

CMPU 240 · Spring 2026

Assignment 7

Submissions due: 7 April, 1:30 p.m.

Corrections due: 14 April, 1:30 p.m.

Exercise 1

In class, we saw that every context-free grammar can be converted to a more restricted format called Chomsky normal form (CNF), which makes it easier for a program to check if a string can be generated by the grammar. (Having a grammar be in CNF can also make your job easier when writing a proof, since there are fewer possibilities to deal with!)

Consider the following grammar:

$$S \rightarrow aACa$$

$$A \rightarrow B \mid a$$

$$B \rightarrow C \mid c$$

$$C \rightarrow cC \mid \varepsilon$$

Why is it *not* in Chomsky normal form? Which of the productions are not allowed?

Exercise 2

Give the table produced by the CYK algorithm when run with input string `abbb` and the following CFG in Chomsky normal form:

$$S \rightarrow AT \mid AB$$

$$T \rightarrow XB$$

$$X \rightarrow AT \mid AB$$

$$A \rightarrow a$$

$$B \rightarrow b$$

Can the string be generated by this given grammar?

Exercise 3

If A and B are languages, define $A \diamond B = \{xy \mid x \in A, y \in B, \text{ and } |x| = |y|\}$. Show that if A and B are *regular* languages, then $A \diamond B$ must be a *context-free* language.

Note: This is *not* just concatenation! You don't need to give a complete proof, but it needs to be convincing.

Exercise 4

How hard is it to search a string for a substring?

Given a string to search for (called the *pattern*) and a string to search in (the *text*), we want to determine whether that pattern appears in the text. To encode this as a language problem, we let $\Sigma = \{a, b, ?\}$ and encode questions of the form “does pattern string s appear in text t ?” as the string $s?t$, e.g.,

“Does abba appear in bbbabba?” would be encoded as abba?bbbabba
 “Does bb appear in aaab?” would be encoded as bb?aaab
 “Does ε appear in bbaa?” would be encoded as ?bbaa

Let $SEARCH = \{s?t \mid s, t \in \{a, b\}^* \text{ and } s \text{ is a substring of } t\}$. Use the Pumping Lemma for Context-free Languages to prove that $SEARCH$ is not context-free.

This should be a formal proof. See the example from class and in the [handout](#) on the Pumping Lemma for CFLS.

Hint: The pattern and the text string you choose must use both as and bs. If we only used a, the language would be context-free.