CS 145 – Foundations of Computer Science

Professor Eric Aaron

Lecture – W F 1:30pm
Lab – F 3:30pm

Lecture Meeting Location: SP 105
Lab Meeting Location: SP 309

Business

• HW6 due already
• HW7 out Monday, Nov. 27; due Dec. 5 / Dec. 6 (see HW sheet)

• Make-up Lecture! Monday, Nov. 27, 1:30pm, in SP105

• Reading: Makinson, Ch. 6
  – We may not cover all the material in chapter 6, but it’s worth reading anyway
Counting Cards

- Number of 5-card hands with 3 of a kind and nothing more (i.e., having 3 of a kind but not 4 of a kind or a full house)?

- Number of 5-card hands with 2 pair and nothing more?

- Number of 5-card hands with a straight (i.e., in rank order)?

  Which 5-card hand is more common, a straight or a flush? Did we just figure that out, in the last two slides?

- How about the number of 8-card hands with 2 pair and one 3-of-a-kind (but no 4-of-a-kind)?

Ranking of Poker Hands

- In the game of poker, there are several kinds of hands with a pair or better… how do they rank relative to each other?
  - On what basis should one kind of poker hand be a winner over a different kind?
  - To the right, on this slide, is the standard rank ordering of poker hands.
  - Do we think the standard ordering is right? How would we confirm it?
  - How would we compute the probability for each of the kinds of hands?
Counting Full Houses

• What’s the number of 5-card hands can be dealt from a standard 52-card deck (standard 4 suits, 13 cards each; no jokers)?
  - \( \binom{52}{5} = \frac{52!}{5! \cdot 47!} = 2598960 \)

• Number of 5-card hands with a full house?
  - One way to think of it: choose rank \( r_1 \) for the triple, then choose 3 cards for it; then choose rank \( r_2 \) for the pair, then choose 2 cards for it
    - So: \( 13 \cdot \binom{4}{3} \cdot 12 \cdot \binom{4}{2} = 13 \cdot 4 \cdot 12 \cdot \frac{(4 \cdot 3)}{2!} = 3744 \)

• Probability of a 5-card hand being a full house?
  - \( \frac{\text{number of full houses}}{\text{number of 5-card hands}} = \frac{3744}{2598960} \)

What are the odds?

Some other probability-related exercises:

• Imagine that there are 10 envelopes in a mailbox, exactly 2 of which contain bills. The owner of the mailbox picks 3 envelopes to open. What is the probability that none of them is a bill?
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Some other probability-related exercises:

• Imagine that there are 10 envelopes in a mailbox, exactly 2 of which contain bills. The owner of the mailbox picks 3 envelopes to open. What is the probability that none of them is a bill?

• There are 7 index cards in a bag, each identical to the others except for the number printed on it: 4 cards have even numbers, 3 cards have odd numbers. (No two numbers are the same.) Someone then picks the cards out of the bag in random order, one at a time. What is the probability that all of the odd numbers are picked before any of the even numbers?

Probability

• To talk about probability more rigorously, some terminology / definitions:
  – We will discuss discrete probability, i.e., probability over finite domains (rather than continuous, infinite domains)
  – Probabilities are considered w.r.t. a sample space, i.e., a finite (non-empty) set S
  – Probabilities of elementary events (i.e., single elements of S) are described by functions called distributions

See Makinson, Ch.6
Probability Distributions

- Given a sample space $S$, a **probability distribution** on $S$ is a function $p: S \rightarrow [0,1]$ such that $\sum \{p(s) | s \in S\} = 1$
  - $[0,1]$ is the **real interval**, the set of all real numbers from 0 to 1, inclusive
  - The summation ($\sum \{p(s) | s \in S\} = 1$) constrains function $p$ so that the sum of the values it assigns add up to exactly 1
    - (Can think of it as the probability of *something* occurring is 1)
- **Exercise:** Let $S = \{a,b,c,d,e\}$. In each case below, is function $p$ a probability distribution?
  1. $p(a) = 0.1$, $p(b) = 0.2$, $p(c) = 0.3$, $p(d) = 0.4$, $p(e) = 0.5$
  2. $p(a) = 0.1$, $p(b) = 0.2$, $p(c) = 0.3$, $p(d) = 0.4$, $p(e) = 0$
  3. $p(a) = 0$, $p(b) = 0$, $p(c) = 0$, $p(d) = 0$, $p(e) = 1$
  4. $p(a) = -1$, $p(b) = 0$, $p(c) = 0$, $p(d) = 1$, $p(e) = 1$
  5. $p(a) = 0.2$, $p(b) = 0.2$, $p(c) = 0.2$, $p(d) = 0.2$, $p(e) = 0.2$

Probability Function

- A probability distribution assigns a probability (in $[0, 1]$) to each single element of $S$ (i.e., each **elementary event**)
- A **probability function** is an extension of a probability distribution, so it applies to all events—i.e., all subsets of $S$—not just all elements of $S$
  - Probability function $p^*: P(S) \rightarrow [0,1]$ assigns values to elements of the power set of $S$
  - $p^*(A) = \sum \{p(a) | a \in A\}$ for non-empty subsets $A$ of $S$
  - $p^*(\emptyset) = 0$
- **Terminology**
  - The pair of a sample space $S$ and a probability function $p^*$ over $S$, $(S, p^*)$, is called a **probability space**
Probability Function

• For simplicity’s sake, we might call a probability function $p$ instead of $p^+$, when it is not harmfully ambiguous to do so.

• Exercises: Let $p: \mathcal{P}(S) \rightarrow [0,1]$ be a probability function over $S$. Then, show that $p$ has the following properties:
  - $p(S) = 1$
  - $p(A \cup B) = p(A) + p(B) - p(A \cap B)$
  - when $A, B$ are disjoint, $p(A \cup B) = p(A) + p(B)$
  - $p(A \cap B) = p(A) + p(B) - p(A \cup B)$
  - $p(S - A) = 1 - p(A)$

(A, B are arbitrarily chosen subsets of S)