Why `(b c)` instead of `(a b c)`?

`(b c)` is the input to the recursive call to member when it finds a match.

Checking whether a list of numbers is increasing

```
(define increasing?
  (lambda (lst)
    ...???))
```

```
> (increasing? '())
#t
> (increasing? '(7))
#t
> (increasing? '(5 8 11 54))
#t
> (increasing? '(1 2 3 2 1))
#f
> (increasing? '(1 2 3 4 5))
#f
```

This is a “for all” question, unlike the “exists” question of member?.

It’s trivially true when the list is empty.

Is it true that all Martians love their children? Sure, because there are no Martians.

We want to return #t unless we can find a counterexample.
Constructing solution to larger problem from solution to smaller one

Argument = (x y₁ y₂ y₃ y₄ ... yₙ)

Case 1: x < y₁
Return: #t if (y₁ y₂ y₃ y₄ ... yₙ) is also increasing, otherwise return #f.

Case 2: x ≥ y₁ – a counterexample!
Return: #f

Recursive definition of increasing?

(define increasing? (lambda (lst)
  (cond
    ;; Base case 1:
    ;; No elements left; trivially true.
    ((null? lst) #t)
    ;; Base case 2:
    ;; One element left; trivially true.
    ((null? (rest lst)) #t)
    ;; Recursive case:
    ;; The first two elements are increasing;
    ;; keep checking.
    (or (null? lst)
        (null? (rest lst))
        (and (< (first lst) (second lst))
          (increasing? (rest lst))))
    ;; Base case 3:
    ;; Found a counterexample.
    (else #f)))))
You may have been wondering...

You've heard of bits and bytes.

We've been talking about symbols and lists.

What's the connection?

How are lists actually stored in the memory of a computer?

Von Neumann architecture

- Central Processing Unit (CPU)
- Random Access Memory (RAM)
- System Bus
- Disk drive

Programs

- (define listify-all (lambda (lst) ...))

Data

- (batman superman ...)

Organization of RAM

<table>
<thead>
<tr>
<th>Address</th>
<th>Contents</th>
<th>Memory is divided into storage locations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>Each location holds some data called its “contents”.</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>Each location has a label called its “address”.</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>Given the address, one can use it to find the location and get the contents.</td>
</tr>
<tr>
<td>2ⁿ−1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Organization of RAM

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</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$2^n - 1$</td>
<td></td>
</tr>
</tbody>
</table>

The address of one location may be stored as the contents of another. The first location is said to "point" to the second. We represent a pointer visually with an arrow from one location to another.

cons, first, and rest

A cons cell:

```
first rest
```

A block of RAM is divided into two parts:

- One part is the "first".
- The other part is the "rest".

Splitting (a (b c)) into firsts & rests

```
(a (b c))
```

```
(a (b c))
```

```
((b c))
```

```
(b c)
```

```
(c)
```

```
() (define expression '(a (b c)))
```

cons cell representation of (a (b c))

```
expression
```

```
(a (b c))
```

```
((b c))
```

```
(b c)
```

```
(c)
```

```
() (define expression '(a (b c)))
```

```
(a)
```

```
(b)
```

```
(c)
```
Representation of lists

(define heroes '(batman superman hulk))

representation of list

(define primes '(1 3 5))

representation of list

Representation of nested lists

(define couples '(((adam eve) (romeo juliet)))

representation of nested list

(define thing '(((alpha bravo)) (charlie delta)))

representation of nested list
What is this?

> (define foobar (cons 'foo 'bar))
> foobar
'(foo . bar)

A “dotted pair”.

What is this?

> (define abc (cons 'a (cons 'b 'c)))
> abc
'(a b . c)

An “improper list”.

Scheme's symbol table

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>year</td>
<td>2019</td>
</tr>
<tr>
<td>class</td>
<td>senior</td>
</tr>
<tr>
<td>gpa</td>
<td>3.75</td>
</tr>
<tr>
<td>heroes</td>
<td></td>
</tr>
</tbody>
</table>

(define year 2019)
(define class 'senior)
(define gpa 3.75)
(define heroes (list 'turing 'knuth 'mccarthy))

aliases

(define foo '(a b))
(define bar foo)

We say that bar is aliasing foo because foo and bar refer to the same structure in memory.
Copying a list

(define copy
  (lambda (x)
    ...?...))

> (copy '(a b c))
'(a b c)
> (copy 'a)
'a

Illustration of copying

(define foo '(a b))
(define bar (copy foo))
Comparing equal? and eq?

The `equal?` procedure tests whether two lists are structurally equivalent.

`equal?` takes more time for longer lists.

The `eq?` procedure tests whether two lists are the same object in memory.

`eq?` takes the same time for all lists.

Equality and aliasing

```
(define foo '(a b))
(define bar foo)
(equal? foo bar) → #t
(eq?foo bar) → #t
```

Equality and copying

```
(define foo '(a b))
(define bar (copy foo))
(equal? foo bar) → #t
(eq? foo bar) → #f
```

Acknowledgments

This lecture incorporates material from:

Tom Ellman