One of the best ways to improve your proof writing is to do some proof reading. In doing so, you’ll get a better perspective on why certain stylistic conventions make proofs easier to understand – and why not following them can make proofs harder to read.

You’ll also get practice tracing through mathematical arguments, which will help expand your repertoire of techniques and give you a better sense of what details to focus on in your reasoning.

For each of the following proofs,

a. **Critique its style.** Go through the provided Guide to Proofs. Identify where the proof violates these guidelines.

b. **Critique its reasoning.** Are the arguments given by these proofs correct? If there are reasoning errors, point them out and demonstrate why they’re incorrect.

c. **Correct the proof.** Using your list of issues, do one of the following:

- If the reasoning is correct but the proof has poor style, simply rewrite the proof to improve its style, keeping the core argument intact.
- If the reasoning is incorrect but the statement being proved is still true, write a new proof of the theorem. Try to modify the original argument as little as possible.
- If the reasoning is incorrect and the statement being proved isn't even true to begin with, briefly explain why the statement isn't true, though no formal disproof is required.

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\[^1\text{Any resemblance to an actual party is purely coincidental. Sorry.}\]
Proof 1

**THEOREM**  The sum of an even integer and an odd integer is odd.

**PROOF**  This proof will talk about adding together different kinds of numbers. An even integer is an integer that can be written as $2k$ for some integer $k$. Therefore, $m = 2k$. Similarly, an odd integer is one that can be written as $2k + 1$ for some integer $k$. So $n = 2k + 1$. $m + n = 2k + 2k + 1 = 4k + 1$. Therefore $m + n$ is odd. ■
Proof 2

**Theorem** Every natural number is odd.

**Proof** Assume for the sake of contradiction that every natural number is even. In particular, that would mean that 137 is even. Since $137 = 2 \cdot 68 + 1$ and 68 is a natural number, we see that 137 is odd. We know that there is no integer $n$ where $n$ is both odd and even. However, $n = 137$ is both even and odd. This is impossible. We've reached a contradiction, so our assumption must have been wrong. Therefore, every natural number is odd. ■
Proof 3

**Theorem**  If $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

**Proof**  Since $A \subseteq B$, it means that some group of the elements of $B$ is the set $A$. Since $A \subseteq C$, it means that some group of the elements of $C$ is the set $A$. Therefore, some group of the elements of $B \cap C$ is the set $A$, so $A \subseteq B \cap C$. $\blacksquare$
Proof 4

**Theorem**  If $A \subset B$ and $A \subset C$, then $A \subset B \cap C$.

**Proof**  Since $A \subset B$, it means that some group of the elements of $B$ is the set $A$, and there are some other elements of $B$. Since $A \subset C$, it means that some group of the elements of $C$ is the set $A$, and there are some other elements of $C$. Therefore, some group of the elements of $B \cap C$ is the set $A$, and there are some other elements of $B \cap C$, so $A \subset B \cap C$. $\blacksquare$