Due October 2, 9:00 a.m.

**Problem 1**

Use the Pumping Lemma to show that the following languages are not regular.

a. \( L = \{ w \in \{a, b\}^* \mid \text{no prefix of } w \text{ has more } bs \text{ than } as \} \)

b. \( L = \{ a^lb^mc^l \mid l, m \geq 0 \} \)

**Problem 2**

Show that the following languages are not regular, using closure properties and, where appropriate, the Pumping Lemma.

a. \( L = \{ a^lb^nc^m \mid l, m, n \geq 0 \text{ and } l + m \neq n \} \)

b. \( L = \{ w \mid n_a(w) \neq n_b(w) \} \), where \( n_a(w) \) is the number of \( as \) in string \( w \) and likewise for \( n_b(w) \) and \( bs \).

**Problem 3**

Indicate whether each of the following languages is regular or not, and prove your answer. You can use the Pumping Lemma or closure properties to prove a language is non-regular. To prove a language is regular, you can use closure properties, give a regular expression for the language, or construct a DFA that accepts it.

a. \( L = \{ uv \mid u, v \in L^R \} \), where \( L \) is regular.

b. \( L = \{uvw^RV \mid u, v, w \in \{a, b\}^+ \} \)

**Problem 4**

Indicate whether each statement below is true or false and justify your answer, e.g., by giving a counterexample.

a. If \( L_1, L_2, L_3, \ldots \) are all regular, then the language \( \bigcup_{i=1}^{\infty} L_i \) is also regular.

b. If \( L_1 \) is regular and \( L_2 \) is not regular, then \( L_1 \cup L_2 \) is not regular.
Optional: Bonus Problem for Extra Credit

Consider the regular expression

\[(a(a \cup b)^* b \cup a)^* b^*\]

Give the minimum-state automaton recognizing the language described by this expression. For full credit, first construct an NFA-\(\epsilon\) recognizing the language, then a DFA, then minimize the DFA using the minimization algorithm. Show the steps in the minimization, i.e., the results of the rounds in which you discover new, distinguishable pairs of states.