What problems are beyond our capacity to solve? In this last assignment, you’ll explore the limits of computation.

**Problem 1**

Let $L$ be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1 \rangle, \langle M_2 \rangle, \ldots \}$, where every $M_i$ is a decider. Use diagonalization to prove that some decidable language $D$ is not decided by any decider $M_i$ whose description appears in $L$.

**Problem 2**

Determine which one of the following languages is decidable and which one is undecidable, where $M$ is a Turing machine. Briefly justify your answer, either sketching how a Turing machine could decide the language or how you could solve a known undecidable problem if a decider existed for the language.

(a) $\{\langle M, a \rangle \mid M$ writes the character $a$ at some point when started on the empty tape$\}$

(b) $\{\langle M \rangle \mid M$ writes a non-blank character when started on the empty tape$\}$

**Problem 3**

Consider the language $A_{\varepsilon TM} = \{\langle M \rangle \mid M$ is a TM that accepts $\varepsilon\}$. Use proof by reduction to show that $A_{\varepsilon TM}$ is undecidable.

**Problem 4**

Let $M_1$ and $M_2$ be two arbitrary Turing machines. Use proof by reduction to show that the problem $L(M_1) \subseteq L(M_2)$ is undecidable.

*Hint: Consider the empty set.*

**Problem 5**

For each of the following languages, explain whether or not Rice’s Theorem applies to those languages.

For those languages for which Rice’s Theorem does not apply, state whether the language is decidable or undecidable and give a one-sentence justification for your answer.

(a) $L = \{\langle M \rangle \mid M$ accepts at least one even-length string$\}$
b. $L = \{\langle M \rangle \mid M$ never prints the tape symbol under the tape head on a transition\} (i.e., it never has a transition where it writes the same symbol it read)

c. $L = \{\langle M \rangle \mid M$ rejects all descriptions of Turing machines\}