CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – W F 1:30pm
Lab – F 3:30pm

Lecture Meeting Location: SP 105
Lab Meeting Location: SP 309

Business

• Full HW1 out today, due Wednesday September 13
  – (i.e., at the beginning of class that day—as always, please see the HW sheet for information about HW deadlines)
• Reading: Please finish Ch. 1 from your textbook
  – Your book talks about sets and diagrams in Ch.1.2.3; it’s interesting, but I won’t cover it in class
• Tomorrow’s Office Hours: one-time change in time / place
  – 1-3pm, Olmsted 258
• Lab Friday
  – If there are problems logging in to machines in SP307 or SP309, please let me know
Business, pt. 2

• Coaching Hours for the semester:
  – Sunday: 7-9pm
  – Monday: 7:30-9pm
  – Tuesday: 7-10pm
  – Wednesday: 7:30-9pm
  – Thursday: 7-10pm

• **Important procedural note** about Coaching Hours:
  – CS145 Coaches should write their names on the board to say they’re working as CS145 Coaches at that time
  – CS145 students should, if they want Coaching help, write their names on the board to form a list, ordered by when they came in to the Hours

These are also posted on the course website, www.cs.vassar.edu/~cs145

Exercises

• **True or False?**
  (a) Whenever $A \subseteq B$ and $B \subseteq C$, $A \subseteq C$
  (b) Whenever $A \subseteq B$ and $C \subseteq B$, $A \subseteq C$
  (c) Whenever $A_1 \subseteq A_2 \subseteq A_3$ \ldots $\subseteq A_n$, and also $A_n \subseteq A_1$, then $A_i = A_j$ for all $i, j$ in $[1..n]$
  (d) $A = B$ iff neither $A \subset B$ nor $B \subset A$
  (e) Whenever $A \subset B$ and $B \subseteq C$, $A \subset C$
Exercise

• True or false: Whenever $A \subset B$ and $B \subseteq C$, $A \subset C$
  
  – Answer: True

• Proof idea:
  
  – $A \subset B$ means two things:
    • P1) For all elements $x$, if $x \in A$ then $x \in B$; and
    • P2) There exists element $y$ in $B$ that is not in $A$—that is, there exists $y$ s.t. $y \notin A$ and $y \in B$
  
  – $B \subseteq C$ means:
    • P3) For all elements $x$, if $x \in B$ then $x \in C$
  
  – From these 3 premises, we need to show $A \subset C$—that is, show that
    • C1) For all elements $x$, if $x \in A$ then $x \in C$; and
    • C2) There exists element $y$ s.t. $y \notin A$ and $y \in C$
  
  – To show C1)....? [Hint: use P1 and P3]
    • For the “for all”, consider arbitrarily chosen element…
    • For the “if—then”, assume the antecedent, show the consequent…
  
  – To show C2)....? [Hint: use witness $y$ from P2, and P3]

The Same Exercise (with a few annotations)

• True or false: Whenever $A \subset B$ and $B \subseteq C$, $A \subset C$
  
  – Answer: True

• Proof idea:
  
  – $A \subset B$ means two things:
    • P1) For all (“for all”: universal quantifier) elements $x$, if $x \in A$ then $x \in B$; and
    • P2) There exists (“there exists”: existential quantifier) element $y$ in $B$ that is not in $A$—that is, there exists $y$ s.t. $y \notin A$ and $y \in B$
  
  – $B \subseteq C$ means:
    • P3) For all elements $x$, if $x \in B$ then $x \in C$
  
  – From these 3 premises, we need to show $A \subset C$—that is, show that
    • C1) For all elements $x$, if $x \in A$ then $x \in C$; and
    • C2) There exists element $y$ s.t. $y \notin A$ and $y \in C$
  
  – To show C1)....? [Hint: use P1 and P3]
    • For the “for all”, consider arbitrarily chosen element…
    • For the “if—then”, assume the antecedent, show the consequent…
  
  – To show C2)....? [Hint: use witness $y$ from P2, and P3]
Defining Sets

• Intuitively, there are multiple ways of defining a set
  – Enumerate all of its individual members
    • E.g., \{2, 3, 5, 7\}, \{1, 2, 3, 5, 8, 13\}
  – Provide a common, defining property
    • E.g., \{n \mid n \text{ is a prime number less than 10}\},
      \{n \mid n \text{ is a Fibonacci number less than 20}\}
  – Either of these is fine, as long as the definition is complete
    and clear in context

(There are other ways, too, but we’ll focus on these for now)

Intersection

• There are some fundamental set operations, i.e., ways of constructing sets
  from other sets
• Example: Intersection
  – Intuitively, an intersection is what two (or more) things have in common
  – For sets A, B, the intersection A \cap B is the set of elements that A and B have in
    common. More formally…
• Definition: The intersection A \cap B of sets A and B is defined by:
  \[ x \in A \cap B \iff x \in A \text{ and } x \in B \]
  – Also written as A \cap B = \{x \mid x \in A \text{ and } x \in B\}
• Examples:
  – What’s \{1,2,3,4\} \cap \{2,4,6,8\}?  
  – What’s \{x \mid x \text{ is odd between 0 and 10}\} \cap \{x \mid x \text{ is prime between 0 and 10}\}
Union

• Another set operation: Union
  – Intuitively, a union is the combination of what two (or more) things have when they’re put together
  – For sets $A$, $B$, the union $A \cup B$ is the set of elements that either $A$ or $B$ contain. (Or both! This is a non-exclusive or.) More formally…
• Definition: The union $A \cup B$ of sets $A$ and $B$ is defined by: $x \in A \cup B$ iff $x \in A$ or $x \in B$
  – Also written as $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
• Examples:
  – What’s $\{1,2,3,4\} \cup \{2,4,6,8\}$?
  – What’s $\{x \mid x \text{ is odd between 0 and 10}\} \cup \{x \mid x \text{ is prime between 0 and 10}\}$

Definitions like the definition of union or of intersection can be important parts of proofs—they can be reasons for proof steps. In your proofs, think about what definitions can be used to justify your proof steps!

Before We Hit Empty: A Logical Progression

• Basic ideas from propositional logic are showing up in “logic boxes” in your reading

• Before we go further into sets, an overview of these basics of boolean expressions and propositional reasoning
Propositions

- Defn: proposition – a statement that has the property of truth or falsity
- Propositions are the key elements to represent, analyze, or explain declarative knowledge

<table>
<thead>
<tr>
<th>Propositions:</th>
<th>Non-Propositions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Washington, D.C. is the capital of the USA.</td>
<td>• What time is it?</td>
</tr>
<tr>
<td>Poughkeepsie is the capital of New York.</td>
<td>• Pass the salt.</td>
</tr>
<tr>
<td>• 1 + 1 = 2</td>
<td>• x + 1 = 2</td>
</tr>
<tr>
<td>• 2 + 2 = 3</td>
<td>• xy + 5z</td>
</tr>
</tbody>
</table>

The first and third of these are true; the second and fourth are false. Presuming values for x, y, z are not given / known.

Propositional operators

- Recall: proposition – a statement that has the property of truth or falsity
  - Often, we use propositional letters (or variables) to represent propositions: e.g., p stands for “Poughkeepsie is the capital of NY”
- There are several operators (sometimes called boolean operators) that can construct new propositions from old ones
  - Negation (“not”): if P is a proposition, not P is a proposition
  - Conjunction (“and”): P and Q
  - Disjunction (“or”): P or Q
  - Implication (“if – then”): if P then Q
  - Equivalence (“is equal / equivalent to”): P iff Q
    - Equivalence can also be written as “if and only if”