Foundations of Computer Science
CS 145-52
Spring 2017
Homework 2

Due AT THE BEGINNING OF CLASS Wednesday, February 22

• A general note: When writing up your homework, please explain your arguments clearly and write neatly. Make sure that your proofs are clear, that each step is justified, and that you don’t skip over too many intuitively obvious steps; use only the definitions seen in class or in the book.

In general, please show your work! Graders may not award credit to incomplete or unclear solutions. Clear communication is the point, on every assignment.

• A general note: Neatly written (or typeset) solutions, with enough blank space on the page to allow graders to write comments before returning the papers, are greatly appreciated!

• A reminder from the syllabus: “On each submitted assignment, you should always cite and acknowledge sources from which you receive assistance, including your textbook, your CS145 Coaches, or your classmates.” Please be sure to do so!

1. Write the full truth tables for the following propositions:

   (a) \( \neg(p \lor q) \leftrightarrow (\neg p \land \neg q) \)
   (b) \((p \rightarrow q) \rightarrow r \rightarrow s\)

2. Power set exercises! Recall that for a set \( A \), the power set \( \mathcal{P}(A) \) is the set of all subsets of \( A \). For each claim below, either give a counterexample that demonstrates that it is false for some sets \( A \) and \( B \), or give a rigorous proof (explicitly citing all of the formal definitions that you use) that the claim is true for all sets \( A \) and \( B \).

   (a) Claim: If \( a \in A \) and \( b \in B \), then \( \{a, b\} \in \mathcal{P}(A \cap B) \).
   (b) Claim: If \( A \) is a proper subset of \( B \), then \( \mathcal{P}(A) \) is a proper subset of \( \mathcal{P}(B) \).
   (c) Claim: \( \mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B) \).

3. Prove that if relations \( R \) and \( S \) are both transitive, then their intersection \( R \cap S \) must also be a transitive relation.

4. Give an example of two relations \( R \) and \( S \), both relations over the set \( A = \{0, 1, 2\} \), such that \( R \) and \( S \) both contain at least three ordered pairs, and \( R \) and \( S \) are both transitive, but their union \( R \cup S \) is not transitive. As always, be sure to thoroughly explain your answer (including explanations of all details that show your answer satisfies the conditions of the exercise)!

   Recall that a relation is a set of ordered pairs. Therefore, be sure to give your answer as a set of ordered pairs.