CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – W F 1:30pm
Lab – F 3:30pm

Lecture Meeting Location: SP 105
Lab Meeting Location: SP 309

Business

• HW1 in already
• HW2-Lookahead out today
  – Full HW2 out soon, due Sept. 27
• Reading: Please finish Ch. 1 from your textbook
  – Your book talks about sets and diagrams in Ch.1.2.3; it’s interesting, but I won’t cover it in class
• Reading: Please read Ch.2.1 and 2.2.2 in your textbook
• Lab0 due by the end of the day Thursday, Sept. 14
  – Must be submitted online (using the submit145 command)
  – Must be checked off by me or a Coach
Business, pt. 2

- A few changes to the Links page from our course website (to make them more consistent with Prof. Hunsberger’s CS101)
  - New download link for DrScheme
    - Does not solve previous problems with DrScheme for Mac OS systems
  - Excellent new Scheme resource!
    - Prof. Hunsberger’s *Introduction to Computer Science via Scheme*
    - (replaces old Scheme summary link from our website)
    - Resource for questions on Scheme syntax and semantics—if you have questions on functions or special forms (e.g., `member`, `map`, `letrec`), start by looking them up here!

Compound propositions and their truth tables

- Just as we use truth tables to understand meanings of propositional operators, we can also use them to understand compound propositions
- The truth table for $(p \lor \neg q) \rightarrow (p \land q)$:

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<th>p</th>
<th>q</th>
<th>\neg q</th>
<th>p \lor \neg q</th>
<th>p \land q</th>
<th>(p \lor \neg q) \rightarrow (p \land q)</th>
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Contrapositive and Proof of “If-then” Propositions

• Material implication review: \( P \rightarrow Q \) is equivalent to \( \neg P \lor Q \)
• Therefore, \( \neg Q \rightarrow \neg P \) is equivalent to \( P \rightarrow Q \)!

  - Note that \( \neg(\neg Q) = Q \)—i.e., double negation leaves the value unchanged. Then:
    \[
    \neg Q \rightarrow \neg P = \neg(\neg Q) \lor \neg P
    = Q \lor \neg P
    = \neg P \lor Q
    = P \rightarrow Q
    \]

• Thus, whenever we’re proving implication \( P \rightarrow Q \), we could equivalently prove \( \neg Q \rightarrow \neg P \)

• Vocabulary: The expression \( \neg Q \rightarrow \neg P \) is called the contrapositive of \( P \rightarrow Q \); the proof technique is proof by contraposition (or proof by contrapositive)

End Of Logical Progression: Back To Sets

• Those fundamental logical / boolean operators are part of the way we reason about many things, including sets

• (There’s an example coming up soon)
Empty Set

- The empty set, written as $\emptyset$, is the (unique) set containing no elements.
- The empty set can be described many ways, sometimes without it being obvious that it’s the empty set. E.g.,
  - $S = \{n \text{ is prime} \mid 24 \leq n \leq 28\}$
  - $G = \{$Grammy awards won by The Beach Boys, Led Zeppelin, The Who, The Doors, Queen, Guns N’ Roses, or Bob Marley during their careers (before 2014)$\}$
- Exercise: Consider a set $S$. If $\emptyset \subseteq S$, what do we know about $S$?
  - Prove your answer; it should contain a proof of an important fact about the empty set!

Disjoint Sets

- Definition: Sets $A$, $B$ are disjoint if there is no element $x$ s.t. $(x \in A \land x \in B)$
  - That is, sets $A$ and $B$ have no element in common
- This idea can be useful when talking about more than 2 sets where no two have an element in common—they’re all disjoint from each other
- Definition: A collection of sets $A_1$, $A_2$, $\ldots$, $A_n$ is pairwise disjoint iff for any $i, j \leq n (i \neq j)$, $A_i$ and $A_j$ are disjoint
Set Difference

• Another set operation: Difference
  – Intuitively, the difference between two things is what’s in the first that’s not in the second
  – For sets A, B, the difference A - B (the book writes it as A \ B) is the set of elements that are in A but not in B. More formally…
• Definition: The difference A - B of sets A and B is defined by:
  \[ x \in A - B \text{ iff } x \in A \text{ and } x \notin B \]
  – Also written as A - B = \{x \mid x \in A \text{ and } x \notin B\}
• Examples:
  – What’s \{1,2,3,4\} - \{2,4,6,8\}?
  – What’s \{x \mid x \text{ is odd between 0 and 10}\} - \{x \mid x \text{ is prime between 0 and 10}\}

Enumerating Subsets

• If you’re given a set, especially a finite set, you might consider all the subsets of that set
  – If S = \{Phil Collins, Peter Gabriel\}, what are all its subsets? How many are there?
  – If S = \{1,2,3\}, what are all its subsets? How many are there?
  – If S = \{2,4,6,8\}, what are all its subsets? How many are there?
• What do you notice about the relationship between the number of elements in a set and the number of subsets it has?
Power Sets

• Given a set $S$, the set of subsets of $S$ is called the **power set** of $S$, sometimes written $\mathcal{P}(S)$
  
  $\mathcal{P}(S) = \{A \mid A \subseteq S\}$

• Questions:
  
  – What’s the power set of $\{a, b\}$?
  
  – Let $S$ be $\{x \mid x$ is between 0 and 10 and $x$ is prime$\}$. What is the size of the power set of $S$?
  
  – What is the size of $\mathcal{P}(\emptyset)$? What is $\mathcal{P}(\emptyset)$?

Complement of a Set

• Another set operation: Complement

  – For set $A$, the complement $\overline{A}$ is the set of elements not in $A$. More formally…

  • Definition: For set $A$, its complement $\overline{A}$ (also written as $A$ with a line over it) is defined by:

    $x \in \overline{A}$ iff $x \notin A$

    – Also written as $\overline{A} = \{x \mid x \notin A\}$

  • When discussing set complement, it is (explicitly or implicitly) in the context of a *universe* $U$ of all elements to be considered

    – So, $\overline{A} = U - A$

• Examples:

  – What’s $\overline{\{1, 2, 3, 4\}}$? [Assume that the complement here is with respect to the natural numbers]
Generalized Union / Intersection

- Consider the $\sum$ summation notation for the sum of $f(i)$ as $i$ goes from 1 to $n$
  - This is a generalization of the addition operator $+$, extending it to apply to a collection of values
  - Each item in that collection is accessed by index
    - e.g., $f(1)$ for index $i=1$, $f(2)$ for index $i=2$, etc.
- The same thing can be done for union $\cup$ or intersection $\cap$

- Exercise: Assume $S_1, S_2, \ldots, S_n$ are finite. What is the size of the (generalized) union of all $S_i$ [i from 1 to n]?
  - How about if $S_1, S_2, \ldots, S_n$ are finite and pairwise disjoint?

Exercise

- Claim: For any sets A and B, $(A \cap B) \subseteq (A \cup B)$.
  - Prove it, or give a counterexample.