Assignment 4

Due October 7, 11:59 p.m.

Problem 1

As in Assignment 3, a relation $R$ over a set $A$ is called Euclidean iff whenever $(a, b)$ and $(a, c)$ are in $R$, then so is $(b, c)$.

a. What does the Euclidean property say when $a = c$?

b. Give an example of a relation over the set $\{1, 2, 3\}$ that is Euclidean but not transitive.

c. Prove that if a relation $R$ over some set $A$ is both Euclidean and reflexive, then it must be symmetric. (Hint: See (a).)

Problem 2

If $f : A \rightarrow B$ and $g : B \rightarrow C$ are both injective functions, then their composition, $g \circ f : A \rightarrow C$ must also be injective. However, if $g$ is not injective, then $g \circ f$ might not be injective either – even if $f$ is. Give an example that demonstrates this possibility. (Use a small set $A$.)