CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – T R 3:10pm
Lab – M 3:10pm

Lecture Meeting Location: OH 162
Lab Meeting Location: SP 309

Business

• HW1 out tonight, due Tuesday, February 13
  – (i.e., at the beginning of class that day—as always, please see the HW sheet for information about HW deadlines)

• See new links from course webpage Additional Notes / Readings for two documents:
  – A document about induction (containing the proof from the last lecture!)
  – A document about the equational proof style we use for inductions

• Reading: Please finish Ch. 1 from your textbook
  – Your book talks about sets and diagrams in Ch.1.2.3; it’s interesting, but I won’t cover it in class
Business, pt. 2

- Coaching Hours will start **tomorrow, Wednesday, Jan. 31**
- Coaching Hours for the semester (held in SP307):
  - Sunday: 7-10pm
  - Monday: 7-9pm
  - Tuesday: 7:30-9pm
  - Wednesday: 6:30-9pm
  - Thursday: 6:30-8pm

**Important procedural note about Coaching Hours:**
- CS145 Coaches should write their names on the board to say they’re working as CS145 Coaches at that time
- CS145 students should, if they want Coaching help, write their names on the board to form a list, ordered by when they came in to the Hours

Business, pt. 3

- No class meeting this Thursday, Feb. 1
  - Will be made up later in the semester
- No Office Hours this Thursday, Feb. 1
- Bonus Coaching Hour, this week only:
  - Friday, Feb. 2, 4-6pm, in SP307
Mathematical In(tro)duction, cont.

• Intuitive idea: Proving that all the dominos will fall
  (1) First, make sure the first domino falls
  (2) Then, make sure they’re all set up such that if you look at any
domino in the chain—say, the k’th domino, for any k—then if the
k’th domino falls, then the (k+1)’st will fall
• This is enough to prove that every domino falls!
  Think about it a bit…
  – The first one falls, because we prove that directly with (1)
  – The second one falls, because we consider (2) with k = 1—it says if
the first domino falls, so does the second
  – Similarly, the third one falls, because we consider (2) with k=2…
  – Similarly, the fourth one falls… (is it clear why?)
  – And we can show that all of them fall! How?

Inductive Proofs

• Our inductive proof, like all inductive proofs, has
four parts
  (1) Write down what we’re trying to prove
    • Be sure to describe the variables! (e.g., “For every number n
greater than 1…”)
  (2) Prove the base case—the first domino falls
  (3) Write the inductive hypothesis—assume domino k falls
  (4) Prove your inductive case—using the inductive
hypothesis, show that domino k+1 falls too

Every step in this can be subtle, or require some thought—
we’ll see examples as the course goes on!
Another Example Induction

- Show that the sum of the first $n$ odd numbers is $n^2$
- Proof by induction! Go through the steps
  - Write down what we’re trying to prove
  - What’s our base case?
  - What’s our inductive hypothesis?
  - How do we use our inductive hypothesis to prove our inductive case?

Getting started with induction can feel a little getting started with recursion—like there’s some magic to it! (“Wait... that’s the whole recursive program?”)

But as with recursion, you learn induction by thinking through several examples. We’ll see more of it as the course goes along!
Subset (the *Inclusion* relation)

- **Definition:** Set A is said to be a *subset* of set B if every element of A is also an element of B
  - *i.e.,* iff (“if and only if”) for all x, if \( x \in A \) then \( x \in B \)
- **Terminology / notation:**
  - We write \( A \subseteq B \) to represent that A is a subset of (and possibly equal to!) B
  - When A is a subset of B, we can also say B is a *superset* of A, or B *includes* A, written as \( B \supseteq A \)

*Your textbook (see Ch. 1.2, pg. 2) draws a distinction between the word *include* and the word *contain* in this context. Please be aware of that distinction. Also, please be aware that it is not always conventionally followed outside of your textbook!*

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**Exercises**

- **True or False?**
  (a) Whenever \( A \subseteq B \) and \( B \subseteq C \), \( A \subseteq C \)
  (b) Whenever \( A \subseteq B \) and \( C \subseteq B \), \( A \subseteq C \)
  (c) Whenever \( A_1 \subseteq A_2 \subseteq A_3 \ldots \subseteq A_n \), and also \( A_n \subseteq A_1 \), then \( A_i = A_j \) for all \( i, j \) in \([1..n]\)
  (d) \( A = B \) iff neither \( A \subset B \) nor \( B \subset A \)
  (e) Whenever \( A \subset B \) and \( B \subseteq C \), \( A \subset C \)
Exercise

• True or false: Whenever $A \subseteq B$ and $B \subseteq C$, $A \subseteq C$
  
  - *Answer*: True
  
  • Proof idea:
    - $A \subseteq B$ means two things:
      • P1) For all elements $x$, if $x \in A$ then $x \in B$; and
      • P2) There exists element $y$ in $B$ that is not in $A$—that is, there exists $y$ s.t. $y \notin A$ and $y \in B$
    - $B \subseteq C$ means:
      • P3) For all elements $x$, if $x \in B$ then $x \in C$
    - From these 3 premises, we need to show $A \subseteq C$—that is, show that
      • C1) For all elements $x$, if $x \in A$ then $x \in C$; and
      • C2) There exists element $y$ s.t. $y \notin A$ and $y \in C$
    - To show C1)...? [Hint: use P1 and P3]
      • For the “for all”, consider arbitrarily chosen element…
      • For the “if—then”, assume the antecedent, show the consequent…
    - To show C2)...? [Hint: use witness $y$ from P2, and P3]

In every proof for CS145, be sure to explicitly say what is to be proved (the proof goal), and explicitly label where the proof starts.

Note: This is not a full proof! It sketches the proof ideas; a full proof would include the details.