

Assignment 4

Submissions due: 24 February, 1:30 p.m.

Corrections due: 3 March, 1:30 p.m.

Exercise 1

Let $\Sigma = \{ (,) \}$. Consider the following language over Σ :

$$L_4 = \{ w \in \Sigma^* \mid w \text{ is a string of balanced parentheses} \}$$

For example,

$$\begin{array}{ll} () \in L_4 &) (\notin L_4 \\ (()) \in L_4 & ((\notin L_4 \\ (()()) \in L_4 & (() \notin L_4 \\ ((())((())) \in L_4 & (())) \notin L_4 \\ \varepsilon \in L_4 & \end{array}$$

I realize it can be confusing to write about the symbols (and). Feel free to substitute 1 and r or a and b if you prefer.

Use the Pumping Lemma to prove that L_4 is nonregular.

Exercise 2

Suppose we want to check whether $x + y = z$, where x , y , and z are all natural numbers. We can formulate this problem as a language using the *unary number system*, where the number n is represented by writing out n 1s.[†] For example, the number 5 would be written as 11111. Then, given the alphabet $\Sigma = \{1, +, =\}$, we can encode $x + y = z$ by writing out x , y , and z in unary. For example:

3 + 4 = 7 would be encoded as 111+1111=1111111

7 + 1 = 8 would be encoded as 1111111+1=11111111

0 + 1 = 1 would be encoded as +1=1

Consider the language $ADD = \{1^m + 1^n = 1^{m+n} \mid m, n \in \mathbb{N}_0\}$. That is, ADD consists of strings encoding two unary numbers and their sum. Use the Pumping Lemma to prove that ADD is not a regular language.

[†] You can think of this like tally marks.