Exercise 1

We proved in class that if \( f : A \to B \) and \( g : B \to C \) are both injective functions, then their composition \( g \circ f : A \to C \) must also be injective. However, if \( g \) is not injective, then \( g \circ f \) might not be injective either, even if \( f \) is. Give an example that demonstrates this possibility.

Exercise 2

A function \( f : A \to A \) is called an involution if \( f(f(x)) = x \) for all \( x \in A \) (that is, \( f \) is its own inverse).

a. Find three examples of involutions from \( \mathbb{Z} \) to \( \mathbb{Z} \). Briefly justify your answers.

b. Prove that if \( f \) is an involution, then \( f \) is a bijection.