CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – T R 3:10pm  
Lab – M 3:10pm

Lecture Meeting Location: OH 162  
Lab Meeting Location: SP 309

Business

• Change lecture rooms for first half of the semester?  
  – We’ll talk about this on Monday

• HW1 due Tuesday, February 13  
  – (i.e., at the beginning of class that day—as always, please see the HW sheet for information about HW deadlines)  
  – In proofs, use formal definitions / notation where appropriate for clarity and rigorous expression of the relevant ideas

• See links from course webpage Additional Notes / Readings for two documents:  
  – A document about induction (containing the proof from the last lecture!)  
  – A document about the equational proof style we use for inductions

• Reading: Please finish Ch. 1 from your textbook  
  – Your book talks about sets and diagrams in Ch.1.2.3; it’s interesting, but I won’t cover it in class
Exercises

• True or False?

(a) Whenever $A \subseteq B$ and $B \subseteq C$, $A \subseteq C$
(b) Whenever $A \subseteq B$ and $C \subseteq B$, $A \subseteq C$
(c) Whenever $A_1 \subseteq A_2 \subseteq A_3 \ldots \subseteq A_n$, and also $A_n \subseteq A_1$, then $A_i = A_j$ for all $i, j$ in $[1..n]$
(d) $A = B$ iff neither $A \subset B$ nor $B \subset A$
(e) Whenever $A \subset B$ and $B \subseteq C$, $A \subset C$

For these exercises, if a statement is True, give an explanation, and if it is False, give a counterexample.

Exercise

• True or false: Whenever $A \subset B$ and $B \subseteq C$, $A \subset C$

– Answer: True

– Proof idea:

  – $A \subset B$ means two things:
    • $P1)$ For all elements $x$, if $x \in A$ then $x \in B$; and
    • $P2)$ There exists element $y$ in $B$ that is not in $A$—that is, there exists $y$ s.t. $y \notin A$ and $y \in B$
  – $B \subseteq C$ means:
    • $P3)$ For all elements $x$, if $x \in B$ then $x \in C$
  – From these 3 premises, we need to show $A \subset C$—that is, show that
    • $C1)$ For all elements $x$, if $x \in A$ then $x \in C$; and
    • $C2)$ There exists element $y$ s.t. $y \notin A$ and $y \in C$
  – To show $C1$)....? [Hint: use $P1$ and $P3$]
    • For the “for all”, consider arbitrarily chosen element...
    • For the “if—then”, assume the antecedent, show the consequent...
  – To show $C2$)....? [Hint: use witness $y$ from $P2$, and $P3$]

In every proof for CS145, be sure to explicitly say what is to be proved (the proof goal), and explicitly label where the proof starts.

Note: This is not a full proof! It sketches the proof ideas; a full proof would include the details.
The Same Exercise (with a few annotations)

- True or false: Whenever $A \subseteq B$ and $B \subseteq C$, $A \subseteq C$

  - Answer: True

- Proof idea:
  - $A \subseteq B$ means two things:
    - P1) For all ("for all"): universal quantifier elements $x$, if $x \in A$ then $x \in B$; and
    - P2) There exists ("there exists"): existential quantifier element $y$ in $B$ that is not in $A$—that is, there exists $y$ s.t. $y \notin A$ and $y \in B$
  - B $\subseteq C$ means:
    - P3) For all elements $x$, if $x \in B$ then $x \in C$
  - From these 3 premises, we need to show $A \subseteq C$—that is, show that
    - C1) For all elements $x$, if $x \in A$ then $x \in C$; and
    - C2) There exists element $y$ s.t. $y \notin A$ and $y \in C$
  - To show C1)....? [Hint: use P1 and P3]
    - For the "for all", consider arbitrarily chosen element…
    - For the "if—then", assume the antecedent, show the consequent…
  - To show C2)....? [Hint: use witness $y$ from P2, and P3]

Defining Sets

- Intuitively, there are multiple ways of defining a set
  - Enumerate all of its individual members
    - E.g., \{2, 3, 5, 7\}, \{1, 2, 3, 5, 8, 13\}
  - Provide a common, defining property
    - E.g., \{n \mid n \text{ is a prime number less than 10}\}, \{n \mid n \text{ is a Fibonacci number less than 20}\}
  - Either of these is fine, as long as the definition is complete and clear in context

(There are other ways, too, but we’ll focus on these for now)
Intersection

- There are some fundamental set operations, i.e., ways of constructing sets from other sets
- Example: Intersection
  - Intuitively, an intersection is what two (or more) things have in common
  - For sets A, B, the intersection $A \cap B$ is the set of elements that A and B have in common. More formally...
- Definition: The intersection $A \cap B$ of sets A and B is defined by: $x \in A \cap B$ iff $x \in A$ and $x \in B$
  - Also written as $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Examples:
  - What’s $\{1,2,3,4\} \cap \{2,4,6,8\}$?
  - What’s $\{x \mid x \text{ is odd between } 0 \text{ and } 10\} \cap \{x \mid x \text{ is prime between } 0 \text{ and } 10\}$

Union

- Another set operation: Union
  - Intuitively, a union is the combination of what two (or more) things have when they’re put together
  - For sets A, B, the union $A \cup B$ is the set of elements that either A or B contain. (Or both! This is a non-exclusive or.) More formally...
- Definition: The union $A \cup B$ of sets A and B is defined by: $x \in A \cup B$ iff $x \in A$ or $x \in B$
  - Also written as $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Examples:
  - What’s $\{1,2,3,4\} \cup \{2,4,6,8\}$?
  - What’s $\{x \mid x \text{ is odd between } 0 \text{ and } 10\} \cup \{x \mid x \text{ is prime between } 0 \text{ and } 10\}$

Definitions like the definition of union or of intersection can be important parts of proofs—they can be reasons for proof steps. In your proofs, think about what definitions can be used to justify your proof steps!