Introduction
"Honestly? I preferred when we didn't talk about the elephant."

Shannon Wheeler, 2011
Professor Jonathan Gordon
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Office hours TBD
Maximum class size: 27
Course website

cs.vassar.edu/~cs145
Prerequisites

CMPU 101: Problem-solving and Abstraction

There are no math prerequisites; high-school algebra is enough!
Lectures:

Monday & Thursday, 3:10–4:25 p.m.
Sanders Physics 105

Lab:

Friday, 11:00 a.m.–1:00 p.m.
Sanders Physics 309
Grading

8–10 homework assignments
Programming and written work
Grading

- Assignments: 40%
- Labs: 20%

10–12 lab exercises
Grading

Assignments: 40%
Labs: 20%
Midterm: 20%

Midterm exam around Spring Break
Grading

Regularly scheduled final exam
Basic math concepts, common in computer science:

*Set theory* – collecting things
*Relations* – comparing things
*Functions* – associating one item with another
*Recursion and induction* – recycling outputs as inputs
*Combinatorics* – counting things
*Probability* – weighing the odds

Proofs:

Is it true? How do I know?
Proof techniques and formal reasoning, using logic.

Racket/Scheme:

Practice applying the concepts
We divide these into topics with names, but they’re rarely used in isolation.

These ideas gain power when they’re used together.
There will be very little to memorize. Instead, there are general concepts to understand and apply.

To really understand the concepts, we need to work with them. We’ll do a bit of this in class, but we’ll do more of it in labs and on assignments (both programming and proofs).
Approaching proof-based mathematics
Most high-school math classes focus on calculation.

This course focuses on argumentation.

Your goal is to see why things are true, not to check that they work in a few cases.
Mental traps to avoid

“Everyone else has been doing math since before they were born, and there is no way I’ll ever be as good as them.”
“A little slope makes up for a lot of y-intercept.”

John Ousterhout
Mental traps to avoid

“A small minority of people are math geniuses and everyone else has no chance at being good at math.”
Never confuse experience for talent.
Have a growth mindset.

Matt Waite: How I faced my fears and learned to be good at math

You might think the principal coder behind PolitiFact took naturally to math. You’d be wrong.

By MATT WAITE @mattwaite Nov. 13, 2013, 10 a.m.

Somewhere in middle school, I had convinced myself that I was bad at math. It was okay: My mom was bad at math too. So were lots of people I looked up to. “Bad at math” was a thing — probably even genetic — and it was okay.

I so thoroughly convinced myself that I was bad at math that I very nearly didn’t graduate from high school. It took tutors and hours a week to squeak through an advanced algebra class my friends had all breezed through on their way to much harder classes.

Welcome to journalism, where “bad at math” isn’t just a destructive idea — it’s a badge of honor.

But it was okay. I was bad at math. They weren’t. Simple as that.
Mental traps to avoid

“Being good at math means being able to instantly solve any math problem thrown at you.”
Simple open problems

Math is often driven by seemingly simple problems that no one knows the answer to.

E.g., the integer brick problem:

Is there a rectangular brick where every line connecting two corners has an integer length?

Having open problems like this drives the field forward – it motivates people to find new techniques and discover new results.
It’s perfectly normal to get stuck or be confused when learning math.

Even seemingly simple problems can take a lot of time and thought.

It’s important to struggle – but also to know when to ask for help.
How are you going to do awesome in this class?
We’ve got a big journey ahead of us.

Let’s get started!
Introduction to set theory
“CMPU 145 students”

“Prime numbers”

“Cute animals”

“Presidents who’ve been impeached”

“The atomic elements”

“US coins”
A **set** is an unordered collection of distinct objects, which may be anything, including other sets.
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Two sets are equal when they have the same contents, ignoring order.
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Sets can’t contain duplicate elements. Any repeated elements are ignored.
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Set membership

Given a set $S$ and an object $x$, we write

\[ x \in S \]

if $x$ is contained in $S$ and

\[ x \notin S \]

otherwise.

If $x \in S$, we say that $x$ is an element of $S$.

Given any object and any set, either that object is in the set or it isn’t!
Sets can contain any number of elements.
Sets can contain any number of elements.

The "empty set" is the set with no elements.

We use this symbol to denote the empty set.
1 \ ? \ {1}

QUESTION. Are these objects equal?
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This is a number.

This is a set. It contains a number.

\[ 1 \quad ? \quad \{1\} \]
QUESTION. Are these objects equal?

This is a number.

This is a set.
It contains a number.

1 ≠ \{1\}
A set like \{1\} that contains exactly one element is sometimes called a *singleton*. 
QUESTION. Are these objects equal?

\[ \emptyset \quad ? \quad \{ \emptyset \} \]
$\emptyset \ ? \ \{\emptyset\}$

**QUESTION.** Are these objects equal?

This is the empty set

This is a set with the empty set in it.
QUESTION. Are these objects equal?

This is the empty set

This is a set with the empty set in it.
More generally no object $x$ is equal to the set containing $x$.

$x \neq \{x\}$

This is $x$ itself

This is a set with $x$ in it.
Infinite sets

Some sets contain *infinitely many* elements!

\( \mathbb{N} = \{0, 1, 2, 3, \ldots\} \) is the set of *natural numbers*.

Note: Some mathematicians don’t include 0 in \( \mathbb{N} \). If you want to make it explicit that you’re including 0, you can write \( \mathbb{N}_0 \).

\( \mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\} \) is the set of *integers*.

The “Z” comes from the German *ganze Zahlen*, “whole number”.

\( \mathbb{R} \) is the set of real numbers.

\( e \in \mathbb{R}, \pi \in \mathbb{R}, 4 \in \mathbb{R}, \text{ etc.} \)
Here are some English descriptions of infinite sets:

“The set of all even natural numbers”

“The set of all real numbers less than 137”

“The set of all negative integers”

To describe complex sets like these mathematically, we’ll use *set-builder notation*. 
Even natural numbers

\{ n \mid n \in \mathbb{N} \text{ and } n \text{ is even} \}
Even natural numbers

\[ \{ n \mid n \in \mathbb{N} \text{ and } n \text{ is even} \} \]
Even natural numbers

\{ n \mid n \in \mathbb{N} \text{ and } n \text{ is even} \}

The set of all \( n \) where
Even natural numbers

\{ n \mid n \in \mathbb{N} \text{ and } n \text{ is even} \}

The set of all \( n \) where \( n \) is a natural number
Even natural numbers

\[ \{ n \mid n \in \mathbb{N} \text{ and } n \text{ is even} \} \]

The set of all \( n \) where \( n \) is a natural number and \( n \) is even
Even natural numbers

\[ \{ n \mid n \in \mathbb{N} \text{ and } n \text{ is even} \} \]

The set of all \( n \) where \( n \) is a natural number and \( n \) is even

\{0, 2, 4, 6, 8, 10, 12, 14, 16, \ldots\}
Set-builder notation

A set may be specified in set-builder notation:

\[ \{ x \mid \text{some property } x \text{ satisfies} \} \]
\[ \{ x \in S \mid \text{some property } x \text{ satisfies} \} \]

For example:

\[ \{ n \mid n \in \mathbb{N} \text{ and } n \text{ is even} \} \]
\[ \{ C \mid C \text{ is a set of US currency} \} \]
\[ \{ r \in \mathbb{R} \mid r < 3 \} \]
\[ \{ n \in \mathbb{N} \mid n < 3 \} \]
Combining sets
Venn diagrams
Venn diagrams

A = \{1, 2, 3\}
Venn diagrams

\[ B = \{3, 4, 5\} \]
Venn diagrams

Union

$A \cup B = \{1, 2, 3, 4, 5\}$
Venn diagrams

Intersection

$A \cap B = \{3\}$
Venn diagrams

\[ A - B = \{1, 2\} \]
Venn diagrams

\[ A \setminus B = \{1, 2\} \]
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