

CMPU 240 · Spring 2026

Assignment 8

Submissions due: 21 April, 1:30 p.m.

Corrections due: 23 April, 1:30 p.m.

Exercise 1

Let $\Sigma = \{a, b\}$. The language $L_1 = \{a^m b^n \mid n = 2m\}$ is context-free, as it can be generated by the grammar

$$S \rightarrow aSbb \mid \varepsilon$$

which adds two bs to the end every time it adds an a to the beginning. Design a Turing machine to recognize L_1 using the same recursive intuition, but rather than generating larger and larger strings every time the rule is applied, the TM should make the input smaller and smaller by erasing characters.

Your solution for this exercise should be submitted as a csv file generated by `tock`, using the provided Colab notebook. Be aware that `tock` does not allow a Turing machine to move left past the beginning of the input, but that shouldn't be necessary for this exercise.

Exercise 2

We've shown that the language $L_2 = \{a^n b^n c^n \mid n \in \mathbb{N}_0\}$ is not context-free and therefore cannot be recognized by a pushdown automaton (nor by a finite automaton). However, it *can* be recognized by a Turing machine!

- a. Give a high-level description of a Turing machine whose language is L_2 . For example, you might write,

The Turing machine repeatedly ... then moves all the way to the right and ... If ..., it accepts the input.

- b. Design the state-transition diagram for a Turing machine for L_2 that works in the way you described in part a.

Your solution for part (b) should be submitted as a csv file generated by tock, using the provided Colab notebook. Be aware that tock does not allow a Turing machine to move left past the beginning of the input, but that shouldn't be necessary for this exercise.

- c. Prove that regardless of what language L is, there is a simple Turing machine M that satisfies properties 2 and 3, without even considering its input.
- d. Suppose L is a language over Σ for which there is a Turing machine M that satisfies properties 1, 2, and 3. Does M 's behavior correspond to what you think it means to solve a problem? Why or why not?