Foundations of Computer Science  
CS 145-52  
Spring 2017  
Homework 1  

Due AT THE BEGINNING OF LECTURE Wednesday, February 8

- A general note: When writing up your homework, please explain your arguments clearly and write neatly. This applies to every homework exercise in this class, and especially to proofs: Make sure that your proofs are clear, that each step is justified, and that you don’t skip over too many intuitively obvious steps; use only the definitions seen in class or in the book.

Graders may not award credit to incomplete or unclear solutions. Clear communication is the point, on every assignment.

- A general note: Neatly written (or typeset) solutions, with enough blank space on the page to allow graders to write comments before returning the papers, are greatly appreciated!

1. Use induction to prove each of the following equations involving non-empty summations. (For each exercise, the first term and pattern of the series determine what is to be proved; the number of sample terms given to illustrate the pattern is not relevant for the proof.) Each solution must be a proof by induction.

Be sure to write your proofs out carefully! For example, in each proof, be sure to explicitly state what statement is being proved, what the base case is, what the inductive hypothesis is, and where the inductive hypothesis is applied. Use summation notation (i.e., Σ notation) for clarity when appropriate, including when writing the statement to be proved, the base case, and the inductive hypothesis.

(a) \[ 2 + 4 + 8 + 16 + \cdots + 2^n = 2(2^n - 1) \]
(b) \[ 2 + 6 + 12 + \cdots + (n^2 - n) = \frac{n(n^2 - 1)}{3}. \]

2. Proof the following claim, using the definitions of set operations from class and the textbook.

Claim: For all sets \( A, B, C \) and \( D \), if \( A \subseteq B \) and \( C \subseteq D \), then \( (A \cup C) \subseteq (B \cup D) \).