Foundations of Computer Science  
CS 145  
Spring 2018  
Homework 2  
Due AT THE BEGINNING OF CLASS Tuesday, February 27

- A general note: When writing up your homework, please explain your arguments clearly and write neatly. Make sure that your proofs are clear, that each step is justified, and that you don’t skip over too many intuitively obvious steps; use only the definitions seen in class or in the book.

In general, please show your work! Graders may not award credit to incomplete or unclear solutions. Clear communication is the point, on every assignment.

- A general note: Neatly written (or typeset) solutions, with enough blank space on the page to allow graders to write comments before returning the papers, are greatly appreciated!

- A reminder from the syllabus: “On each submitted assignment, you should always cite and acknowledge sources from which you receive assistance, including your textbook, your CS145 Coaches, or your classmates.” Please be sure to do so!

1. Write the full truth tables for the following propositions, following the form presented in class:
   - (a) \( \neg(p \land q) \leftrightarrow (\neg p \lor \neg q) \)
   - (b) \((p \rightarrow q) \rightarrow r \rightarrow s \)

2. Power set exercises! Recall that for a set \( A \), the power set \( \mathcal{P}(A) \) is the set of all subsets of \( A \). For each claim below, either give a counterexample that demonstrates that it is false for some sets \( A \) and \( B \), or give a rigorous proof (explicitly citing all of the formal definitions that you use) that the claim is true for all sets \( A \) and \( B \).

   Moreover, in all proofs of implications (including those as parts of proving biconditionals) for these exercises, please use direct proofs; do not use proof by contrapositive for these exercises.

   - (a) Claim: If \( a \in A \) and \( b \in B \), then \( \{a, b\} \in \mathcal{P}(A \cap B) \).
   - (b) Claim: If \( A \) is a proper subset of \( B \), then \( \mathcal{P}(A) \) is a proper subset of \( \mathcal{P}(B) \).
   - (c) Claim: \( \mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B) \).

3. Give an example of two relations \( R \) and \( S \), both relations over the set \( A = \{0, 1, 2\} \), such that \( R \) and \( S \) both contain at least three ordered pairs, are not intransitive (see below), and are transitive, but their union \( R \cup S \) is not transitive. Please say what \( R \cup S \) is as part of your answer, and as always, be sure to thoroughly explain your answer (including explanations of all details that show your answer satisfies the conditions of the exercise)!

Recall that a relation is a set of ordered pairs. Therefore, be sure to explicitly give the relations your answer as sets of ordered pairs.

Note: As presented in lecture notes and on page 43 of your textbook, a relation \( R \) is intransitive iff whenever \( (a, b) \in R \) and \( (b, c) \in R \), then \( (a, c) \notin R \).
Optional Practice Exercise

The exercise below is not to be submitted and will not be graded. It is, however, an excellent practice exercise, covering essential material for the course. You are encouraged to work on it and discuss it freely with classmates, Coaches, and your Prof.; a full solution will be provided as part of the HW2 solution set.

• Prove that if relations $R$ and $S$ are both transitive, then their intersection $R \cap S$ must also be a transitive relation.