Chapter 11

Trees

Terminology

- Definition of a general tree
  - A general tree T is a set of one or more nodes such that T is partitioned into disjoint subsets:
    - A single node r, the root
    - Sets that are general trees, called subtrees of r
- Trees have hierarchical structure
  - parent/child/sibling
  - root/leaf
  - ancestor/descendant
  - subtree

Terminology

- Definition of a binary tree
  - A binary tree is a set T of nodes such that either
    - T is empty, or
    - T is partitioned into three disjoint subsets:
      - A single node r, the root
      - Two possibly empty sets that are binary trees, called left and right subtrees of r

Terminology

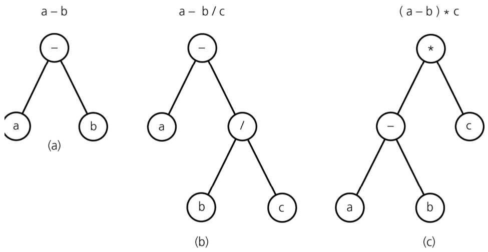


Figure 11-4

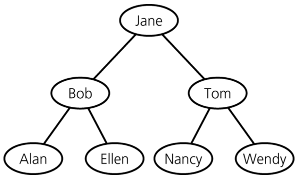
Binary trees that represent algebraic expressions

Terminology

- A binary search tree
  - A binary tree that has the following properties for each node n
    - n's value is greater than all values in its left subtree T_L
    - n's value is less than all values in its right subtree T_R
    - Both T_L and T_R are binary search trees

Figure 11-5

A binary search tree of names



Terminology

- The height of trees
  - Level of a node n in a tree T
    - If n is the root of T, it is at level 1
    - If n is not the root of T, its level is 1 greater than the level of its parent
  - Height of a tree T defined in terms of the levels of its nodes
    - If T is empty, its height is 0
    - If T is not empty, its height is equal to the maximum level of its nodes

Terminology

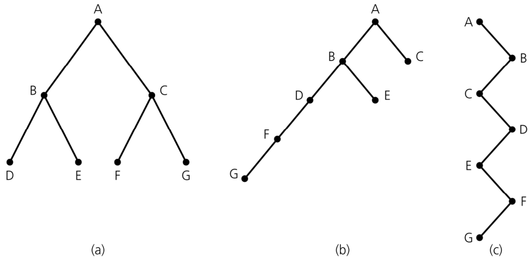


Figure 11-6

Binary trees with the same nodes but different heights

Terminology

- Full, complete, and balanced binary trees
  - Recursive definition of a full binary tree
    - If T is empty, T is a full binary tree of height 0
    - If T is not empty and has height h > 0, T is a full binary tree if its root’s subtrees are both full binary trees of height h – 1

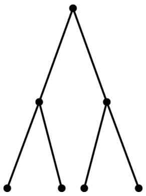


Figure 11-7

A full binary tree of height 3

Terminology

- Complete binary trees
  - A binary tree T of height h is complete if
    - All nodes at level h – 2 and above have two children each, and
    - When a node at level h – 1 has children, all nodes to its left at the same level have two children each, and
    - When a node at level h – 1 has one child, it is a left child

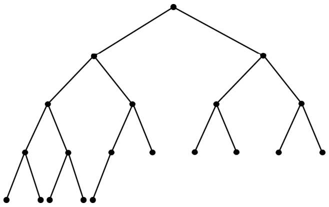


Figure 11-8

A complete binary tree

Terminology

- Balanced binary trees
  - A binary tree is balanced if the height of any node’s right subtree differs from the height of the node’s left subtree by no more than 1
- Full binary trees are complete
- Complete binary trees are balanced

Terminology

- Summary of tree terminology
  - General tree
    - A set of one or more nodes, partitioned into a root node and subsets that are general subtrees of the root
  - Parent of node n
    - The node directly above node n in the tree
  - Child of node n
    - A node directly below node n in the tree
  - Root
    - The only node in the tree with no parent

Terminology

- Summary of tree terminology (Continued)
  - Leaf
    - A node with no children
  - Siblings
    - Nodes with a common parent
  - Ancestor of node n
    - A node on the path from the root to n
  - Descendant of node n
    - A node on a path from n to a leaf
  - Subtree of node n
    - A tree that consists of a child (if any) of n and the child’s descendants

Terminology

- Summary of tree terminology (Continued)
  - Height
    - The number of nodes on the longest path from the root to a leaf
  - Binary tree
    - A set of nodes that is either empty or partitioned into a root node and one or two subsets that are binary subtrees of the root
    - Each node has at most two children, the left child and the right child
  - Left (right) child of node n
    - A node directly below and to the left (right) of node n in a binary tree

Terminology

- Summary of tree terminology (Continued)
  - Left (right) subtree of node n
    - In a binary tree, the left (right) child (if any) of node n plus its descendants
  - Binary search tree
    - A binary tree where the value in any node n is greater than the value in every node in n's left subtree, but less than the value of every node in n's right subtree
  - Empty binary tree
    - A binary tree with no nodes

Terminology

- Summary of tree terminology (Continued)
  - Full binary tree
    - A binary tree of height h with no missing nodes
    - All leaves are at level h and all other nodes each have two children
  - Complete binary tree
    - A binary tree of height h that is full to level h – 1 and has level h filled in from left to right
  - Balanced binary tree
    - A binary tree in which the left and right subtrees of any node have heights that differ by at most 1

Possible Representations of a Binary Tree

- An array-based representation of a complete tree
  - If the binary tree is complete and remains complete
    - A memory-efficient array-based implementation can be used

Possible Representations of a Binary Tree

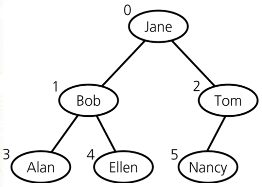
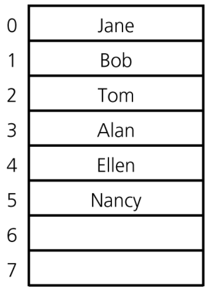


Figure 11-12

Level-by-level numbering of a complete binary tree



0	Jane
1	Bob
2	Tom
3	Alan
4	Ellen
5	Nancy
6	
7	

Figure 11-13

An array-based implementation of the complete binary tree in Figure 10-12

Possible Representations of a Binary Tree

- A reference-based representation
  - Java references can be used to link the nodes in the tree

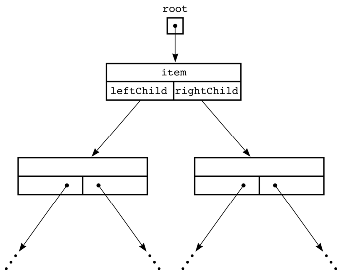


Figure 11-14

A reference-based implementation of a binary tree

A Comparison of Sorting Algorithms

	Worst case	Average case
Selection sort	n^2	n^2
Bubble sort	n^2	n^2
Insertion sort	n^2	n^2
Mergesort	n * log n	n * log n
Quicksort	n^2	n * log n
Radix sort	n	n
Treesort	n^2	n * log n
Heapsort	n * log n	n * log n

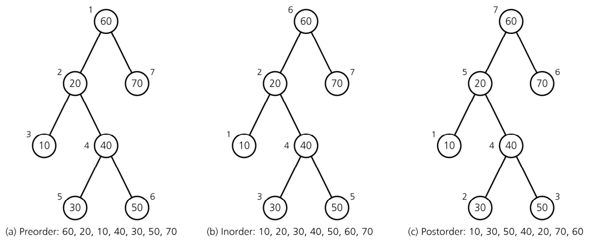
Figure 10-22

Approximate growth rates of time required for eight sorting algorithms

Traversals of a Binary Tree

- A traversal algorithm for a binary tree visits each node in the tree
- Recursive traversal algorithms
  - Preorder traversal
  - Inorder traversal
  - Postorder traversal
- Traversal is O(n)

Traversal of a Binary Tree



(a) Preorder: 60, 20, 10, 40, 30, 50, 70

(b) Inorder: 10, 20, 30, 40, 50, 60, 70

(c) Postorder: 10, 30, 50, 40, 20, 70, 60

(Numbers beside nodes indicate traversal order.)

Figure 11-10

Traversals of a binary tree: a) preorder; b) inorder; c) postorder

Heaps

- A heap is a complete binary tree
  - That is empty

  or

  - Whose root contains a search key greater than or equal to the search key in each of its children, and
  - Whose root has heaps as its subtrees

Heaps

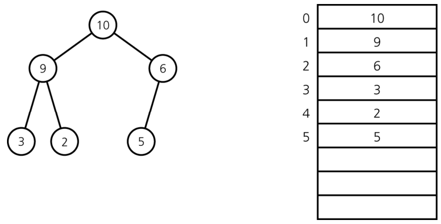
- Maxheap
  - A heap in which the root contains the item with the largest search key
- Minheap
  - A heap in which the root contains the item with the smallest search key

Heaps: An Array-based Implementation of a Heap

- Data fields
  - `items`: an array of heap items
  - `size`: an integer equal to the number of items in the heap

Figure 12-11

A heap with its array representation



0	10
1	9
2	6
3	3
4	2
5	5