Business

- HW1 in already
- HW2-Lookahead out today
  - Full HW2 out soon, due Feb. 27
- Reading: Please finish Ch. 1 from your textbook
  - Your book talks about sets and diagrams in Ch.1.2.3; it’s interesting, but I won’t cover it in class
- Reading: Please read Ch.2.1 and 2.2.2 in your textbook
- Options for making up missed classes
- Change lecture rooms for first half of the semester
  - Now in SP105 for all lectures until Break (or until further notice)
- Plans for labs, this week and next
Empty Set

• The empty set, written as $\emptyset$, is the (unique) set containing no elements. (Do you see what makes it unique?)

• The empty set can be described many ways, sometimes without it being obvious that it’s the empty set. E.g.,
  - $S = \{n \text{ is prime } | 24 \leq n \leq 28\}$
  - $G = \{\text{Grammy awards won by The Beach Boys, Led Zeppelin, The Who, The Doors, Queen, Guns N’ Roses, or Bob Marley during their careers (before 2014)}\}$

• Exercise: Consider a set $S$. If $\emptyset \subseteq S$, what do we know about $S$?
  - Prove your answer; it should contain a proof of an important fact about the empty set!

Disjoint Sets

• Definition: Sets $A$, $B$ are disjoint if there is no element $x$ s.t. $(x \notin A \land x \notin B)$
  - That is, sets $A$ and $B$ have no element in common

• This idea can be useful when talking about more than 2 sets where no two have an element in common—they’re all disjoint from each other

• Definition: A collection of sets $A_1$, $A_2$, ..., $A_n$ is pairwise disjoint iff for any $i, j \leq n \ (i \neq j)$, $A_i$ and $A_j$ are disjoint
Set Difference

- Another set operation: Difference
  - Intuitively, the difference between two things is what’s in the first that’s not in the second
  - For sets A, B, the difference A - B (the book writes it as A \ B) is the set of elements that are in A but not in B. More formally…
- Definition: The difference A - B of sets A and B is defined by:
  \[ x \in A - B \iff x \in A \text{ and } x \not\in B \]
  - Also written as A - B = \{x \mid x \in A \text{ and } x \not\in B\}
- Examples:
  - What’s \{1,2,3,4\} - \{2,4,6,8\}?  
  - What’s \{x \mid x \text{ is odd between 0 and 10}\} - \{x \mid x \text{ is prime between 0 and 10}\}

Enumerating Subsets

- If you’re given a set, especially a finite set, you might consider all the subsets of that set
  - If S = \{Phil Collins, Peter Gabriel\}, what are all its subsets? How many are there?
  - If S = \{1,2,3\}, what are all its subsets? How many are there?
  - If S = \{2,4,6,8\}, what are all its subsets? How many are there?
- What do you notice about the relationship between the number of elements in a set and the number of subsets it has?
Power Sets

- Given a set S, the set of subsets of S is called the *power set* of S, sometimes written $\mathcal{P}(S)$
  - $\mathcal{P}(S) = \{A \mid A \subseteq S\}$

- Questions:
  - What’s the power set of $\{a,b\}$?
  - Let S be $\{x \mid x$ is between 0 and 10 and x is prime.$\}$. What is the size of the power set of S?
  - What is the size of $\mathcal{P}(\emptyset)$? What is $\mathcal{P}(\emptyset)$?

Complement of a Set

- Another set operation: Complement
  - For set A, the complement $-A$ is the set of elements not in A. More formally…

- Definition: For set A, its complement $-A$ (also written as A with a line over it) is defined by:
  - $x \in -A$ iff $x \notin A$
  - Also written as $-A = \{x \mid x \notin A\}$

- When discussing set complement, it is (explicitly or implicitly) in the context of a *universe* U of all elements to be considered
  - So, $-A = U - A$

- Examples:
  - What’s $-\{1,2,3,4\}$? [Assume that the complement here is with respect to the natural numbers]
Generalized Union / Intersection

- Consider the $\sum$ summation notation for the sum of $f(i)$ as $i$ goes from 1 to $n$
  - This is a generalization of the addition operator $+$, extending it to apply to a collection of values
  - Each item in that collection is accessed by index
    - e.g., $f(1)$ for index $i=1$, $f(2)$ for index $i=2$, etc.
- The same thing can be done for union $\cup$ or intersection $\cap$

- Exercise: Assume $S_1$, $S_2$, $\ldots$, $S_n$ are finite. What is the size of the (generalized) union of all $S_i$ [i from 1 to n]?
  - How about if $S_1$, $S_2$, $\ldots$, $S_n$ are finite and pairwise disjoint?

Exercise

- Claim: For any sets $A$ and $B$, $(A \cap B) \subseteq (A \cup B)$.
  - Prove it, or give a counterexample.
Ordered Pairs and n-tuples

• By design / definition, sets are good for asking questions of membership, but not for questions of relative ordering of elements
• A different structure, an n-tuple, represents elements and their ordering

• Definition: An ordered pair is a pair of elements expressed in parenthesis—e.g., (0,0), (9,17), (Stephen Colbert, James Corden)
  – Order matters, so (17, 9) is not the same as (9,17)
  – Similarly, order matters, so (0,0) does not contain redundant elements—the 0s are distinct from each other, by position
  – How could we state the criterion for identity for ordered pairs?
• This generalizes to n-tuples with more than 2 elements
  – E.g., (0,0,0),(3,4,5) are both 3-tuples; (72, 86, 94, 86, 76, 66, 72) is a 7-tuple