Exercise 1: Reading propositions

Consider the following two statements:

\( p: \) It is Friday.
\( q: \) I go to lab.

Express the following in plain English.

a. \( p \Rightarrow q \)

b. \( \neg p \Rightarrow \neg q \)

c. \( p \lor \neg q \)

d. \( p \land q \)
Exercise 2: Set theory and propositional logic

Imagine we have two sets, $A$ and $B$, and some object, $x$. We can introduce two propositional variables:

$a$, which states that $x \in A$, and

$b$, which states that $x \in B$.

By combining $a$ and $b$ in different ways using logical connectives, we can express different claims about how $x$ relates to $A$ and $B$.

a. Write a statement in propositional logic using the variables $a$ and $b$ that says $x \in A \cap B$.

b. Write a statement in propositional logic using the variables $a$ and $b$ that says $x \in A \cup B$.

c. Write a statement in propositional logic using the variables $a$ and $b$ that says $x \in A - B$.

d. Write a statement in propositional logic that says $x \in A \Delta B$. Your solution should contain no more than two connectives (including any repetitions).

Recall that $\Delta$ is the symmetric difference of two sets, defined in class.
Exercise 3: Where there's smoke

Consider the following two statements:

\[ s: \text{There is smoke.} \]
\[ f: \text{There is fire.} \]

Decide whether each of the following is valid (i.e., a tautology), unsatisfiable (i.e., a contradiction), or neither (i.e., contingent), and describe why.

a. \( s \Rightarrow s \)

b. \( s \Rightarrow f \)

c. \( s \land \neg s \)

d. \( s \lor f \lor \neg f \)
Exercise 4: Magical resolution

Consider the following statements:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

a. Use the provided propositional variables to rewrite the statements above as propositions. You can take “immortal” to be equivalent to “not mortal”.

b. Convert the propositions into conjunctive normal form (CNF) and split into terms (the inner disjunctions of variables or their negations). You should have six terms.

c. Use resolution to prove g is true, i.e., the unicorn is magical.