CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – T R 3:10pm
Lab – M 3:10pm

Lecture Meeting Location: OH 162
Lab Meeting Location: SP 309

Business

• HW5 due April 16 / April 17 (as always, see assignment sheet for exact deadlines)
• Exams back today

• Reading: Ch.4.1-4.6
  – Our coverage of the material will be different from that in the textbook, but it’s good to see the textbook’s presentation, as well
• (Next reading: Ch. 5)

• Document on structural induction available from course website (follow the Additional Notes link)
Proving Statements about Recursively Defined Sets

• A related kind of induction is called *structural induction*, which can be used to prove claims about all items constructed by a recursive definition.

  • To prove property \( P \) holds for all elements of a recursively defined set:
    - Base case(s): Show that \( P \) holds for every element in the basis for the recursive definition.
    - Inductive case(s): Show that every *constructor* in the definition preserves property \( P \).

• Recall the definition of *transitive closure*:
  - \( R^0 = R \);
  - \( R^{n+1} = R_n \cup \{(a,c) \mid \exists x \text{ s.t. } (a,x) \in R_n \text{ and } (x,c) \in R\} \);

• Claim: In the above definition, \( R \) is a subset of \( R_i \) for all \( i \). Prove by structural induction.

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• Recall our recursive definition of propositional logic expressions
  - Base: Given an initial set \( A \) of propositional letters (e.g., \( p, q, r, \ldots \)), all elements of \( A \) are propositional logic expressions
  - Induction: If \( P, Q \) are propositional logic expressions, then the following are also propositional logic expressions (note that the parentheses are part of the expressions)
    * \( \neg P \); \( P \land Q \); \( P \lor Q \); \( P \rightarrow Q \); \( P \leftrightarrow Q \)  
    
  **(Note: 5 constructors)**

• Claim: All propositional logic expressions contain an even number of parentheses. (We consider 0 to be an even number.) Prove by structural induction.